Analog Matching of Colored Sources to Colored Channels

Yuval Kochman[1] and Ram Zamir
Dept. Electrical Engineering - Systems, Tel Aviv University

***Abstract*— Uncoded transmission provides a simple, delay-less and robust scheme for communicating a Gaussian source over a filter channel under the mean squared error (MSE) distortion measure. Unfortunately, its performance is usually inferior to the all-digital solution, consisting of a rate-distortion code for the source followed by a capacity achieving code for the channel. The performance loss of uncoded transmission comes from the fact that except for very special cases, it is impossible to achieve simultaneous matching of source to channel and channel to source by linear means. We show that by combining prediction and modulo-lattice arithmetic, we can match any stationary Gaussian source to any inter-symbol interference, colored-noise Gaussian channel, hence we achieve Shannon's optimum attainable performance $R(D) = C$. This scheme is based upon a novel analog modulo-lattice solution to the joint source-channel coding problem for a Gaussian Wyner-Ziv source and a dirty-paper channel.**

I. Introduction

We consider the case where a Stationary Gaussian source S needs to be transmitted over a stationary Gaussian channel. Using the separation principle, one can construct an all-digital system, consisting of source and channel coding schemes. For known sources and channels, this scheme is optimal although it suffers from large delay and complexity. However, if the channel noise level turns out to be higher than expected the reconstruction may suffer from very large distortion, while if the channel has lower noise level than expected, there is no improvement in the distortion. In contrast, it is well known that if we are allowed one channel use per source sample, the source is white (source samples are i.i.d.) and the channel is white as well (additive noise channel with i.i.d. noise samples), then analog transmission is optimal in the sense of mean squared error. In such a system, the encoder will consist only of multiplication by a constant factor that adjusts the source to the channel power constraint, while the decoder is simply a multiplication by another constant factor, which materializes Wiener estimation of the source from the channel output. Such a system achieves the Shannon bound $R(D) = C$, and moreover it does so with *low complexity* (two multiplications per sample), *zero delay* and *full robustness* (the encoder does not need to know the channel noise level).

While analog single-letter codes are very appealing, in most cases they are sub-optimal. In [7], the question of optimality of single-letter codes is investigated, and source-channel pairs that admit such a solution are termed *probabilistically matched*. It is not hard to verify, that the example above is the only case where a Gaussian source is probabilistically matched to a power-constrained Gaussian channel under MSE criterion. If we allow delay in the system while still seeking a linear time-invariant (LTI) solution (c.f. a scheme involving filters), a somewhat wider class of source-channel pairs match, but even this relaxed demand is met only in very special cases, since an LTI system can not allocate power from one frequency band to another. Seeing that it is generally not possible to gain the full advantages of codeless schemes, it is natural to search at least for "soft" transmission schemes. Such schemes will be optimal for a specific scenario, but their resulting distortion will improve when the system noise level decreases.

Simple practical cases, in which any pure analog LTI scheme will suffer from very poor performance, include disjoint source and channel frequency bands, bandwidth expansion and bandwidth compression. Many solutions have been offered for such scenarios, especially for the special case of bandwidth expansion. Most of these solutions share, explicitly or implicitly, the *hybrid analog-digital* (HDA) approach, allocating different power and bandwidth resources to analog and digital source representations. For treatment of the bandwidth expansion problem, see for example [16], [12], [15]. A recent work [13] generalizes the discussion to sources with memory, while the channel is still white.

