# Non-Rigid Parallax for 3D Linear Motion*

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## Abstract

We consider the problem of reconstructing the location of a moving 3D point seen from a monocular moving camera. Since the point is moving while the camera is moving, then even if the camera motion is known, it is impossible to reconstruct the 3D location of the point under general circumstances. However, we show that if the point is moving along a straight line, then the parameters of the line (and hence the 3D position of the point at each time instance) can be uniquely recovered, and by linear methods, from at least 5 views. Consequently, we propose a new approach for dealing with dynamic scenes (rich with moving objects) in which once the camera motion is recovered, the 3D trajectory (straight line) of the moving target can be recovered — even when the moving target consists of a single point.

## 1 Introduction

We consider the problem of recovering structure and motion of dynamically moving points seen from a moving camera. Consider Fig. 1 which contains a static scene with a moving object. Given enough static scene points one can use robust techniques to remove the outliers (in this case the outliers include the points arising from the moving car) and compute the motion of the camera (the two camera matrices from 3D to the two 2D images). Our problem is then to recover the 3D motion of the car. One possible approach is to perform a second round of robust estimation on outlier points in order to recover a new set of camera matrices derived solely from the points on the moving object — and then to somehow represent the car's motion relative to the camera motion. This alternative maybe undesirable in cases where the moving object occupies a relatively small region in the image (like in this example), and what if our moving object is a single point? We therefore wish to develop an alternative scheme described next.

In our approach we consider the problem of a moving 3D point seen by a moving camera. Clearly, under general circumstances (point generally moving in 3D), one cannot recover the 3D location of the moving point because in order to that it is required to have at least two views of the point while it is static and the camera is moving. However, if we constrain the type of allowable trajectories of the point, for example, assume the 3D point is moving along some (unknown) 3D line, then the problem is solvable in a relatively straightforward manner. Hence, we define our problem as follows (see Fig. 2):

**Problem Definition 1 (Non-Rigid Parallax)** *Given a scene of dynamically moving 3D points, each point is moving along some unknown 3D line, seen from a moving camera whose motion is known (or can be recovered from static scene points or through other means), reconstruct the trajectory (the 3D line) of each moving point from the known 2D matches across the views.*

## 2 Non-Rigid Parallax

The first question to consider is *how many views are necessary for a unique solution?*. One can easily show that 4 views provide two solutions, thus 5 views provide a unique solution. The argument goes as follows: the camera center and the image point define a 3D line which we will call a ray. Therefore if we have $k$ views we have $k$ rays, and we wish to find the minimal $k$ such that if the $k$ rays form a linear line complex (intersecting some 3D line $L$), it is unique. Clearly, if $k = 2$ we have infinite lines $L$ intersecting the 2 rays. For $k = 3$, for each point along the first ray there is a unique line incident to it and to the other two rays (the point and the second ray define a plane which intersects the third ray uniquely) — hence we still have infinite lines $L$ (see Fig. 3). For $k = 4$, three of the rays define a ruled quadric which intersects the fourth ray in two distinct points — thus we have two solutions for $L$, and thus, for $k = 5$ we have a unique solution. The argument that one needs at least 4 lines for a finite number of solutions for a common in-

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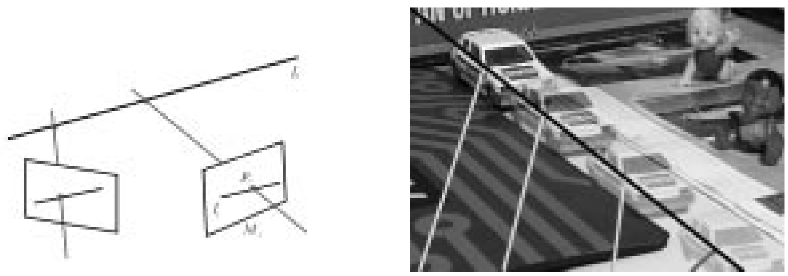
(a) (b) (c)

Figure 1: Three frames from a sequence in which the car is moving independently of the scene. The camera is moving to the right.



(a) (b)

Figure 2: Illustration of Non-Rigid Parallax. Schematic illustration (a) of A point moving along a 3D line and is projected on the image plane of a moving camera. Since the 3D point is moving one can not use triangulation to recover its coordinates. A sketch of this principle is seen in (b) where the car is moving along a straight line while the camera is moving. The lines drawn from the car are the optical rays of a single point on the car as seen by the moving camera.

tersecting line is well known and is also used in graphics algorithms for synthetic illumination and visibility computations (cf. [3]).

