Experimental Approaches in Computer Science

Dror Feitelson
Hebrew University

Lecture 11 – Experimental Algorithmics
Traditional approach:

"Quicksort is $O(n^2)$"

- What this means:
  Quicksort will perform at most $cn^2$ operations when sorting $n$ numbers, regardless of programming language, programmer experience, or the specific input sequence

- This is a universal truth

- Note, however, that
  - $c$ is not specified
  - This is the worst case, and might not be representative of common cases
Misleading complexity analysis – only huge $n$:

**Minimum spanning tree**

- Prim's algorithm has complexity $O(E \ lg \ V)$
  
  [Start with any node, and iteratively add lowest cost edge to a node that is not in the tree yet. Complexity depends on appropriate data structure]

- Fredman and Tarjan have an algorithm with complexity $O(E \ lg^* \ V)$
  
  [Very complicated]

- In practice, the improvement is only seen for dense graphs with more than 1000000 nodes
  
  [Moret & Shapiro, DIMACS 1994]
Misleading complexity analysis – drowned by constant factor:

- **Test if one graph is a minor of another**
  - Robertson & Seymour give a cubic algorithm
  - However, it has a constant of $10^{150}$

**Sorting network**

- Ajtai, Komlos, and Szemeredi show an optimal $O(\log n)$-depth construction
- Based on expander graphs
- Huge constants make it impractical
Misleading complexity analysis – worst case is uncommon

**Linear programming**

- The simplex method has an exponential worst case running time
- However, it has a low running time for practically all naturally occurring inputs

Similar situations exist for many **NP-complete** problems

- Approximations may be available for most inputs
- The problematic inputs may be rare and uninteresting
Missing complexity analysis does not necessarily imply bad performance

Minimize edge crossings when drawing bipartite graphs

- Problem is NP-complete
- Algorithm with no known constant approximation ratio leads to better results than algorithm with proven low constant approximation ratio

[Demetrescu & Finocchi, ALENEX 2000]
Misleading complexity analysis – does not take mundane implementation issues into account

**Locality and cache effects**
- May have dramatic effect on performance
- However, can be very hard to predict and analyze
- Partly due to complex parameterization of cache structures (set sizes, associativity, multiple levels)
Naive optimizations – implicit assumptions may be wrong

**Code structure vs. cache effects**

- Optimizations often geared to reduce instruction counts so as to accelerate execution
- This may lead to smaller code blocks and using less data in each basic block
- May result is reduced cache locality and subsequent longer running time
The experimental approach

- Emphasize real-world results, including constants
  - How much time will it really run?
- Emphasize common case rather than worst case
  - Also, how common is the common case?
  - Show distribution rather than just one data point
- Price is possible dependence on platform being used
  - Might restrict applicability
  - Typically good enough for qualitative comparisons
Basic methodology: Use a good implementation

• When studying an algorithm, make it relevant
  – Finding that an inefficient implementation is bad is not interesting
  – Studying a bad implementation is misleading and confounds the issues

• When comparing options, make it fair
  – Want to compare algorithms, not implementations
  – Need to invest similar amounts of effort
Basic methodology: Use representative input instances

- Behavior on random inputs may differ from behavior on real ones
  - Real world inputs may tend to be more structured
  - Such structure may provide opportunity for special optimizations
  - Or such structure may be harder to handle

- Similar to need for representative workloads
Basic methodology: Perform good measurements

• Use repetitions and calculate confidence intervals
  – Repetitions are over multiple representative inputs
  – Need to note whether distribution is bell-shaped or has a tail

• Remove outliers?
  – Not if they are important real cases
  – Yes if they reflect interference with measurement
Basic methodology: Report full details

• Allow for reproducibility
  – If others can reproduce it this increases our confidence
  – Not the same as replication, where others simply run your code on your inputs

• Include platform details, implementation details