In this paper we present a new soft transmission scheme, that is general for all source spectra and all channel spectra. Unlike existing HDA solutions, our scheme does not involve splitting the source or channel into frequency bands, or using a superposition of digital encoders. Rather, our solution treats the source and channel in the *time domain*. We use ideas of prediction, inspired by precoding [17] and differential pulse code modulation (DPCM)-like quantizers [10] , combined with the concept of solving side-information problems by modulo operations w.r.t. a multi-dimensional lattice [20]. In the context of channel coding, the combination of precoding and modulo-lattice transmission is optimal for colored Gaussian channels [20, Section VII-B]. As for source coding, there has been much interest in Wyner-Ziv (WZ) video coding, exploiting the dependence between consecutive frames at the decoder rather than at the encoder (see for example [14]). On the more theoretical side, it is shown in [19] that a DPCM-like encoder using prediction to exploit the source memory achieves the Gaussian-quadratic rate-distortion function (RDF), and a scheme where prediction is used in the decoder only relates to the DPCM scheme the same way that a precoder scheme relates to an optimal feed forward equalizer - decision

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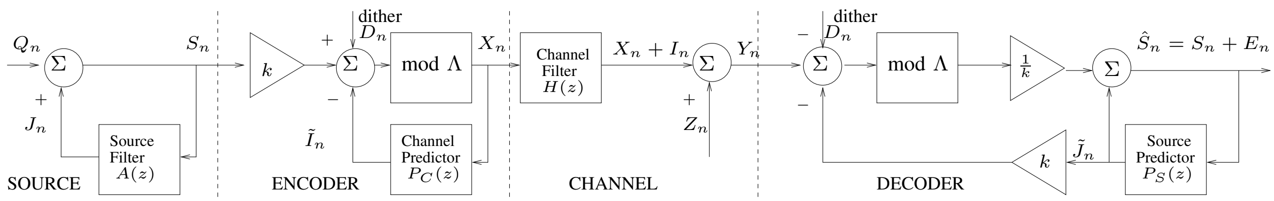

Fig. 1. A Simple High-SNR Scheme

feedback equalizer (FFE-DFE) scheme [2]. We incorporate these ideas into a joint source/channel coding scheme.

In order to demonstrate our basic ideas, we start in Section II with describing our scheme in the high-SNR case. In Section III we show a solution to the Gaussian-Quadratic joint source/channel writing on dirty paper (WDP) and Wyner-Ziv (WZ) problems, which also serves to present elements needed for extending the analog matching scheme to general SNR. Finally in Section IV we arrive at the general-SNR analog matching scheme.

II. Analog Matching for High SNR

Figure 1 illustrates our scheme in the limit of high SNR. The high-SNR assumption simplifies the scheme considerably, allowing to concentrate on the core of the scheme.

Let the source be a discrete-time stationary Gaussian process S_n with power spectrum $S_s(e^{j2\pi f})$, which can be viewed as an AR process constructed from i.i.d. innovations Q_n, by a (possibly infinite-order) filter $A(z)$:

$$S_n = Q_n + S_n * a_n \stackrel{\Delta}{=} Q_n + J_n \quad , \tag{1}$$

where a_n is the (causal) impulse response of the filter $A(z)$ and $*$ denotes convolution. The channel is defined[2] by some monic, minimum-phase causal channel filter $H(z)$ and additive white Gaussian noise Z_n of power N, which will be included in the channel for our purpose.:

$$Y_n = X_n * h_n + Z_n \stackrel{\Delta}{=} X_n + I_n + Z_n \quad , \tag{2}$$

and an input power constraint P. The decoder produces an estimate $\hat{S}_n$ of the source, and the distortion D is defined as the mean square estimation error $D = E\{E_n^2\}$ where $E_n = \hat{S}_n - S_n$. The optimum performance $R(D) = C$ is given in the high SNR limit by:

$$D = \frac{N}{P} P_e\Big(S_S(e^{j2\pi f})\Big) = \frac{N}{P} E\{Q_n^2\} \quad , \tag{3}$$

where the entropy-power of a spectrum $S(e^{j2\pi f})$ is given by:

$$P_e\Big(S(e^{j2\pi f})\Big) = \exp \int_{-\frac{1}{2}}^{\frac{1}{2}} \ln\Big(S(e^{j2\pi f})\Big) df \quad . \tag{4}$$

