# Sequential Partition Mechanism for Strongly Budget-Balanced Redistribution

# (Extended Abstract)

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## ABSTRACT

We propose a new class of strategy-proof and strongly budget-balanced redistribution mechanisms called the sequential partition mechanism (SPM). Recently, studies on redistribution mechanisms have attracted increased attention in the research area of mechanism design to achieve a desirable social decision among self-interested agents. However, since no redistribution mechanism can simultaneously satisfy Pareto efficiency, strategy-proofness, individual rationality, and is strongly budget-balanced, we need to sacrifice one of these properties.

In the SPM, agents and items are divided into groups, and then a strategy-proof mechanism is sequentially applied to each group. The payments in each group are distributed among agents in the remaining groups in a predefined way. The auctioneer can dynamically determine how to divide agents and items and which mechanism to apply, based on the results of previous auctions. As an instance of the SPM, we introduce the redistribution mechanism based on a take-it-or-leave-it auction (RM-TLA) mechanism. The RM-TLA does not require agents to reveal a bidding price. Thus, the agents only have to accept/reject the offered price. Furthermore, we show that we can set the optimal reserve price so that the expected social surplus is maximized if an auctioneer knows the distribution of an agent's valuation in advance.

## Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent Systems*; K.4.4 [**Computers and Society**]: Electronic Commerce

## General Terms

Economics, Theory

## Keywords

Mechanism design, redistribution, budget-balance, strategy-proofness

## 1. INTRODUCTION

Various studies on the design of mechanisms or institutions implementing desirable collective decisions among agents have been conducted in multiagent systems. Recently, studies on redistribution mechanisms have attracted increased attention among those studies [1, 3, 4]. For example, if a group of colleagues jointly owns a car and an auction is executed to decide who will use the car on a certain day, the problem how to deal with the winner's payment occurs. To solve this problem, a redistribution mechanism decides not only the winner and the payment but also the redistributed payments to participants. Unfortunately, no redistribution mechanism can simultaneously satisfy strategy-proofness, strong budget-balance, individual rationality, and Pareto efficiency. Thus mechanisms proposed so far sacrifice one of these properties.

Cavallo [1] proposed a mechanism in which the winner and payment are decided by a Vickrey auction and then each agent's redistribution payment is calculated by the amount of the second highest bid among all bids, excluding her own bid, divided by the number of bids. This mechanism is not strongly budget-balanced. Guo and Conitzer [3] defined a class of linear allocation mechanisms in which the payments and social surplus are represented by linear equations of bidding prices. Also, they developed a new class of strongly budget-balanced redistribution mechanisms called the partition mechanism as an instance of linear allocation mechanism.

We propose a new class of strategy-proof redistribution mechanisms, which satisfy strong budget-balance as a hard constraint, called the sequential partition mechanism (SPM). The SPM is a different class of mechanisms from that of linear allocation mechanisms. In the SPM, agents and items are divided into groups, and a strategy-proof auction is sequentially applied to each group. In a predetermined way, the collected payments for a group are distributed among the remaining agents who have not yet participated in an auction.

Furthermore, we introduce a new redistribution mechanism based on take-it-or-leave-it auction (RM-TLA) as an instance of the SPM. The RM-TLA is an intuitive mechanism in which the auctioneer sequentially offers a reserve price for each agent and the offered agent only accepts/rejects it if the reserve price is lower/higher than her value. If the agent accepts it, she pays the reserve price. The payment is redistributed among the remaining agents to whom it has not yet been offered by the auctioneer. Preserving the privacy of bidding prices is a desirable property, since the bidding price is crucial private information. In the RM-TLA, reserve prices directly affect efficiency. We can calculate the optimal reserve price for each round so that the expected social surplus is maximized if the auctioneer knows distributions for agent's valuations.

## 2. SPM

In this section, we explain a new class of redistribution mechanisms called the SPM and its main theoretical properties.

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## 2.1 SPM procedure

We define the SPM procedure as follows:

1. Agents are divided into groups $A_1, A_2, \ldots, A_k$.
2. Goods are also divided into groups $B_1, B_2, \ldots, B_k$. Goods can be heterogeneous or identical.
3. A (possibly inefficient) strategy-proof mechanism $M_1$ is applied to $A_1$ and $B_1$.
4. The payments are distributed among agents in $A_2, \ldots, A_k$ in a predefined way.
5. Then, a (possibly inefficient) strategy-proof mechanism $M_2$ is applied to $A_2$ and $B_2$ + unsold goods. The payments are distributed among $A_3, \ldots, A_k$.
6. Repeat this process for $A_i$ and $B_i$ until the final round is achieved or all the items are sold.
7. Finally, the unsold goods and $B_k$ are allocated to $A_k$ in a predefined way for free.

