Chapter 1

OPTICS FOR OMNISTEREO IMAGING

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Abstract

Omnistereo Panoramas use a new scene-to-image projection, the *circular projection*, that enables stereo in a full 360^o panoramic view. Circular projections are necessary since it is impossible to create two stereo panoramic images using the perspective projection.

The circular projection that can generate omnistereo panoramas can be obtained by mosaicing images taken with a rotating video camera. But the capture with a rotating camera is limited to static scenes. In this paper two optical systems are presented that perform the circular projection in optics. One system uses a spiral mirror, and the other system uses a lens.

1. INTRODUCTION

OmniStereo Panoramas [5, 4, 9, 14] use a new scene to image projection that enables simultaneously both (i) stereo and (ii) a complete panoramic view. Viewers of stereo panoramas have the ability to view in stereo all directions.

Short introductions are given in this section to panoramic imaging and to stereo imaging. Section 2. presents circular projection: the multiple viewpoint projection used to create stereo panoramas. Section 3. describes a method to obtain circular projection using a rotating cameras. Section 4. presents definitions of curves useful for the derivation of optics for stereo panoramas. Section 5. presents the derivation of a mirror that obtains circular projections in optics using differential principles. Section 6. presents the derivation of a mirror that obtains circular projections using wave propagation. Section 7. presents a lens systems that can obtain a circular projection.

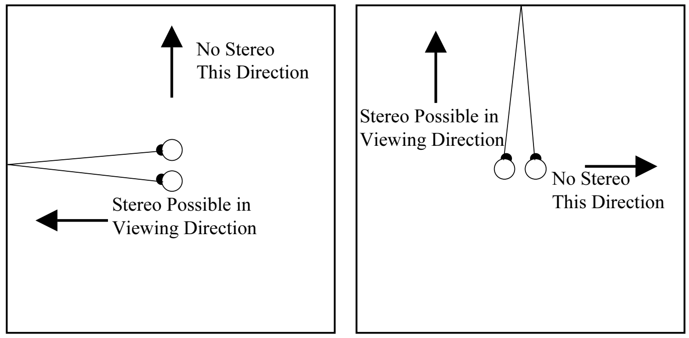

Figure 1.1 No arrangement of two single-viewpoint images can give stereo in all viewing directions. For upward viewing the two cameras should be separated horizontally, and for sideways viewing the two cameras should be separated vertically.

1.1 PANORAMIC IMAGES

A panoramic image is a wide field of view image, up to a full view of 360^o. Traditional panoramic images have a single viewpoint, also called the "center of projection" [7, 3, 15]. Panoramic images can be captured by panoramic cameras using special mirrors or lenses [8, 6], or by mosaicing a sequence of images from a rotating camera [15, 10, 11].

1.2 VISUAL STEREO

A stereo pair consists of two images of a scene from two different viewpoints. The disparity, which is the angular difference in viewing directions of each scene point between the two images, is interpreted by the brain as depth. Fig. 1.1 describes a conventional stereo setting. The disparity is a function of the point's depth and the distance between the eyes (*baseline*). Maximum disparity change, and hence maximum depth separation, is along the line in the scene whose points have equal distances from both eyes ("principal viewing direction"). No stereo depth separation exists for points along the extended baseline.

2. CIRCULAR PROJECTIONS

Regular images are created by perspective projections: scene points are projected onto the image surface along projection lines passing through a single point, called the "optical center" or the "viewpoint". Multiple viewpoint projections use different viewpoints for different viewing direction, and were used mostly for special mosaicing applications. Effects that can be created with multiple viewpoint projections and mosaicing are discussed in [16, 12].

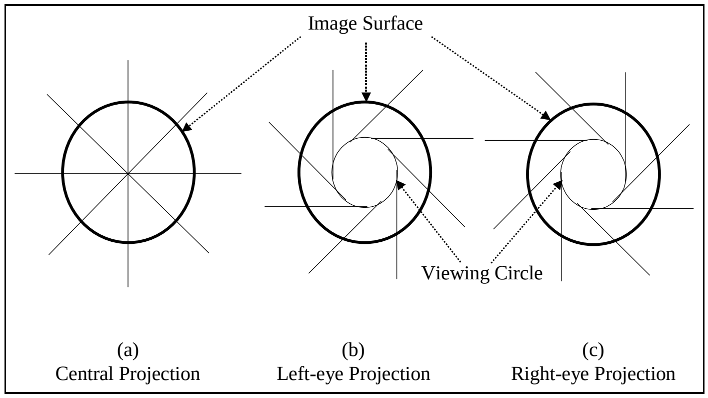

Figure 1.2 Circular projections. The projection from the scene to the image surface is done along the rays tangent to the viewing circle. (a) Projection lines perpendicular to the circular imaging surface create the traditional single-viewpoint panoramic image. (b-c) Families of projection lines tangent to the inner viewing circle form the multiple-viewpoint circular projections.

Omnistereo imaging uses a special type of multiple viewpoint projections, *circular projections*, where both the left-eye image and the right-eye image share the same cylindrical image surface. To enable stereo perception, the left viewpoint and the right viewpoint are located on a circle (the "viewing circle") inside the cylindrical image surface, as shown in Fig. 1.2. The viewing direction is defined by a line tangent to the viewing circle. The left-eye projection uses the rays on the tangent line in the clockwise direction of the circle, as in Fig. 1.2.b. The right-eye projection uses the rays in the counter clockwise direction as in Fig. 1.2.c. Every point on the viewing circle, therefore, defines both a viewpoint and a viewing direction of its own.

The applicability of circular projections to panoramic stereo is shown in Fig. 1.3. From this figure it is clear that the two viewpoints associated with all viewing directions, using the "left-eye" projection and the "right-eye" projection, have maximal stereo baseline for all directions. The vergence is also identical for all viewing directions [13], unlike regular stereo that has a preferred viewing direction.

3. OMNISTEREO MOSAICING

Representing all stereoscopic views with only two panoramic images presents a contradiction, as described in Fig. 1.1. When two ordinary panoramic images are captured from two different viewpoints, the disparity and the stereo perception will degrade as the viewing direction becomes closer to the baseline until no stereo will be apparent.