The modulo-Λ operation is defined as follows: Let the basic lattice cell be the interval $\nu_0 = (-\frac{\Delta}{2}, \frac{\Delta}{2})$. The modulo operation adds an integer multiplication of Δ s.t. the output lies within ν_0. If the dither D_n is independent of the input and uniform on ν_0, then choosing $\Delta = \sqrt{12P}$ ensures that the power constraint is satisfied [18]. Define the predictor outputs:

$$\begin{aligned} \tilde{I}_n &= X_n * p_{c_n} \\ \tilde{J}_n &= \hat{S}_n * p_{s_n} \quad , \end{aligned} \tag{5}$$

and then the encoder and decoder are respectively:

$$X_n = \Big[k S_n - \tilde{I}_n + D_n\Big] \mod \Lambda \tag{6}$$

$$\hat{S}_n = \frac{[Y_n - k\tilde{J}_n] \mod \Lambda}{k} + \tilde{J}_n \quad . \tag{7}$$

Combining (1), (6) and (2), the channel output is:

$$Y_n = [k(Q_n + J_n) - \tilde{I}_n + D_n] \mod \Lambda + I_n + Z_n \quad .$$

Substituting this in (7) and using the fact that

$$(a \mod \Lambda + b) \mod \Lambda = (a+b) \mod \Lambda$$

we have that:

$$\hat{S}_n = \frac{\Big[k(Q_n + J_n - \tilde{J}_n) + I_n - \tilde{I}_n + Z_n\Big] \mod \Lambda}{k} + \tilde{J}_n \quad . \tag{8}$$

The key feature of the scheme, is that the source predictor can cancel the effect of the source AR filter, while the channel predictor can cancel the effect of the channel ISI. To see this, we note that if we choose:

$$\begin{aligned} P_c(z) &= H(z) - 1 \\ P_s(z) &= A(z) \quad , \end{aligned} \tag{9}$$

then:

$$\begin{aligned} \tilde{I}_n &= I_n \\ \tilde{J}_n &= J_n + E_n * a_n \quad . \end{aligned} \tag{10}$$

Substituting this back in (8), we find that

$$\hat{S}_n = \frac{\Big[k(Q_n - E_n * a_n) + Z_n\Big] \mod \Lambda}{k} + \tilde{J}_n \quad . \tag{11}$$

Under the high SNR conditions, the noise Z_n and reconstruction error E_n are very small comparing to kQ_n, and if

$$k^2 E\{Q_n^2\} \ll P \tag{12}$$

then the modulo operation in (11) will have no effect, with high probability. Under this assumption we will have:

$$\hat{S} = S_n + \frac{Z_n}{k} \tag{13}$$

and $D = \frac{N}{k^2}$. We see that for achieving the distortion of (3) we need to satisfy (12) with equality, so that the scheme

[2] Under the Paley-Wiener conditions, a Gaussian channel of any transfer function and noise spectrum can be transformed into such a channel. This can be achieved by a whitened matched-filter at the receiver front-end (see for example [8])

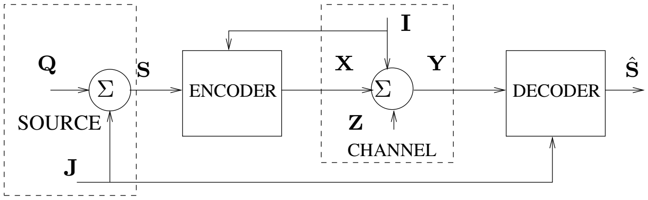

Fig. 2. The Wyner-Ziv / Dirty Paper Coding Problem

presented here achieves the optimum up to a constant margin factor. It will become evident in the sequel that this factor can be eliminated by using a high dimensional modulo-lattice operation. Note that using a "naive" coding scheme, only adjusting the source power to the channel power constraint as in the memoryless case, we would get the distortion $\frac{N}{P}E\{S_n^2\}$, which can be much higher than the optimum distortion when the source is highly predictable.

