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Abstract—

For each picture a signature is generated by operating on it with different masks. The operations a are generalization to grey level images of "shrink" and "expand" of binary pictures using Serra's morphological methods [5]. The signatures are sets of numbers, and can be used to analyze and discriminate textures. It is shown that this family of operators includes as private cases several currently used texture descriptors. A metric between matrices is also proposed that improved classification over ordinary distance.

1. INTRODUCTION

Serra defines dilation, erosion, opening and closing of binary pictures and uses these operators to measure morphological properties. These operations will be generalized to grey level pictures and used to analyze and classify textures.

1.1. BINARY DEFINITIONS.

Let P be a binary picture, the 1's represent the object, the o's the background. If $a = (a_x, a_y)$ are the coordinates of a pixel then P_a is P translated by a, i.e. $P_a(x - a_x, y - a_y) = P(x, y)$.

Dilation : Let A, B be binary pictures. The dilation of A by B, D_{AB}, is defined as a binary pictures s.t. $D_{AB}(a) = 1$ iff B_a and A have a non empty intersection of their respective objects. Or if $f_p(a)$ is the characteristic predicate of P, being true iff a is in the object of P then $f_{D_{AB}}(a) = \bigvee_b f_A(b) \wedge f_B(b - a)$ (similar to convolution). For example if $A = \begin{smallmatrix}1\\1\end{smallmatrix}$ and $B = 1\ 1$ then $D_{AB} = \begin{smallmatrix}1 & 1\\1 & 1\end{smallmatrix}$.

Erosion : The erosion of A by B, E_{AB}, is defined as the binary picture in which $E_{AB}(a) = 1$ iff the object of B_a is totally included in the object of A. Erosion is related to dilation through the following formulas

$$\overline{E_{AB}} = D_{\overline{A}B} \ ; \ \overline{D_{AB}} = E_{\overline{A}B}.$$

$\overline{X}$ being complement (object and background changing places). So we get $f_{E_{AB}}(a) = \bigwedge_b f_A(b) \vee \neg f_B(b - a)$ where f the characteristic predicate.

For example, if $A = 1111$, $B = 111$ then $E_{AB} = 11$. Dilation is used for enlarging and smoothing the objects. For example with simple B's we can compute the convex hull of A using dilation [5]. Erosion is used for enlarging and smoothing the background.

Opening and closing are two ways to compute "smooth" approximations of A. The closing of A by B, C_{AB}, is defined as $C_{AB} = E_{D_{AB}B}$. A is dilated with B and the result is eroded by B. The opening of A by B is defined by $O_{AB} = D_{E_{AB}B}$. A is eroded by B and the result is dilated by B.

Opening and closing are related through

$$\overline{O_{AB}} = C_{\overline{A}B} \ , \ \overline{C_{AB}} = O_{\overline{A}B}.$$

We also have:

$$E_{AB} \subseteq C_{AB} \subseteq A \subseteq O_{AB} \subseteq D_{AB}$$

$X \subseteq Y$ meaning $X(l) \leq Y(l)$ for all pixels l. In many cases all these inclusions are strict, but if A is convex and B convex and open with respect to A [5], then $C_{AB} = A = O_{AB}$.

2. GREY LEVEL GENERALIZATIONS

To generalize the operators from binary functions to the grey level pictures, we give new meaning to dilation and erosion using fuzzy logic. In the binary case dilation is defined by $f_{D_{AB}}(a) = \bigvee_b f_A(b) \wedge f_B(b - a)$, f being the characteristic predicate. In fuzzy logic V is sup and $\wedge$ is min: So we will define D_{AB}, where A, B are grey level pictures with values between 0 and 1 as

$$D_{AB}(a) = \sup_b \{min(A(b), B(b-a))\}.$$

In the case that A and B are binary, this new definition is equivalent to the original definition. In the case that A is grey-level and B binary, then $D_{AB}(a)$ is the maximum of all the values of A that coincide with the 1's of B_a. We will mostly talk about the case where A is grey-level and B binary.

Similarly we define E_{AB} where A, B are grey level pictures as

$$E_{AB}(a) = \inf_b \{\max(A(b), 1 - B(b - a))\}.$$

In the case that both A and B are binary this definition is the same as the original. In the case that A is a grey level function and B is binary, $E_{AB}(a)$ is the minimum of all the values of the pixels of A that coincide with the 1's of B_a.

Using the new definitions for dilation and erosion, opening and closing are defined in the same way as the binary case: $O_{AB} = D_{E_{AB}B}$ and $C_{AB} = E_{D_{AB}B}$.

Peleg et al[4] used these inf, sup (min,max) operators with B being a digital ball of radius 1, generalizing shrink and expand to the grey level case.

3. GENERATION OF DESCRIPTORS

In this section we develop families of operators on a picture (or part of a picture) that generate matrices used as descriptors.