OmniStereo panoramas can be obtained by mosaicing images captured by a single rotating camera [9, 5, 14]. This is done by simulating a "slit camera" as

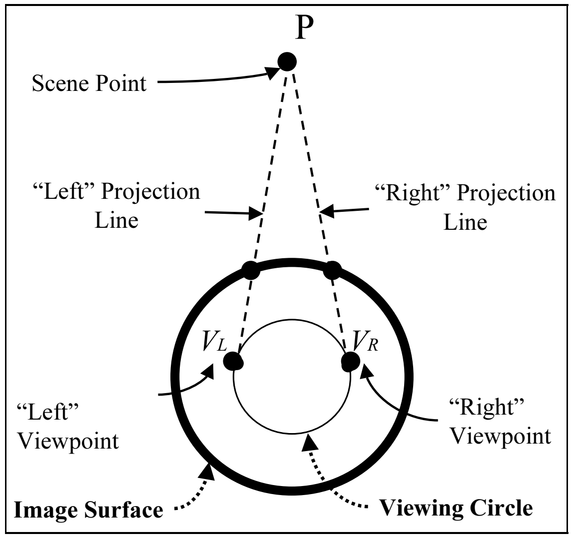

Figure 1.3 Viewing a scene point with "left-eye" and "right-eye" projections. The two viewpoints for these two projections are always in optimal positions for stereo viewing.

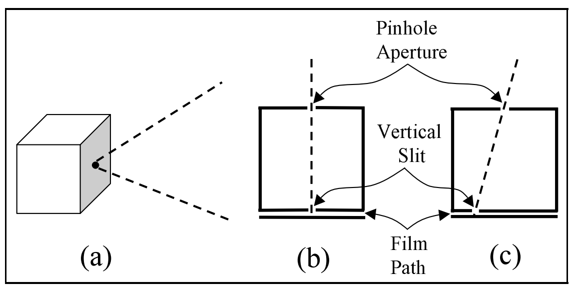

Figure 1.4 Two models of slit cameras. (a) Side view. (b-c) Top view from inside the camera. While the camera is moving, the film is also moving in the film path. The locations of the aperture and the slit are fixed in each camera. (b) A vertical slit at the center gives a viewing direction perpendicular to the image surface. (c) A vertical slit at the side gives a viewing direction tilted from the perpendicular direction.

shown in Fig 1.4. In such cameras the aperture is a regular pinhole as shown in Fig 1.4.a, but the film is covered except for a narrow vertical slit. The plane passing through the aperture and the slit determines a single viewing direction for the camera. The camera modeled in Fig 1.4.b has its slit fixed at the center, and the viewing direction is perpendicular to the image surface. The camera modeled in Fig 1.4.c has its slit fixed at the side, and the viewing direction is tilted from the perpendicular direction.

When a slit camera is rotated about a vertical axis passing through the line connecting the aperture and the slit, the resulting panoramic image has a single viewpoint (Fig 1.2.a). In particular, a single viewpoint panorama is obtained with rotations about the aperture. However, when the camera is rotated about a vertical axis directly behind the camera, and the vertical slit in not in the center, the resulting image has multiple viewpoints. The moving slit forms a cylindrical image surface. All projection lines, which are tilted from the cylindrical image surface, are tangent to some *viewing circle* on which all viewpoints are located. The slit camera in Fig. 1.4.c, for example, will generate the circular projection described in Fig. 1.2.b.

For omnistereo panoramas we use a camera having two slits: one slit on the right and one slit on the left. Both slits have the same distance from the image center. The two slits, which move together with the camera, form a single cylindrical image surface just like a single slit. The two projections obtained on this shared cylindrical image surface are exactly the circular projections shown in Fig 1.2. Therefore, the two panoramic images obtained by the two slits enable stereo perception in all directions.

Stereo panoramas can be created with video cameras in the same manner as with slit cameras, by using vertical image strips in place of the slits [9]. The video camera is rotated about an axis behind the camera as shown in Fig 1.5. The panoramic image is composed by combining together narrow strips, which together approximate the desired circular projection on a cylindrical image surface.

In manifold mosaicing [11, 10] each image contributes to the mosaic a strip taken from its center. The width of the strip is a function of the displacements between frames. Stereo mosaicing is very similar, but each image contributes **two** strips, as shown in Fig. 1.5. Two panoramas are constructed simultaneously. The left panorama is constructed from strips located at the right side of the images, giving the “left-eye” circular projection. The right panorama, likewise, is constructed from strips located at the left side of the images, giving the “right-eye” circular projection.

4. CURVES FOR OMNISTEREO OPTICS

Optics can be used for imaging omnistereo panoramas. The optical design follows the principle of caustic curves.

Definition 1 *The* **envelope** *of a set of curves is a curve C such that C is tangent to every member of the set.*

An envelope can be thought of as a way of deriving a new curve based on a set of curves. In figure Fig. 1.6 an example of an envelope is given. A Cardioid can be defined as the envelope of circles with centers on the base circle C and each of the circles of this family touching a given point P on the base circle C.

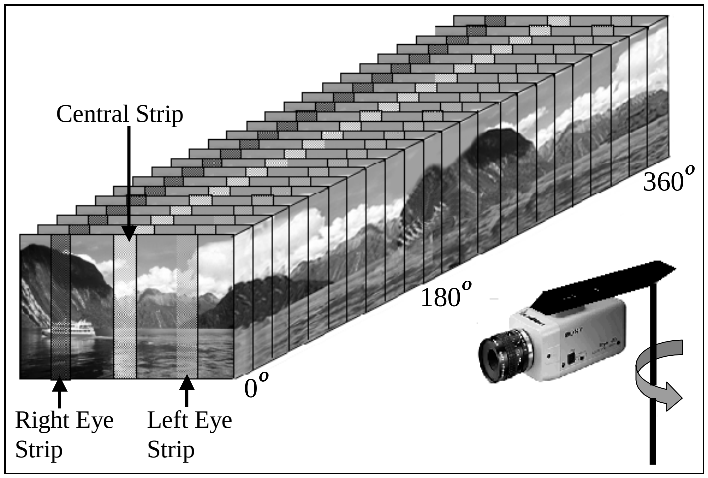

Figure 1.5 Stereo Panoramas can be created using images captured with a regular camera rotating about an axis behind it. Pasting together strips taken from each image approximates the panoramic image cylinder. When the strips are taken from the center of the images an ordinary panorama is obtained. When the strips are taken from the left side of each image, the viewing direction is tilted counter clockwise from the image surface, obtaining the right-eye panorama. When the strips are taken from the right side of each image, the left-eye panorama is obtained.