III. Analog Joint Gaussian Wyner-Ziv Source and Dirty Paper Channel Coding

Consider the joint source-coding problem for the the Gaussian Wyner-Ziv source and dirty paper channel with MSE distortion measure depicted in Figure 2. The source and channel are defined by

$$S_n = Q_n + J_n \quad , \tag{14}$$

$$Y_n = X_n + Z_n + I_n \tag{15}$$

respectively, where Q_n and Z_n are discrete-time i.i.d. Gaussian process with power σ_Q^2 and N respectively. The channel input power constraint is P. J_n is some arbitrary sequence, which forms the source side information known at the decoder only. I_n is another arbitrary sequence which forms the channel side information, known at the encoder only. The sequences Q_n, Z_n, I_n, J_n are all mutually independent. We seek the encoder and decoder pair that will produce the best estimate of S_n in the mean squared error sense. We arrive at a solution which, beyond solving this joint source-channel side-information problem, includes elements needed for the analog matching scheme to work for general SNR.

This is a special case of the joint source-channel problem for WZ source and Gel'fand-Pinsker channel, discussed in [11]. The main result therein is that the separation principle holds for the problem, i.e. the combination of WZ source coding scheme and GP channel coding scheme achieves minimum distortion. They also present optimality conditions for joint single-letter codes, which can be seen as an extension to the results of [7] to side-information problems. In the Gaussian case presented here, single-letter codes are not feasible, but we show that the only non-linear, vector component required is a modulo-lattice operation. By the separation principle, optimal performance is given by $R_{WZ}(D) = C_{WDP}$, yielding:

$$D_{OPT} = \frac{N}{P+N}\sigma_Q^2 \quad . \tag{16}$$

We include here a brief summary of lattice properties needed for this work. Let Λ be a K-dimensional lattice, defined by the generator matrix $G \in \mathbb{R}^{K\times K}$. The lattice includes all points $\{G\cdot \mathbf{i} : \mathbf{i} \in \mathbb{Z}^K\}$ where $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$. The nearest neighbor quantizer associated with Λ is defined by $Q(\mathbf{x}) = \arg\min_{\mathbf{l}\in\Lambda} \|\mathbf{x} - \mathbf{l}\|$ where $\|\cdot\|$ denotes Euclidean norm. The modulo-lattice operation is defined by: $\mathbf{x} \mod \Lambda = \mathbf{x} - Q(\mathbf{x})$. Let the basic Voronoi cell of Λ be $\nu_0 = \{\mathbf{x} : Q(\mathbf{x}) = \mathbf{0}\}$. If the dither vector $\mathbf{d}$ is independent of $\mathbf{x}$ and uniformly distributed over ν_0, then $[\mathbf{x}+\mathbf{d}] \mod \Lambda$ is uniformly distributed over ν_0 as well, and independent of $\mathbf{x}$. The normalized second moment of a lattice is:

$$G(\Lambda) = \frac{1}{K}\frac{\int_{\nu_0}\|\mathbf{x}\|^2 d\mathbf{x}}{[\int_{\nu_0} d\mathbf{x}]^{1+\frac{2}{K}}} \quad .$$

We define a sequence of *good* lattices Λ_K as a sequence of lattices s.t. the normalized second moment is asymptotically optimal: $\lim_{K\to\infty} G(\Lambda_K) = \frac{1}{2\pi e}$ and at the same time the probability of a Gaussian i.i.d. sequence with the same second moment as ν_0 to fall within ν_0 approaches one as $K \to \infty$. Such lattices exist [4], and we will assume throughout the paper the use of such lattices.

