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NOTE

Elimination of Seams from Photomosaics*

SHMUEL PELEG

Computer Vision Laboratory, Computer Science Center, University of Maryland, College Park, Maryland 20742†

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When aerial photographs are combined into a photomosaic, seams are often apparent between the parts. The seams are caused by gray level differences due to the different conditions under which the parts were recorded. A relaxation method is described to obtain a function such that when this function is subtracted from the original image the seams are eliminated, but the details are not affected or blurred.

1. INTRODUCTION

Aerial photographs are often combined into photomosaics. Separately recorded photographs are aligned and combined to cover the entire desired region. Since the parts are recorded under different conditions, including weather, lighting, film, processing, and noise, they have different gray level characteristics. This may cause seams to be apparent between two different parts. The seams can be very noticeable, and they often interfere with the perception of the details of the picture.

When the difference between two adjacent parts is primarily a constant shift of the gray level, the gray level of one part can be changed to neutralize that shift. The case where the two parts have an overlapping area was studied by Milgram [1, 2]. Milgram suggested that histogramming the common area of the two parts can help to find the gray level shift.

The algorithm presented in this paper does not assume any simple difference such as a gray level shift, or use overlapping regions. It also allows an arbitrary number of parts to be treated, unlike Milgram's algorithm that applies to the case of two parts.

To eliminate the seams in a picture $g(x,y)$, we are looking for a *seam-eliminating function* (SEF) $s(x,y)$ such that s is everywhere constant except on the seams. Furthermore, where (x,y) and (x',y') are neighboring points on the opposite sides of a seam, $g(x,y) - s(x,y) = g(x',y') - s(x',y')$. Given such a function s, $g - s$ will be a picture without any seams, and with all its details preserved. An ideal function s as described above exists only when the differences between the parts are pure gray level shifts.

Such an ideal case is very rare, so we have to change the requirement that the SEF s is everywhere constant. Instead, we require s to be the smoothest possible function, having the minimal variance. Subtracting this SEF s from the original

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†Present address: Ben Gurion University of the Negev, Beersheva, Israel.

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picture g will result in a seamless picture, with smooth and gradual gray level changes within the parts. This gradual and smooth change will not affect the details nor blur the picture near the seams.

2. THE SEAM-ELIMINATING FUNCTION (SEF)

Two properties are desired in a seam-eliminating function s. First, when subtracting s from the original picture g, the seams between the parts should disappear. Second, the SEF s should be as smooth as possible, so the details of the picture will not be affected. To find such a function we first compute the difference conditions required at the seams, and then find the smoothest function satisfying these conditions.

The seam conditions of the SEF s are computed by requiring the final gray levels on the two sides of a seam to be identical. This means that for all pairs of points (x,y) and (x',y') that are neighboring points on the opposite sides of a seam, we should have $g(x,y) - s(x,y) = g(x',y') - s(x',y')$. From this condition we can derive the difference conditions on s:

$$s(x,y) - s(x',y') = g(x,y) - g(x',y') \tag{1}$$

for all pairs of neighboring points that are on opposite sides of a seam. We denote this difference by $d(x,y,x',y')$. For all other pairs of points d is defined to be zero.

The SEF s will be the smoothest function satisfying (1). Since (1) imposes only difference conditions on s, s is not uniquely determined. To be unique, s should be assigned some boundary values. There are several ways to select the points to be fixed and their assigned values. For every selection of fixed points a different SEF will result; thus the selection of fixed points is important.

An important special case of seam elimination is when a mosaic is composed of two parts with one seam between them. If one of the parts is known to have better gray level properties than the other, the gray levels of its points can be preserved by fixing the value of the SEF to zero over this part. Only the points of the other part will change their gray levels when the SEF is subtracted. In this case the SEF has a fixed boundary value of zero over the preferred part, and fixed values along the other side of the seam as required by the difference conditions (1). Since the fixed boundary conditions themselves satisfy the difference conditions, only the boundary conditions are needed when looking for the smoothest function s satisfying them. And since the SEF is fixed to zero over the preferred part, only points of the nonpreferred part are considered.

When two parts are involved but neither one is preferred, each should be affected by an equal change. In this case we set the fixed boundary conditions for the SEF s along the seam, and let the absolute values of s be equal for opposite neighbors. As before, if $g(x,y)$ and $g(x',y')$ are the gray levels of two neighbors on the opposite side of a seam, then the fixed boundary values will be

$$s(x,y) = \frac{g(x,y) - g(x',y')}{2}$$

and

$$s(x',y') = -s(x,y). \tag{2}$$

These values, of course, satisfy the difference conditions (1). Again, since the boundary values themselves satisfy the difference conditions, only the boundary values have to be considered when looking for a smooth function. In practice, the SEF is obtained on each part separately using the boundary values at its seam points. When the two parts have the same shape, it is possible to find the value of s on one part from the value on the other part by symmetry.