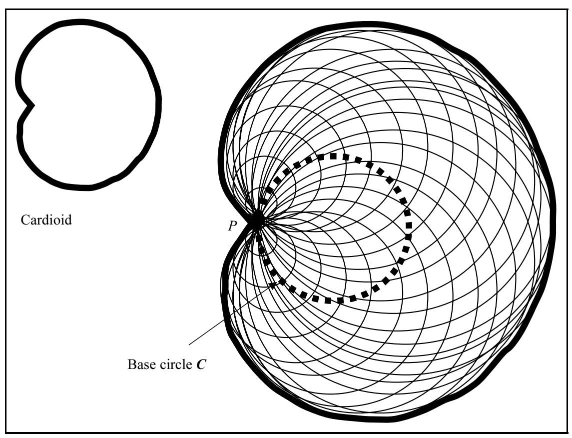

Figure 1.6 A cardioid is an envelope to the following set of circles: each circle in this set has it's center on the base circle C and is touching a given point P on this circle.

Definition 2 *An* **Involute** *of a curve is another curve orthogonal to the tangents to the given curve.*

Definition 3 *An* **Evolute** *is the envelope of the normals of a given curve.*

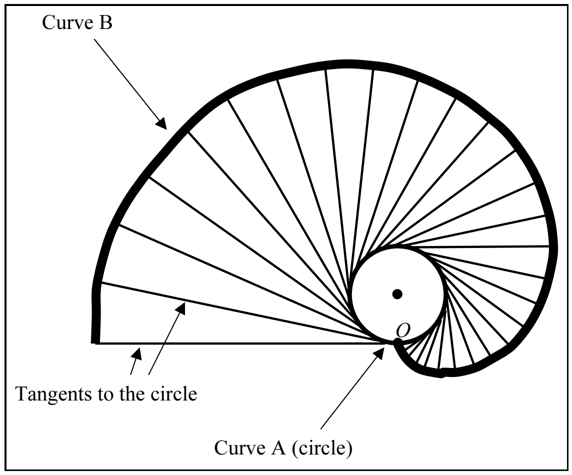

Figure 1.7 Example of Involute of a circle. We can notice (i) Curve B is an Involute to the circle (orthogonal to the tangents of the circle) (ii) Curve A (circle) is the envelope of the normals to curve B - the evolute of curve B.

We should notice that if curve A is the evolute of curve B, then curve B is an involute of curve A. An example of the above definitions can be seen in Fig. 1.7.

Definition 4 *A* **Caustic** *is the envelope of rays emanating from a point source and reflected (or refracted) by a given curve.*

A caustic curve caused by reflection is called a catacaustic, and a caustic curve caused by refraction is called a diacaustic [17].

Cardioid can also be defined as the catacaustic of a circle with light point source on the circle, as demonstrated in Fig. 1.8.

In Fig. 1.19 the catacaustic curve given the mirror and the optical center is a circle. In Fig. 1.21 and in Fig. 1.22, the diacaustic curve given the lens and the optical center is a circle.

5. SPIRAL MIRROR, I

Regular cameras are designed to have a single viewpoint ("optical center"), following the perspective projection. In this section we show how to create images having circular projections using a regular camera and a spiral shaped mirror.

The shape of the spiral mirror can be determined for a given optical center of the camera O, and a desired viewing circle V. The tangent to the mirror at every point has equal angles to the optical center and to the tangent to the circle (See Fig. 1.9). Each ray passing through the optical center will be reflected by

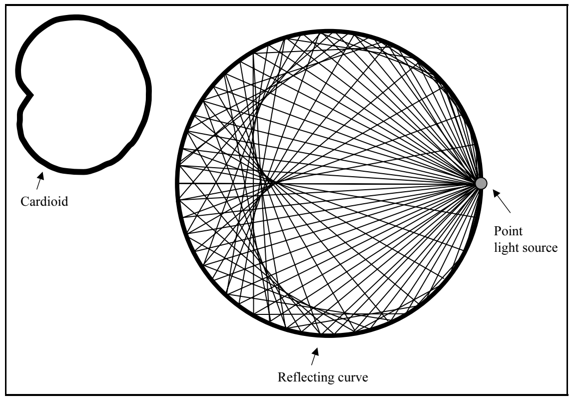

Figure 1.8 A Cardioid is the caustic of a circle with point light source on the circle. The cardioid is generated inside the circle and passing through the point light source.

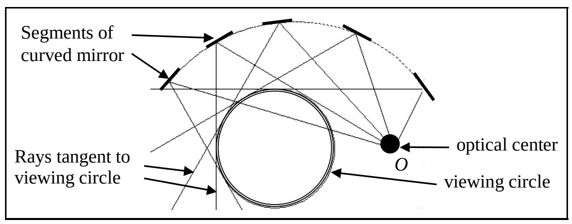

Figure 1.9 The spiral mirror: All rays passing through the optical center O will be reflected by the mirror to be tangent to the viewing circle V. This implies that rays tangent to the viewing circle will be reflected to pass through the optical center.

this mirror to be tangent to the viewing circle. This is true also in reverse: all rays tangent to the circle will be reflected to pass through the optical center. The mirror is therefore a curve whose catacaustic is a circle.