In the joint source-channel coding system depicted in Figure 3, a K-dimensional vector encoder:

$$\mathbf{X} = [k\mathbf{S} + \mathbf{D} - \alpha\mathbf{I}] \mod \Lambda \tag{17}$$

translates a source sequence $\mathbf{S}$ to a channel input $\mathbf{X}$, and a vector decoder:

$$\hat{\mathbf{S}} = \frac{\alpha}{k}\{[\alpha\mathbf{Y} - \mathbf{D} - k\sqrt{\beta}\mathbf{J}] \mod \Lambda\} + \mathbf{J} \quad , \tag{18}$$

produces the estimate $\hat{\mathbf{S}}$ from the channel output $\mathbf{Y}$. The dither vector $\mathbf{D}$ is uniformly distributed over ν_0 and independent of $\mathbf{S}$. For this system we have the following optimality result:

Theorem 1: Let the system be defined by (14),(15),(17) and (18) with dimension K. Let the square distortion be D_K. For $\alpha = \frac{P}{P+N}$ and k approaching $k_0 \stackrel{\Delta}{=} \sqrt{\frac{\alpha P}{\sigma_Q^2}}$, and for a sequence of good lattices Λ_K,

$$\lim_{K\to\infty} D_K = D_{OPT} \quad ,$$

where D_{OPT} was defined in (16).

Proof: The sequence at the input of decoder modulo is given by:

$$\begin{aligned} \mathbf{T} &= \alpha(\mathbf{X}+\mathbf{Z}+\mathbf{I}) - \mathbf{D} - k\mathbf{I} \\ &= k\mathbf{Z} - (1-\alpha)\mathbf{X} + \alpha\mathbf{Z} - \mathbf{M} \quad , \end{aligned} \tag{19}$$

where $\mathbf{M}$ is the displacement caused by the encoder dithered modulo operation. We want to ensure correct decoding, which is the event where the displacement of the dithered decoder modulo exactly cancels the dithered encoder modulo, i.e.

$$\mathbf{T} + \mathbf{M} = k\tilde{\mathbf{Z}} - (1-\alpha)\mathbf{X} + \alpha\mathbf{Z} \in \nu_0 \quad . \tag{20}$$

Since $\mathbf{X}$ and $\mathbf{Z}$ are independent, the power of $\mathbf{T}$ is minimized by the Wiener coefficient $\alpha = \frac{P}{P+N}$, yielding $\mathbf{T}+\mathbf{M} = k\tilde{\mathbf{Q}} + \mathbf{N}^{eq}$ with $E\{(N_n^{eq})^2\} = \frac{PN}{P+N} = \alpha N$. By the properties of dithered modulo-lattice operation, $\mathbf{X}$ is independent of $\tilde{\mathbf{Q}}$ and uniformly distributed over ν_0. Exists an exponentially good sequence of lattice Λ_K such that the probability of correct

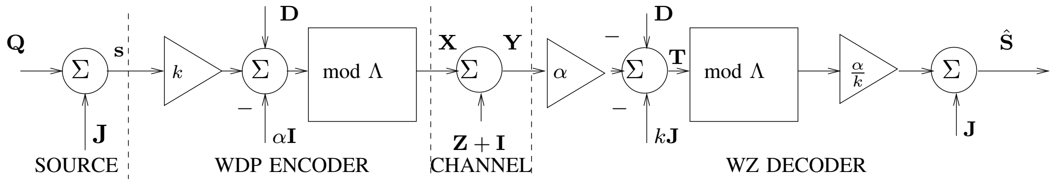

Fig. 3. Analog Wyner-Ziv / Dirty Paper Coding Scheme

decoding approaches 1 as $K \to \infty$ (see [6, Section III-B] for discussion about the case of a combination of a Gaussian part, and a part uniformly distributed over the basic lattice Voronoi cell, such as N_n^{eq}). Thus we need only demand that $k^2\sigma_Q^2 + E\{(N_n^{eq2}\} \leq P$, or: $k < k_0$. Under this condition:

$$\hat{\mathbf{S}} = \frac{\alpha}{k}(\mathbf{T}+\mathbf{M}) + \mathbf{J} = \mathbf{S} + \frac{\alpha}{k}\mathbf{N}^{eq} + (1-\alpha)\mathbf{Z} \quad . \tag{21}$$

Choosing k approaching k_0 we have:

$$D = \frac{\alpha^2 E\{(N_n^{eq})^2\}}{k_0^2} + (1-\alpha)^2\sigma_Q^2 = D_{OPT}$$

■

Remarks:

1. A solution to the same problem using previously known results: By [11], one can build separate source and channel encoder/decoder pairs. By [20], each of these pairs can be based on a nested lattice. In comparison, our scheme does not require a nested lattice (it uses a lattice parallel to the coarse lattice of nested schemes), and in addition the source and channel lattices collapse into a single one.