Let P be a grey level picture. Define $\sigma(P)$ as the sum of the grey level values of the pixels in P. Let $\{M_{r,\omega}\}$ be a set of masks with parameters $r \in R$, $\omega \in \Omega$. We will take, for example, $M_{r,\omega}$ being a straight line segment of length r and angle ω ($r \geq 0$, $0 \leq \omega < \pi$). For example, $M_{3,\frac{\pi}{4}} = \begin{matrix} & & 1 \\ & 1 & \\ 1 & & \end{matrix}$ and $M_{2,0} = 11$. Using the set of masks $\{M_{r,\omega}\}$ we define four matrices for a picture P.

(1) The dilation matrix $D = (\sigma(D_{P M_{r,\omega}}))$. The r , ω entry of the matrix being the sum of the pixels in the picture generated by the dilation of P by $M_{r,\omega}$

(2) The erosion matrix $E = (\sigma(E_{P,M_{r,\omega}}))$

(3) The closing matrix $C = (\sigma(C_{P M_{r,\omega}}))$

(4) The opening matrix $O = (\sigma(O_{P M_{r,\omega}}))$

Other choices for $M_{r,\omega}$ are possible. An example is a binary picture of a rectangle (including its interior) whose sides are parallel to the axis and whose diagonal is of length r and angle ω . For example,

$$M_{3,\frac{\pi}{4}} = \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \text{ and } M_{4,arctg\frac{1}{3}} = \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{matrix}.$$

Given a picture P, for any type of masks, $M_{r,\omega}$ and sets R and Ω of length and angles, the four matrices can be generated. Every matrix measures some property of P. The use of these matrices will be described in the following section.

4. SIGNATURES

We can use one or more of the erosion, dilation, opening and closing matrices in order to define a signature for a picture. Features could be;

(1) The matrix itself.

(2) Sums of rows: When $M_{r,\omega}$ are line segments this corresponds to the overall effect of all operators of equal length (in rectangles: equal diameter).

(3) Sums of columns: When $M_{r,\omega}$ are line segments this corresponds to the overall effect of all operators of the same orientation (in rectangles: similar rectangles).

Measuring the different responses to different masks demonstrates the effect of small changes in mask size. Derivatives could be used to find critical size masks.

(4) Derivatives by rows $(\,)_{r,\omega}-(\,)_{r,\omega-1}$.

(5) Derivatives by columns $(\,)_{r,\omega}-(\,)_{r-1,\omega}$.

In order to compare two pictures we compare some of their respective signatures. This can be done by summing up all the absolute differences of their corresponding feature matrices.

However, better results (but using more computation) can be obtained with generalizations of the methods suggested in [8].

Let L be a set of labels with a metric μ. Let $f_i{:}L \Longrightarrow N$ be functions from L to the natural numbers such that $\sum_{l \in L} f_i(l) = M_i$ for each i. We define a metric ρ on the f_i as follows. Define UF(f_i), the unfolding of f_i, as the multiset of elements of L each l appearing $f_i(l)$ times (The multiset UF(f_i)has M_i elements). A matching of UF(f_i) and UF(f_j) is a 1-1 pairing of elements m $\in$ UF(f_i) and n $\in$ UF(f_j). In order to allow matching between any two multisets even when they do not have equal number of elements, the smaller multiset is padded by a default element. The cost of a matching is the sum of the costs over all pairs in the matching, μ(m,n). The metric ρ will be defined as $\rho(f_i,f_j)$ is the cost of the minimal matching of UF(f_i) and UF(f_j). It can be proved [6] that this defines a metric.

The pairwise metric μ can be defined in several ways. When L is the set of indices of a matrix we can take μ to be the city block distance between the indices, d_4. If L is a set of vectors, we can take μ to be the absolute value of the difference vector. If L is a set of angles, μ can be the the absolute value of the difference between the angles.

We will compute, for example, the distance between two 3x3 matrices,

$$f_1 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } f_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

then UF(f_1) = {(1,1)(1,1)(1,3)} and UF(f_2) = {(2,1)(2,2)(3,1)}, with μ being d_4, then the minimal pairing is [(1,1),(2,1)], [(1,1),(2,2)], [(1,3),(2,2)] and $\rho(f_1,f_2)$ = μ[(1,1),(2,1)] + μ[(1,1),(2,2)] + μ[(1,3),(2,2)] = 1+2+2 = 5 .

5. RELATION TO OTHER METHODS

Several known operators are instances of the general approach of grey level erosion and dilation described in this paper.

5.1. *Run Lengths*

A grey level run length primitive is a maximal collinear connected set of pixels all having the same grey level. A grey level run length primitive can be characterized by its length, its angle and its grey level. Galloway [1,2] used these primitives to categorize textures.

If we take for $M_{r,\omega}$ line segments of length r and angle ω and P a binary picture, then the second derivatives by columns of $E(M_{r,\omega})$ are exactly the run lengths of the ones in P. Each maximal run length of k 1's adds 2 to the k-1's 3 to the k-2's and so on. After the first derivative, all are equal to 1, and after the second, only the k length stays.