The conditions at a surface patch of the spiral shaped mirror are shown in Fig. 1.10. For simplicity the optical center is located at the center of the viewing circle of radius R, and the mirror is defined by its distance $r(\theta)$ from the optical center. A ray passing through the optical center hits the mirror at an angle α to the normal, and is reflected to be tangent to the viewing circle.

Let the radius of the viewing circle be R, and denote by $\overline{r}(\theta)$ the vector from the optical center and the mirror at direction θ (measured from the x-axis). The distance between the camera center and the mirror at direction θ will therefore be $r = r(\theta) = \mid \overline{r} \mid$. The ray conditions can be written as:

$$R = \mid \overline{r} \mid \sin(2\alpha) = \mid \overline{r} \mid 2sin(\alpha)\cos(\alpha), \qquad (1.1)$$

Where,

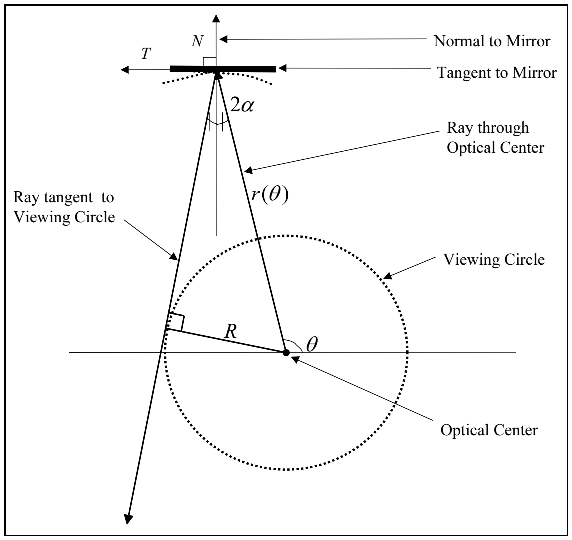

Figure 1.10 Differential conditions at a mirror patch: The optical center is at the center of the viewing circle of radius R, and the mirror is defined by its distance $r(\theta)$ from the optical center. A ray passing through the optical center hits the mirror at an angle α to the normal, and is reflected to be tangent to the viewing circle.

$$\sin(\alpha) = \frac{|N \times \overline{r}|}{|\overline{r}| \cdot |N|}$$
$$\cos(\alpha) = \frac{N^\top \overline{r}}{|\overline{r}| \cdot |N|} \tag{1.2}$$

substituting Eq. 1.2 into Eq. 1.1 we get

$$R \mid \overline{r} \mid \cdot \mid N \mid^2 = 2|N \times \overline{r}| \cdot (N^\top \overline{r}) \tag{1.3}$$

Expressing the Normal to the mirror patch (N) and the tangent (T), we get

$$\overline{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$$

$$T = \begin{pmatrix} x\prime \\ y\prime \end{pmatrix} = \begin{pmatrix} r\prime\cos\theta - r\sin\theta \\ r\prime\sin\theta + r\cos\theta \end{pmatrix}$$

$$N = \begin{pmatrix} -y\prime \\ x\prime \end{pmatrix} = \begin{pmatrix} -r\prime\sin\theta - r\cos\theta \\ r\prime\cos\theta - r\sin\theta \end{pmatrix}$$

$$|N \times \overline{r}| = \det \begin{pmatrix} -r\prime\sin\theta - r\cos\theta & r\prime\cos\theta - r\sin\theta \\ r\cos\theta & r\sin\theta \end{pmatrix} \tag{1.4}$$

$$= -rr\prime$$

$$N^{\top}\overline{r} = -r^2$$

$$|\ \overline{r}\ | = r$$

$$|\ N\ |^2 = r\prime^2 + r^2$$

Substituting the above in Eq. 1.3 we get the differential equation (for $r = r(\theta)$) in polar coordinates:

$$\begin{aligned} R(r\prime^2 + r^2)r &= 2(-rr\prime)(-r^2) \\ R(r\prime^2 + r^2) &= 2r^2 r\prime \end{aligned} \tag{1.5}$$

substituting $\rho = \rho(\theta)$ to be $\frac{r(\theta)}{R}$ we get

$$(\tfrac{\partial\rho}{\partial\theta})^2 + \rho^2 = 2\rho^2 \tfrac{\partial\rho}{\partial\theta} \tag{1.6}$$

This second degree equation in $\frac{\partial\rho}{\partial\theta}$ has two possible solutions:

$$\frac{\partial\rho}{\partial\theta} = \left\{ \begin{array}{c} \rho^2 + \rho\sqrt{\rho^2 - 1} \\ \rho^2 - \rho\sqrt{\rho^2 - 1} \end{array} \right\}. \tag{1.7}$$

The curve is obtained by integration on θ. The solution which fits our case is:

$$\theta = \rho + \sqrt{\rho^2 - 1} + \arctan(\frac{1}{\sqrt{\rho^2 - 1}}) \tag{1.8}$$

With the constraint that $\rho > 1$.

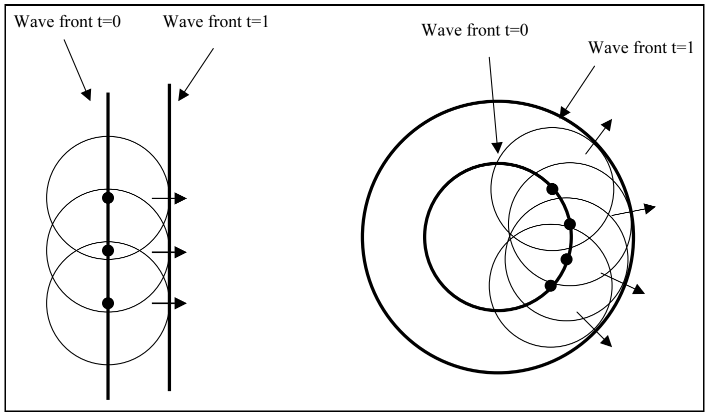

Figure 1.11 Wave propagation in the plane: At any instant of time every point of the wavefront is the origin of a secondary wave. On the left straight wavefront and on the right circle wave front. The wavefront is orthogonal the all the rays of light.