2. In our system the process $\mathbf{Q}_n$ occupies at the lattice input only a power of αP, strictly smaller than the channel power P. This differs from the capacity-achieving nested lattice strategy of [6]. In [5] it is shown that if the "data" (fine lattice) occupies a portion of power γP with $\alpha \leq \gamma \leq 1$, capacity is still achieved. In [1] a similar observation is made, and a code of power αP is presented as a preferred choice, since it allows easy iterative decoding between the information-bearing code and the coarse lattice. In the case of analog transmission, however, we have no choice: Optimality under a continuous distortion measure requires that the equivalent channel is additive rather than modulo-additive, thus necessarily $\gamma = \alpha$.

IV. Analog Matching for general SNR

The high-SNR scheme of Section II and the side-information scheme of Section III bare close similarity. Evidently, we can identify the WZ side-information as the output of the source predictor, while the WDP side-information is the channel ISI. As mentioned in Section I, such connections were already made in channel and source coding.

The generalization of the scheme presented in Section II to general SNR requires the following considerations:

1. MMSE estimation. While the predictors in the high-SNR scheme of Section II perfectly cancel the source and channel memory, optimality in the general case requires to avoid noise amplification. Hence, the channel predictor is replaced by the DFE of the optimal MMSE FFE-DFE solution [2], while the source predictor is replaced by a "noisy predictor" [19].

2. Deflated lattice input. As shown in Section III, correct lattice decoding requires that the total of encoder lattice input and equivalent channel noise power be less than the lattice cell power. In order to ensure that, the encoder gain k must be adjusted as to "leave room" for the noise.

3. Spectral shaping. In order to achieve the full Gaussian channel capacity, the signal at the channel input must have the channel water-filling spectrum [3]. Similarly, when quantization is equivalent to additive white Gaussian noise (AWGN) channel, in order to achieve the Gaussian-quadratic RDF the signal at the quantizer input must have the source reverse water-filling spectrum. The shaping filter which achieves this spectrum is called pre-filter, while an MMSE filter following the quantizer is called post-filter [18].

Combining these ingredients, the general-SNR scheme is identical to the scheme depicted in Figure 1, except for the addition of pre- and post-filters as the encoder first stage and decoder last stage respectively, a shaping filter as the encoder last stage and a linear filter as the decoder first stage. If the channel capacity is C and $D_{OPT} = R^{-1}(C)$, then the pre- and post-filters are defined by

$$\begin{aligned} |F_1(e^{j2\pi f})|^2 &= |F_2(e^{j2\pi f})|^2 = \max\Big(1 - \frac{S_S(e^{j2\pi f})}{\Theta}, 0\Big) \\ F_1(e^{j2\pi f}) &= F_2(e^{j2\pi f})^* \quad , \end{aligned} \tag{22}$$

where Θ corresponds with the reverse water-filling solution for S at distortion level D_{OPT}. The channel shaping filter gives a white process of power P the channel water-filling spectrum. The linear filter consists of a whitened matched-filter for the channel (see for example [8]), together with the linear part of the optimal channel MMSE FFE-DFE solution [2].

For optimality, we need high lattice dimension. We assume that we have K independent source-channel pairs in parallel, which allows a K-dimensional dithered modulo-lattice operation across these pairs. Other operations are done in parallel.

Also note, that without the high-SNR assumption it is not possible in general to represent the source as an AR process, thus the "source" part of Figure 1 is not relevant.

