Digital libraries Exercise #2

May 2004

1. A fair coin has two sides – head and tail. Let the random variable $X$ denote the number of flips of the coin until head shows the first time.

(i) What is the entropy $H(X)$? (Hints: $\sum_{n=1}^{\infty} r^n = r/(1 - r)$ and $\sum_{n=1}^{\infty} nr^n = r/(1 - r)^2$.)

(ii) Assume A has flips the coin. B does not see the result but he is allowed to ask questions of the form: “is the number of flips (i.e. the value of $X$) in the set $S$?” ($S$ is defined by B). Suggest an “efficient” sequence of questions for B, to find the value of $X$. What is the expectation of the number of questions needed to find the value?

(iii) How is this question related to Golomb code?

2. Prove: If $H(Y|X) = 0$, then for each $x$ such that $p(x) > 0$ there is only one value of $Y$ with positive probability (thus, $Y$ is a function of $X$).

3. Given the joint distribution:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
</tbody>
</table>

compute the values of:

$H(X), H(Y), H(X|Y), H(Y|X), H(X,Y), I(X;Y)$.

4. Implement the gamma compression, and add it to your project. Add a flag to allow a user to either use or not use it.