6. SPIRAL MIRROR II

The spiral mirror can also be represented by a parametric equation. We can derive this equation using analytic geometry and the theory of wave propagation. We are looking for the equation of a curve (mirror) where given a point light source (the optical center), the light rays of the source will be reflected such that the envelope of the system of reflected rays (the catacaustic) is a circle - the viewing circle.

Wave propagation (Huygens) : Huygens represents light rays propagation as a wave front. This means that at any instant every point on the wave front is the origin of a secondary wave (wavelet) which propagates outwards as a spherical wave. Similarly we can define a wavefront as an orthogonal trajectory to a system of light rays (these orthogonal trajectories always exist in the plane). Two simple examples of this wave propagation are shown in Fig. 1.11

We will use a little known principle that was first published by the Belgian mathematician Timmermans in the 1830's and was brought to our knowledge by Eisso Atzema (University of Maine).

Definition 5 Lemma (Timmermans' Principle) *Let W_{in} be an incoming wavefront and R the reflecting curve. Now construct a family of circles with centers on R and tangent to W_{in}. Then the envelope of this family of circles will be degenerate with W_{in} being one part and the other part of the envelope is the outgoing wavefront W_{out}.*

A simple example of this principle is demonstrated in figure Fig. 1.12. We will use Timmerman's principle in reverse: knowing the incoming wavefront and the outgoing wavefront we can construct the reflecting surface (the mirror).

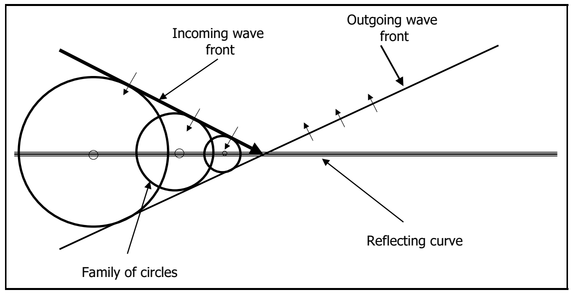

Figure 1.12 Demonstration of Timmerman's principle for straight mirror surface. The outgoing wavefront can be constructed using the envelope of circle with centers on the mirror and touching the incoming wavefront.

The incoming wavefront W_{in} will be at the optical center. Let the outgoing wavefront W_{out}, be the involute of the viewing circle. According to the definition of an involute, W_{out} is now orthogonal to the the tangents to the viewing circle. This means that the rays represented by the wavefront W_{out} are all tangent to the viewing circle as shown in Fig. 1.13. To construct a mirror whose catacaustic is the viewing circle we need a mirror that will have W_{in} and W_{out} as wavefronts. Fig. 1.14 shows a family of circles tangent to W_{in} and W_{out}. The mirror is the curve passing through the centers of these circles, as shown in Fig. 1.15and Fig. 1.16.

To compute the shape of the mirror we therefore need to construct the involute to the viewing circle, and find the positions of the centers of circles passing through the optical center and tangent to the involute.

In order to construct the involute, we rotate a point X along the circle starting from point a, as in Fig. 1.17. For every point X on the circle we compute the tangent to the circle, and place a point S on the tangent such that $\overline{XS}$ equals the distance $\overline{aX}$ along the circle. The points $\{S\}$ are on the involute. A point on the reflecting curve (mirror) is computed by finding the center Q of the circle whose diameter is $\overline{XS}$ and is passing through the optical center P. This center Q can be found as the point of intersection of $\overline{XS}$ and the perpendicular bisector of $\overline{PS}$, as described in Fig. 1.17. This is true since Q is the center of this circle, therefore $\overline{QS}$=$\overline{QP}$. As X moves along the circle, Q will trace out the reflecting curve.

Let us define S and P for a circle with radius R that is assumed to have the origin for its center. Assuming that $X = (R/cos(t), R/sin(t))$, we find

$$S = \begin{pmatrix} r\cos t + rt\sin t \\ r\sin t - rt\cos t \end{pmatrix}$$

$$P = (p_1, p_2)$$

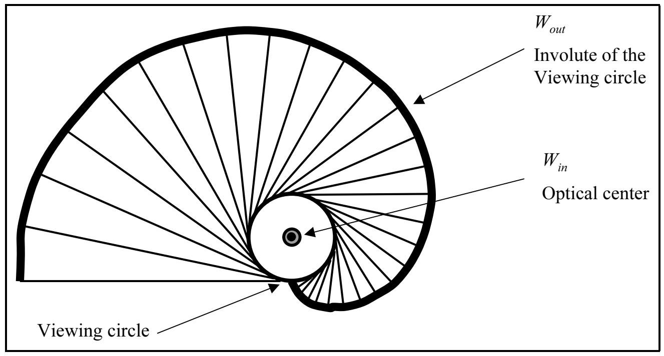

Figure 1.13 Constructing The reflecting curve. The incoming wavefront W_{in} is the optical center, the outgoing wavefront W_{out} is the involute of the viewing circle.

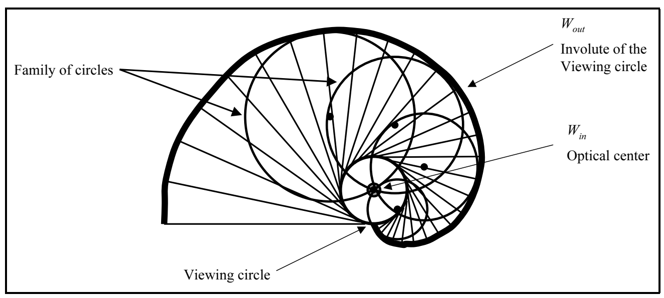

Figure 1.14 Constructing The reflecting curve. Build a family of circles all passing through W_{in} and tangent to W_{out}.