For this scheme we have the optimality Theorem, the proof for which will be included in a full paper to appear:

Theorem 2: For the system described above with dimension K, let the square distortion be D_K. For an appropriate choice of k and for a sequence of good lattices Λ_K,

$$\lim_{K\to\infty} D_K = D_{OPT} = R^{-1}(C) \quad .$$

Remarks:

1. If we want to work with a dimension $K > 1$, but we only have one source and one channel, we face a fundamental problem: The filters' states should be updated every sample,

while the modulo operation are done block-wise. This can be overcome, at the price of large delay, by applying the interleaving approach developed in [9] for FFE-DFE receivers, and used for precoding in [20, Section VII-B], to our scheme.

2. The optimality of the scheme is based upon correct decoding at the decoder lattice. In practice, when the dimension K is finite, the probability for incorrect decoding is not negligible. In the case of incorrect decoding, the distortion is bounded as well, because of the bounded modulo output, and has approximately the source power. By taking small enough k we can make the error probability as small as desired, but the distortion in the case of correct decoding will grow inversely proportional to k^2. One can find the k that minimizes the overall distortion for a given lattice.

3. Denote the pre-filtered version of the source by U_n, and the reconstruction before post-filtering by V_n. The key to understanding the scheme, is that between U_n and V_n there is an equivalent AWGN channel. This equivalent noise, composed of the channel AWGN and self noise (X_n), is scaled down by k. The factor k, in turn, is set by the variance of prediction error of U_n from V_n, for correct decoding. It turns out, that k can be chosen such that correct decoding holds and the AWGN between U_n and V_n has variance Θ corresponding with the source water-filling solution. Since the optimal predictor of U_n from V_n is identical to the optimal predictor of V_n from itself, which is the one-step predictor for the spectrum $S_V(e^{j2\pi f}) = S_U(e^{j2\pi f}) + \Theta$. The resulting prediction error of U_n is [19]:

$$\sigma^2_{U_n|V_{n-1},V_{n-2},\ldots} = P_e\Big(S_U(e^{j2\pi f}) + \Theta\Big) - \Theta \quad . \tag{23}$$

4. Note that the only "digital" component in this scheme is the modulo-lattice operation. This is equivalent to the coarse lattice in nested-lattice schemes, or to shaping needed for capacity-approaching and RDF-approaching schemes. While we do use this component, we do not use a fine lattice, equivalent to the words of a codebook. In other words, when transmitting a Gaussian source through a Gaussian channel, the *only* digital coding needed is for shaping.

We conclude by specializing analog matching to the case of bandwidth expansion. Let the source and channel be both white, with $\rho > 1$ channel uses per source sample (ρ not necessarily integer). We choose to work with a sampling rate corresponding with the channel bandwidth, thus in our discrete-time model the channel is white, but the source is band-limited to a frequency of $\frac{1}{2\rho}$. As a result, the linear filter $L(e^{j2\pi f})$ becomes a scalar and the channel predictor $P_c(z)$ becomes zero. The channel shaping filter is not needed, as the white modulo output already matches the channel water-filling spectrum, and the source pre- and post-filters become both ideal low-pass filters of width $\frac{1}{2\rho}$ and height $\sqrt{1-\frac{D}{\sigma_S^2}} = \sqrt{1-\left(1+\frac{P}{N}\right)^{-\rho}}$. The noise between U_n and V_n is $\Theta = \rho D$, and substituting in (23) we see that the part of U_n unpredictable at the decoder has variance

$$\sigma^2_{U_n|V_{n-1},V_{n-2},\ldots} = \rho\sigma_S^2\frac{P}{N}\Big(1+\frac{P}{N}\Big)^{-\rho} \quad .$$

The resulting distortion achieves the optimum:

$$D = \sigma_S^2\Big(1+\frac{P}{N}\Big)^{-\rho} \quad .$$

For this special case, the issue of improvement of distortion with SNR has been addressed in previous work, especially [12], [13] with an outer bound derived in [15]. Using our scheme, it is possible to decrease distortion for better SNR by changing only the post-filter in the decoder. Investigating the performance of such a system, or other variations of our basic scheme, is left for future research.