In the general case when several parts are involved, it is usually impossible to fix the values along all seams in a consistent manner, especially when a part participates in more than one seam. In this case, we cannot fix the values along seams, but only use the difference conditions $d(x,y,x',y')$. However, we still need some fixed boundary values to uniquely determine the SEF. In this general case, one part is usually chosen to have a fixed zero value for the SEF. If one part is distinguished as having best gray level properties, we fix the SEF to zero over it. Otherwise, a part is chosen that is in the center of the picture to help the algorithm converge faster.

3. RELAXATION

To find the smooth SEF described in the previous section, we employ an iterative relaxation algorithm as used in the solution of partial differential equations [3]. The initial guess for s, s^0, is zero everywhere. At every iteration we compute s^{i+1} from s^i as follows: For all points (x,y) which have fixed boundary values, $s^{i+1}(x,y) = s(x,y)$. For all other points

$$s^{i+1}(x,y) = \text{Average}\left\{s^i(x',y') + d(x,y,x',y')\right\}, \tag{3}$$

where the average is taken over all points (x',y') that are adjacent to (x,y). In our experiments we regarded the immediate four-neighbors of a point as adjacent. Note that $d(x,y,x',y')$ is zero for all pairs (x,y) and (x',y') that are not on opposite sides of a seam.

Expression (3) replaces the value of $s(x,y)$ by the average of its neighbors. When a neighbor is across a seam, the d value is nonzero and takes into account the seam difference.

Since the discrete Laplacian for a function f at a point (x,y) can be defined by

$$\nabla^2(x,y) = f(x,y-1) + f(x-1,y) + f(x+1,y) + f(x,y+1) - 4f(x,y),$$

iterative replacement of the value at a point by its neighborhood average (modulo the differences d) will result in a function having a zero Laplacian. Functions with zero Laplacian are called *harmonic*, and are very smooth. Using the d's assures that the function will have the right differences at the seams.

The convergence of this relaxation algorithm is very slow. To speed it up, we notice that although it is defined in (3) as a parallel algorithm, we execute it sequentially on a sequential machine. Using the new values obtained during the process immediately speeds up the convergence. Further speedup can be obtained by working on blocks of points. Only after values for blocks are obtained do we work on individual points for the final smoothing.

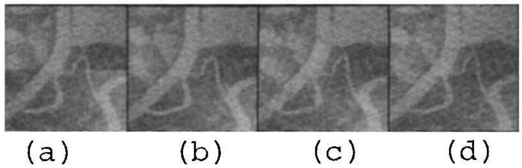

FIG. 1. Seam elimination with two parts. (a) Initial picture. (b)–(d) After 50, 100, 200 iterations, with the upper region unchanged.

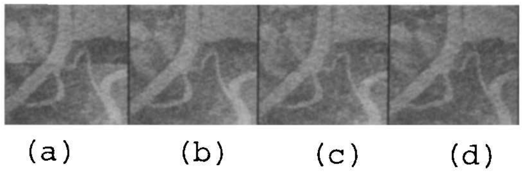

FIG. 2. Same as Fig. 1, with both regions changed equally.

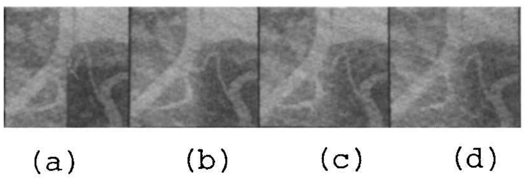

FIG. 3. Seam elimination with three parts. (a) Initial picture. (b)–(d) After 50, 100, 200 iterations with the lower left region unchanged.

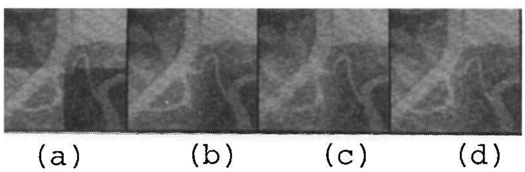

FIG. 4. Same as Fig. 3, with the upper region unchanged.

4. EXAMPLES

Figure 1a shows an original two-part mosaic. The other pictures in Fig. 1 show iterations of the seam elimination process with the upper region fixed. Figure 2 shows the same image, with the two regions changed equally to get a seamless picture. Figures 3 and 4 show a three-part mosaic. In Fig. 3 the lower left part was constant, while in Fig. 4 the upper part was constant. It is seen in all examples that the seam disappears and the resulting picture is very acceptable.

5. CONCLUDING REMARKS

The algorithm described in this paper successfully eliminates seams from photomosaics. The algorithm does not assume any specific types of gray level differences among the parts, or require overlapping areas. It is also defined for any number of parts having arbitrary shapes. One drawback of this algorithm is that two opposite points at a seam are set to the same gray level. This is desirable when the seam is not located along an edge between two regions, but will fail when it is.

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