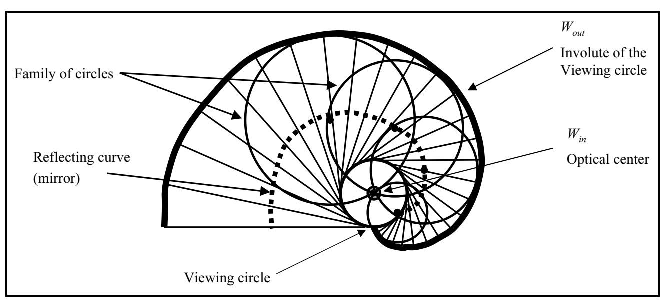

Figure 1.15 Constructing The reflecting curve. The reflecting curve (mirror) R passes through the centers of this family of circles.

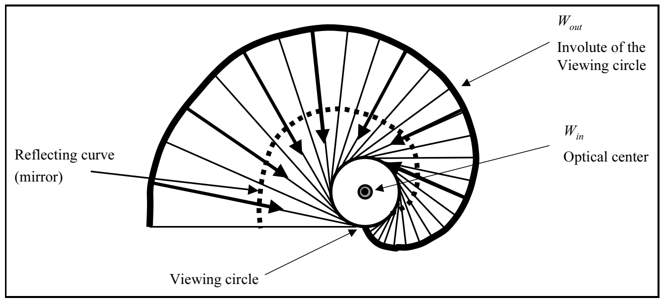

Figure 1.16 Constructing The reflecting curve. The outgoing wavefront of R is W_{out}. Therefore the normals to the wavefront are the rays coming from W_{in} and reflected by R.The envelope of the normals of W_{out} is by definition the evolute of W_{out}. Since W_{out} was constructed as the involute to the viewing circle, it's evolute is the viewing circle.

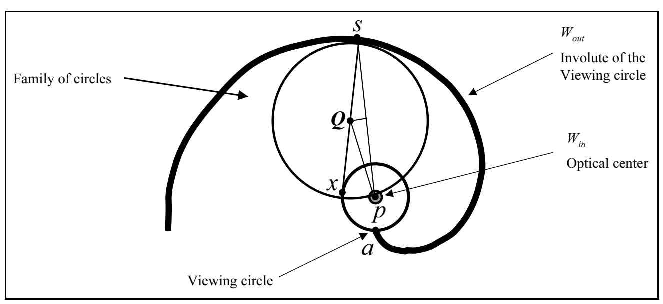

Figure 1.17 Constructing The reflecting curve. We find the reflecting curve equation by constructing the center Q of the circle perpendicular to $\overline{XS}$ and passing through P. This center Q is found as the point of intersection of $\overline{XS}$ and the perpendicular bisector of $\overline{PS}$

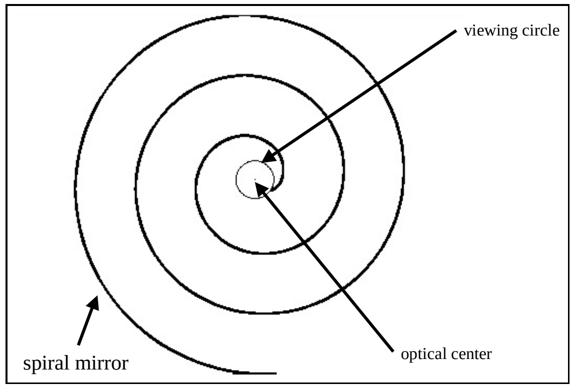

Figure 1.18 A spiral shaped mirror extended for three full cycles. The catacaustic curve of this spiral is the small inner circle.

From S, P we can get the equations for the tangent line to the circle at X as well as the equation for the perpendicular bisector to $\overline{PS}$ and to find the point of intersection O of the two:

Given the position of the camera (p_1, p_2) and the radius R of a viewing circle centered around the origin, points $(x(t), y(t))$ on the mirror can be represented as a function of a parameter t:

$$\begin{aligned} x(t) &= \frac{\sin(t)(R^2+{p_1}^2-R^2t^2+{p_2}^2)-2p_2R-2R^2t\cos(t)}{2(-p_2\cos(t)-Rt+\sin(t)p_1)} \\ y(t) &= \frac{-\cos(t)(R^2+{p_1}^2-R^2t^2+{p_2}^2)+2p_1R-2R^2t\sin(t)}{2(-p_2\cos(t)-Rt+\sin(t)p_1)} \end{aligned} \tag{1.9}$$

When the camera is positioned at the origin, e.g. in the center of the viewing circle, the equations above simplify to:

$$\begin{aligned} x &= \frac{R(-\sin(t)+2t\cos(t)+t^2\sin(t))}{2t} \\ y &= \frac{-R(-\cos(t)-2t\sin(t)+t^2\cos(t))}{2t} \end{aligned} \tag{1.10}$$

A curve satisfying these conditions has a spiral shape, and Fig 1.18 shows such a curve extended for three cycles. To avoid self occlusion, a practical mirror will use only segments of this curve.

A spiral shaped mirror where the optical center is located at the center of the viewing circle is shown in Fig. 1.19.

The configuration where the optical center is at the center of the viewing circle is also convenient for imaging together the left image and the right image. Such a symmetric configuration is shown in Fig. 1.20. This configuration has a mirror symmetry, and each mirror covers 132 degrees without self occlusions. An *Omni Camera* [8, 6] can be placed at the center of the viewing circle to

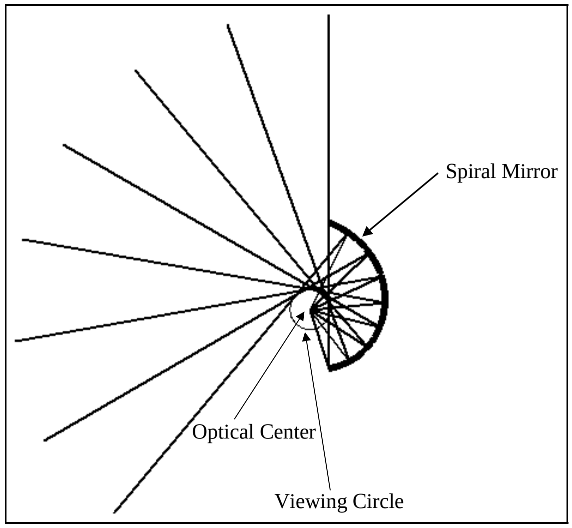

Figure 1.19 A spiral mirror where the optical center is at the center of the viewing circle.