Having an idea of the minimal number of views required for a solution, we next consider the problem of recovering the piercing line $L$ given $k \geq 5$ views, known $3 \times 4$ projection matrices (describing camera motion) $M_i$ and the projections $p_i$, $i = 1, ..., k$ of the moving point along the line $L$.

Let $P, Q$ be any two points on the line $L$, and let $l_i$ be the projection of $L$ on view $i$. Clearly, $p_i^\top l_i = 0$ because $p_i$ is incident to the line $l_i$ in the image plane. We can represent $l_i$ by the cross product of the projections of $P$ and $Q$:

$$l_i \cong (M_i P) \times (M_i Q)$$

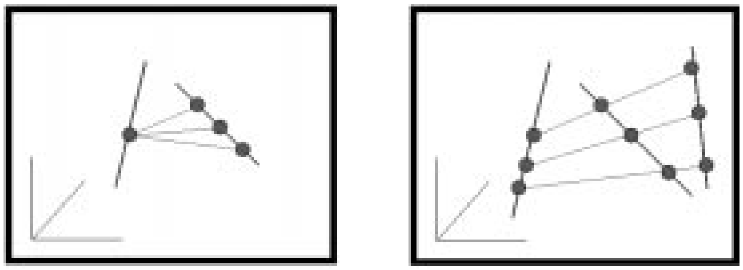
because $M_i$ projects 3D points onto view $i$. A convenient way to simplify this expression is to represent the line $L$ using Plucker coordinates: $L = P \wedge Q$ which is a vector of six components comprising of the six determinants of the $2 \times 2$ minors of the $2 \times 4$ matrix whose rows are the vectors $P, Q$. The Plucker representation of the line is defined up to scale and is independent of the choice of $P, Q$. The space of all 3D lines are therefore embedded in a 5-dimensional projective space and subject to a quadratic constraint (i.e., not every six tuple correspond to a real line). Thus the six Plucker coordinates describe a four parameter space which confirms basic intuition that a 3D line could be parameterized by four numbers (such as by intercept on two standard planes).

In [2] it was pointed out that using Plucker representation one can readily transform $M_i$ into a $3 \times 6$ matrix $\tilde{M}_i$ that satisfies a line projection matrix relation: $l_i \cong \tilde{M}_i L$ where $L$ is represented by its six Plucker coordinates. The rows of $\tilde{M}_i$ are defined by the $\wedge$ ("meet") operation on the pairs of rows of $M_i$. We have therefore established the following linear constraint on the unknown Plucker vector $L$:

$$p_i^\top \tilde{M}_i L = 0$$

which is linear in the parameters of $L$. Thus 5 views provide a unique solution and more than 5 views provide a least-squares solution. Note that one can readily see why 4 views provide two solutions for $L$: in that case we have a two dimensional null space spanned by $v_1, v_2$, thus $L = v_1 + \lambda v_2$. The scalar $\lambda$ can be found from the quadratic constraint that all Plucker line must obey, thus we obtain a second-order constraint on $\lambda$ which provides two solutions for $L$.

Fig. 4 shows an example of recovering the 3D line of a single point tracked along the moving car. Note that once the 3D line $L$ is recovered it is a simple matter to reconstruct the actual 3D points as they were moving in space, simply by intersecting each ray with the line $L$.



(a) (b)

Figure 3: Figures (a) and (b) show that one can define infinite number of lines that intersect two or three given lines, respectively. Given four lines, one can only define two distinct lines that intersect the four lines. Five lines define a unique line in space that intersect them all.


Figure 4: Projecting the 3D line defined by the moving car on one of the images of the sequence.

We have also extended the basic approach to deal with curved trajectories by allowing the 3D point to move along a conic. We show that the 3D conic can be recovered uniquely from 8 views, and the algorithm for recovering the conic uses numerical optimization (Gauss-Newton iterations) [1].

## 3 Summary

We have introduced a new approach for handling scenes with dynamically moving objects viewed by a monocular moving camera. In a general situation in which both the camera and the target are moving without any constraint the problem is not solvable, i.e., one cannot recover the 3D motion of the target even when the camera ego-motion is known. We show that by assuming the target is moving along a straight 3D line the problem of recovering the target's trajectory is uniquely solved given at least five views of the moving target.

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