Each mechanism $M_i$ can be predefined, or it is dependent only to results of previous auctions. Also, $A_i$ and $B_i$ can be determined dynamically according to the result of previous auctions.

## 2.2 Properties

The SPM can satisfy the following properties:

**Strategy-Proofness:** each mechanism $M_i$ is strategy-proof and the predefined redistribution rule is independent of any declared valuations.

**Strong Budget-Balance:** all payments are distributed among the remaining agents. If unsold items remains at the final round, they are allocated to the final agent for free.

# 3. RM-TLA

In this section, we introduce the RM-TLA as an example of the SPM. For conventional auctions, the TLA mechanism was proposed [5]. To the best of our knowledge, no redistribution mechanism utilizing the TLA has been proposed so far. Due to space limitations, we only explain the RM-TLA for a single item, but it can also handle multi-unit auctions.

## 3.1 RM-TLA procedure

1. $A_i = \{i\}$, that is, agents are randomly sorted. $B_1$ is a set of the only item, and $B_i = \emptyset$ for $i \neq 1$.
2. $M_i$ is a take-it-or-leave-it auction with reserve price $r_i$. In more details, the auctioneer offers $r_i$ for agent $i$ and then she only accepts/rejects it if $r_i$ is lower/higher than her true value.
3. Apply $M_i$ for $A_i$ and $B_i$ and repeat this process for until the final round is achieved or the item is sold.
4. Finally, the item is allocated to $A_k$ for free.

The RM-TLA is strategy-proof since the TLA is strategy-proof [5] and the redistributed payment is decided independently of any agents.

## 3.2 Optimal Reserve Price

If the types of agents are symmetric and a valuation is chosen from known distribution function $F$, we can optimize $r_i$ so that expected social surplus is maximized by setting $r_i$ equal to the expected social surplus for the remaining rounds. Assume that $S_{i+1}$ is the expected social surplus for the remaining rounds from the $i+1$-st round and $G(r) = \int F(r)dr$. We can calculate expected social surplus $S_i(r_i)$ when a reserve price is set $r_i$at the $i$-th round as follows:

$$\begin{aligned} S_i(r_i) &= (1 - F(r_i)) \int_{r_i}^{1} \frac{xf(x)}{1 - F(r_i)} dx + F(r_i) S_{i+1} \\ &= -r_i F(r_i) + S_{i+1} F(r_i) + G(r_i) - G(1) + 1 \end{aligned}$$

We can maximize an expected social surplus at the $i$-th round when we set $r_i = S_{i+1}$, since we can obtain a first differential equation such as $S_i'(r_i) = (S_{i+1} - r_i) f(r_i)$.

Furthermore, the RM-TLA can achieve the optimal allocation in the following case: If an auctioneer knows that there is an agent whose valuation exceeds $r$ among the agents, he can sell the item to the agent with $r$ by setting a reserve price $r$ for each agent. On the other hand, the partition mechanism cannot realize optimal allocation in all cases.

| $n$ | Order statistic ($\frac{n}{n+1}$) | TLA | Faltings ($\frac{n-1}{n+1}$) |
|---|---|---|---|
| 2 | 0.667 (100.0%) | 0.625 (93.8%) | 0.500 (75.0%) |
| 3 | 0.750 (100.0%) | 0.695 (92.7%) | 0.667 (88.9%) |
| 4 | 0.800 (100.0%) | 0.742 (92.7%) | 0.750 (93.8%) |
| 5 | 0.833 (100.0%) | 0.775 (93.0%) | 0.800 (96.0%) |
| ... | ... | ... | ... |
| 10 | 0.909 (100.0%) | 0.861 (94.7%) | 0.900 (99.0%) |

**Table 1: Comparison of expected social surplus**

## 3.3 Expected Social Surplus

We theoretically compared the expected social surplus obtained by the RM-TLA with that obtained by the Faltings mechanism [2], which is as an instance of partition mechanisms [3].

Assume there are $n$ agents and a single item. Each agent's valuation are uniformly chosen from $[0, 1]$. As shown in Table 1, if 2 or 3 agents exist, the RM-TLA can obtain better expected social surplus than the Faltings mechanism. Otherwise, the RM-TLA is lower than Faltings, but the difference is bounded within 6%.

# 4. CONCLUSION

We proposed a new class of strongly budget-balanced and strategy-proof redistribution mechanism, called the SPM, that can dynamically adjust several parameters to improve efficiency. Also, we developed the RM-TLA as an instance of the SPM. The advantages of the RM-TLA are its intuitive rules and privacy preserving property so that bidders need not declare their bidding prices.

Our future work includes extending the idea of the RM-TLA to combinatorial auctions.

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