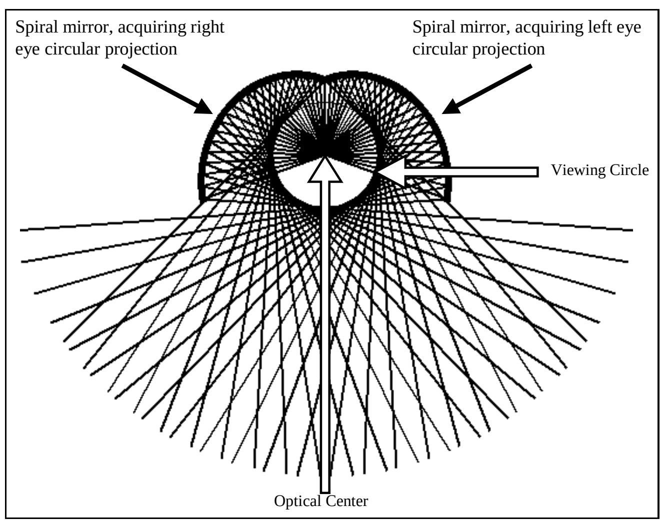

Figure 1.20 Two spiral shaped mirrors sharing the same optical center and the viewing circle. One mirror for the left-circular-projection and one for the right-circular-projection.

capture both the right image and the left image. Since this setup captures up to 132 degrees, three such cameras are necessary to cover a full 360 degrees.

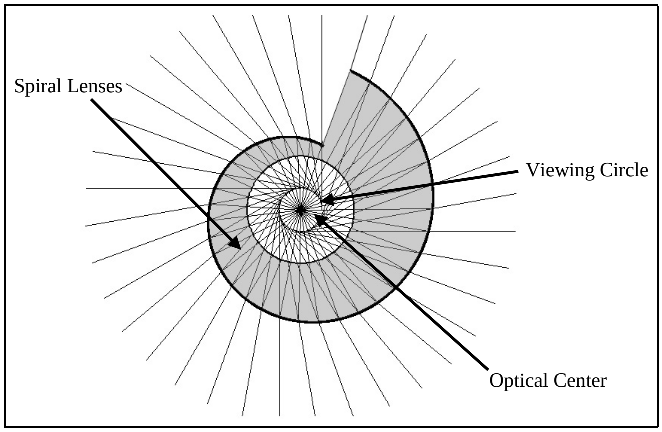

Figure 1.21 A spiral shaped lens. The diacaustic of the lens' outer curve is a circle (the viewing circle). Capturing the panorama can be done by an omnidirectional camera at the center of the viewing circle.

7. A SPIRAL LENS

Circular projections can also be obtained with a lens whose diacaustic is a circle: the lens refracts the rays getting out of the optical center to be tangent to the viewing circle, as shown in Fig. 1.21. A lens can cover up to 360 degrees without self occlusion depending on the configuration. The spiral of the lens is different from the spiral of the mirror. We have not yet computed an explicit expression for this curve, and it is generated using numerical approximations.

It is possible to simplify the configuration and use multiple identical segments of a spiral lens, each capturing a small angular sector. Fig. 1.22 presents a configuration of fifteen lenses, each covering 24 degrees. The concept of switching from one big lens to multiple smaller lenses that produce the same optical function was first used in the Fresnel lens. In practice, a Fresnel-like lens can be constructed for this purpose having thousands of segments. A convenient way to view the entire panorama is by placing a panoramic omnidirectional camera [8, 6] at the center of the lens system as shown in Fig. 1.23.

The requirement that the cylindrical optical element (e.g. as in Fig. 1.22) just bends the rays in the horizontal direction is accurate for rays that are in the same plane of the viewing circle. But this is only an approximation for rays that come from different vertical directions. Examine, for example, Fig. 1.24. Let us examine the rays for viewpoint R. Ray A is in the horizontal plane that includes the viewing circle V. It is deflected by the Fresnel lens into ray a, and passes through the center O of the viewing circle, the location of the optical center of the panoramic camera. Ray B, which also passes through viewpoint R, but from a higher elevation, is also deflected by the same horizontal angle,

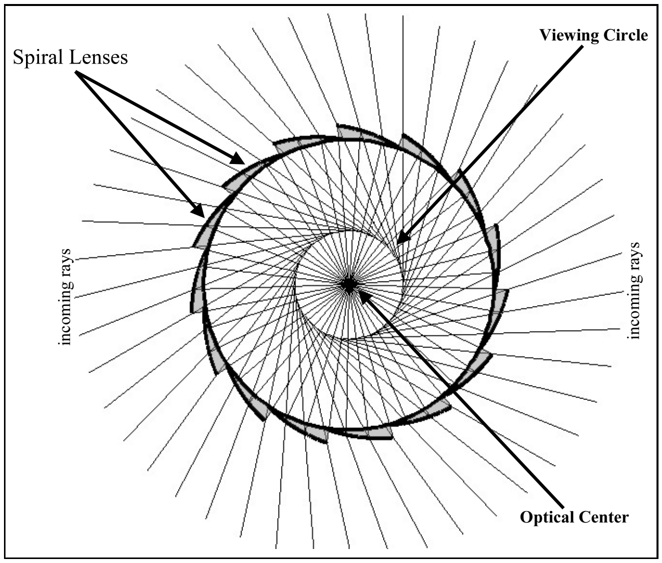

Figure 1.22 A collection of identical short spiral lens positioned on a circle. A Fresnel-like lens can be built with thousands of lens segments. Capturing the panorama can be done by an Omni Camera at the center of the viewing circle.