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References

[1] A. Bennatan, D. Burshtein, G. Caire, and S. Shamai. Superposition Coding for Side-Information Channels. *IEEE Trans. Info. Theory*, IT-52:1872–1889, May 2006.
[2] J.M. Cioffi, G.P. Dudevoir, M.V. Eyuboglu, and G.D. J. Forney. MMSE Decision-Feedback Equalizers and Coding - Part I: Equalization Results. *IEEE Trans. Communications*, COM-43:2582–2594, Oct. 1995.
[3] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. Wiley, New York, 1991.
[4] U. Erez, S. Litsyn, and R. Zamir. Lattices Which are Good for (Almost) Everything. *IEEE Trans. Info. Theory*, IT-51:3401–3416, Oct. 2005.
[5] U. Erez, S. Shamai, and R. Zamir. Capacity and Lattice Strategies for Cancelling Known Interference. *IEEE Trans. Info. Theory*, IT-51:3820–3833, Nov. 2005.
[6] U. Erez and R. Zamir. Achieving 1/2 log(1+SNR) on the AWGN Channel with Lattice Encoding and Decoding. *IEEE Trans. Info. Theory*, IT-50:2293–2314, Oct. 2004.
[7] M. Gasptar, B. Rimoldi, and Vetterli. To Code or Not to Code: Lossy Source-Channel Communication Revisited. *IEEE Trans. Info. Theory*, IT-49:1147–1158, May 2003.
[8] R. D. Gitlin, J. F. Hayes, and S. B. Weinstein. *Data Communications Principles*. Plenum Pub Corp, 1992.
[9] T. Guess and M. Varanasi. An Information-Theoretic Framework for Deriving Canonical Decision-Feedback Receivers in Gaussian Channels. *IEEE Trans. Info. Theory*, IT-51:173–187, Jan. 2005.
[10] N. S. Jayant and P. Noll. *Digital Coding of Waveforms*. Prentice-Hall, Englewood Cliffs, NJ, 1984.
[11] N. Merhav and S. Shamai. On Joint Source-Channel Coding for the Wyner-Ziv Source and the Gel'fand-Pinsker Channel. *IEEE Trans. Info. Theory*, IT-40:2844–2855, Nov. 2003.
[12] U. Mittal and N. Phamdo. Hybrid Digital-Analog (HDA) Joint Source-Channel Codes for Broadcasting and Robust Communications. *IEEE Trans. Info. Theory*, IT-48:1082–1103, May 2002.
[13] V. M. Prabhakaran, R. Puri, and K. Ramchandran. A Hybrid Analog-Digital Framework for Source-Channel broadcast. In *Proceedings of the 43rd Annual Allerton Conference on Communication, Control and Computing*, 2005.
[14] R. Puri and K. Ramchandran. PRISM: A 'Reversed' Multimedia Coding Paradigm. In *Proc. IEEE Int. Conf. Image Processing, Barcelona*, 2003.
[15] Z. Reznic, M. Feder, and R. Zamir. Distortion Bounds for Broadcasting with Bandwidth Expansion. *IEEE Trans. Info. Theory*, submitted.
[16] S. Shamai, S. Verdú, and R. Zamir. Systematic lossy source/channel coding. *IEEE Trans. Info. Theory*, 44:564–579, March 1998.
[17] M. Tomlinson. New automatic equalizer employing modulo arithmetic. *Elect. Letters*, 7:138–139, March 1971.
[18] R. Zamir and M. Feder. Information rates of pre/post filtered dithered quantizers. *IEEE Trans. Info. Theory*, pages 1340–1353, Sept. 1996.
[19] R. Zamir, Y. Kochman, and U. Erez. Achieving the Gaussian Rate Distortion Function by Prediction. In *ISIT-2006, Seattle, WA*, 2006.
[20] R. Zamir, S. Shamai, and U. Erez. Nested Linear/Lattice Codes for Structured Multiterminal Binning. *IEEE Trans. Info. Theory*, IT-48:1250–1276, June 2002.