5.2. *Autocorrelation*

The autocorrelation function is commonly used in texture analysis utilizing the linear dependence of the pixels of a picture [2,7]. The autocorrelation of P is $A(r,\omega) = \sum_a P(a)P_{r,\omega}(a)$, where $P_{r,\omega}$ is P transalated by $r\cos\omega, r\sin\omega$.

The autocorrelation of a binary picture P will be the erosion matrix of P by $M_{r,\omega}$ $E,M_{r,\omega}$ being the endpoints of line segments of length r and angle ω. (Multiplying two binary pictures is the same as taking min.)

5.3. *Fractal Analysis*

In order to classify textures, Peleg et al[3] defined the upper and the lower fractal dimensions of a picture by looking at the grey levels as elevation What they actually computed are the derivatives of the dilation matrix D (and the erosion matrix E) of P by squares of different sizes: $M_{r,\omega}$ being rectangles with diagonals of length r and angle $\frac{\pi}{4}$.

6. COMPUTATION

In order to compute $(\sigma(D_{M_{r,\omega}}))$ $r \in R, \omega \in \Omega$ we do not always have to transform the picture separately for each mask and then sum up the pixel values. We will give two different methods to speed up computation in the case of $M_{r,\omega}$ being line segments of different lengths but the same angle (similar things can be done with rectangles [6]).

If x is a pixel then $D_{P\,M_{r,\omega}}(\mathrm{x}) = \max(D_{P\,M_{r-1,\omega}}(x), x_{r+1,\omega})$ $x_{\alpha,\beta}$ being the value of the pixel at length α and angle β from x. In order to compute $D_{P\,M_{r,\omega}}(x)$ we compute with no extra effort $D_{P\,M_{i,\omega}}(x)$ $i<r$ and we can sum up their values to the proper sum of dilations. Computing $\sigma(D_{P\,M_{i,\omega}})$ for all $i \leq r$ is the same complexity as computing $\sigma(D_{P\,M_{r,\omega}})$, being of the order of $O(|P|r)$.

A different method is to precompute for each pixel the closest pixels on the same line that are bigger or equal to it in value. Knowing this we can compute for exactly how many different length line segments this pixel is the max of the segment. The precomputing takes $O(|P|\log|P|)$ so that it is worthwhile only if r is big (details in [6]).

The same can be done for erosion. Unfortunately we do not know of any methods to speed up the computations of sums of openings and closings.

7. EXPERIMENTS

In order to test our methods we experimented on eight different textures using two different pictures of each texture and using a window of 32x32 from each picture. The textures were bark, grass, sand, raffia, textile, water, wood and wool. For each window we computed a number of matrices and features and used these features in order to classify the different textures.

Using only one of the dilation, erosion, opening and closing matrices, we get between one and two misclassifications; a misclassification being when two different textures are closer together than two pictures of the same texture. But when we used the dilation matrix together with the erosion matrix or the opening matrix together with the closing matrix we get 100% classification.

The following are some significant observations made:

(a) Using the more complicated metric we consistently get better results, but not by much.

(b) If we use two matrices, it is best to use either of the pairs erosion-dilation or opening-closing, as they are complementary.

8. CONCLUSION

In this paper, we have proposed a new set of features that have been shown experimentally to be of value in texture analysis. We believe that these features measure meaningful parameters at different resolution levels and are of practical value. We have also suggested a metric over matrices which is better for our purposes than the sum of differences.

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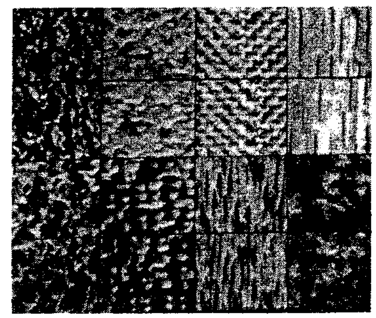

fig. 1 The pictures used in the experiments.

2	17	54	[illegible]	48	44	66	34
26	9	28	4	22	17	40	12
56	36	1	[illegible]	6	20	12	20
21	5	33	2	27	22	44	12
48	21	6	[illegible]	1	7	17	14
35	15	22	[illegible]	16	11	33	10
62	42	6	[illegible]	11	16	6	36
26	7	31	[illegible]	25	20	42	1

Table 1 Distance between erosion matrices using absolute value, smallest distance in each row is underlined (rows and columns are same textures but different pictures).

4	50	104	[illegible]	94	89	129	42
54	16	54	[illegible]	44	39	79	12
105	67	3	[illegible]	8	35	28	59
46	9	63	3	53	48	88	21
78	60	10	[illegible]	1	29	34	52
70	32	38	[illegible]	29	22	63	25
116	82	16	[illegible]	22	50	12	14
46	114	31	[illegible]	52	46	87	2

Table 2 Distance between erosion matrices using improved metric, smallest distance in each row is underlined (rows and columns are same textures but different pictures).