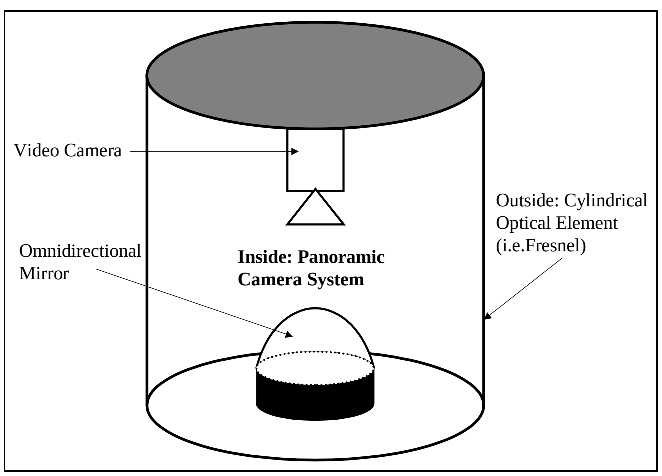

Figure 1.23 An omnidirectional camera at the center of the viewing circle enables the creation of a full 360 degrees left-image or a right-image.

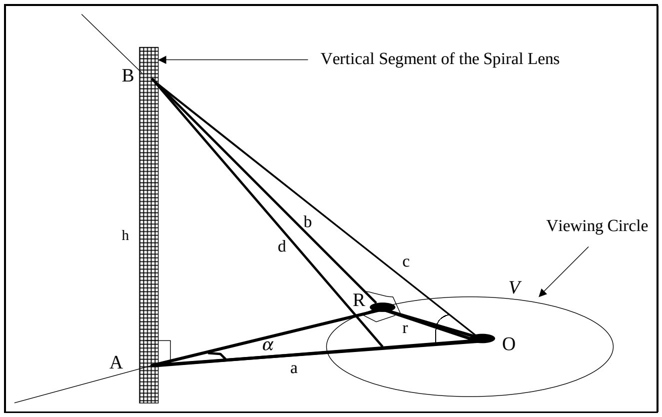

Figure 1.24 Vertical deflection of rays is necessary in order to assure that every viewing direction will have a single viewpoint on the viewing circle.

but will not reach O. Instead, Ray B is deflected into ray d, which can intersect the horizontal plane closer or further to the Fresnel lens than O. In order that ray B will be deflected into Ray c, that intersects O, the Fresnel lens should deflect it also in the vertical direction. Each elevation should have a different vertical deflection. A possible arrangement is that the cylindrical Fresnel lens has vertical elements on one side that take care of the horizontal deflection (which is constant), and on the other side it has horizontal elements that take care of the horizontal deflection (which is different for every elevation).

8. CONCLUDING REMARKS

The theory of omnistereo imaging has been presented. This includes the special circular projection that can provide panoramic stereo in 360 degrees, and several methods to realize this projection. The simplest method that was presented to create omnistereo panoramas is mosaicing. Mosaicing by pasting strips from a rotating camera is applicable to static scenes. In addition, two optical systems, having no moving parts, were presented for capturing stereo panoramic video. One system is based on spiral mirrors, and the second system is based on spiral lenses. While not constructed yet at the time of writing this paper, the optical systems represent the only known possibilities to capture real-time movies having the stereo panoramic features. Omnistereo panoramas can also be rendered from models of virtual scenes.

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References

[1] *IEEE Conference on Computer Vision and Pattern Recognition*, San Juan, Puerto Rico, June 1997.

[2] *Seventh International Conference on Computer Vision*, Kerkyra, Greece, September 1999. IEEE-CS.

[3] S.E. Chen. Quicktime VR - an image-based approach to virtual environment navigation. In *SIGGRAPH'95*, pages 29–38, Los Angeles, California, August 1995. ACM.

[4] Ho-Chao Huang and Yi-Ping Hung. Panoramic stereo imaging system with automatic disparity warping and seaming. *Graphical Models and Image Processing*, 60(3):196–208, May 1998.

[5] H. Ishiguro, M. Yamamoto, and S. Tsuji. Omni-directional stereo. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 1992.

[6] T. Kawanishi, K. Yamazawa, H. Iwasa, H. Takemura, and N. Yokoya. Generation of high-resolution stereo panoramic images by omnidirectional sensor using hexagonal pyramidal mirrors. In *14th International Conference on Pattern Recognition*, pages 485–489, Brisbane, Australia, August 1998. IEEE-CS.

[7] S. Mann and R. Picard. Virtual bellows: Constructing high quality stills from video. In *First IEEE International Conference on Image Processing*, volume I, pages 363–367, Austin, Texas, November 1994.

[8] S.K. Nayar. Catadioptric omnidirectional cameras. In *IEEE Conference on Computer Vision and Pattern Recognition* [1], pages 482–488.

[9] S. Peleg and M. Ben-Ezra. Stereo panorama with a single camera. In *IEEE Conference on Computer Vision and Pattern Recognition*, pages 395–401, Ft. Collins, Colorado, June 1999.

[10] S. Peleg and J. Herman. Panoramic mosaics by manifold projection. In *IEEE Conference on Computer Vision and Pattern Recognition* [1], pages 338–343.

[11] S. Peleg, B. Rousso, A. Rav-Acha, and A. Zomet. Mosaicing on adaptive manifolds. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 22:1144–1154, October 2000.

[12] Paul Rademacher and Gary Bishop. Multiple-center-of-projection images. In *SIGGRAPH'98*, pages 199–206, Orlando, Florida, July 1998. ACM.

[13] H. Shum, A. Kalai, and S. Seitz. Omnivergent stereo. In *Seventh International Conference on Computer Vision* [2], pages 22–29.

[14] H. Shum and R. Szeliski. Stereo reconstruction from multiperspective panoramas. In *Seventh International Conference on Computer Vision* [2], pages 14–21.

[15] R. Szeliski. Video mosaics for virtual environments. *IEEE Computer Graphics and Applications*, 16(2):22–30, 1996.

[16] D.N. Wood, A. Finkelstein, J.F. Hughes, C.E. Thayer, and D.H. Salesin. Multiperspective panoramas for cel animation. In *SIGGRAPH'97*, pages 243–250, Los Angeles, California, August 1997. ACM.

[17] Robert C. Yates. *A Handbook on Curves and Their Properties, rev. ed.* National Council of Teachers of Mathematics, 1952, reprinted 1974.