

# Digital Communication in the Modern World

## Physical Layer: Fourier Series; Physical Media

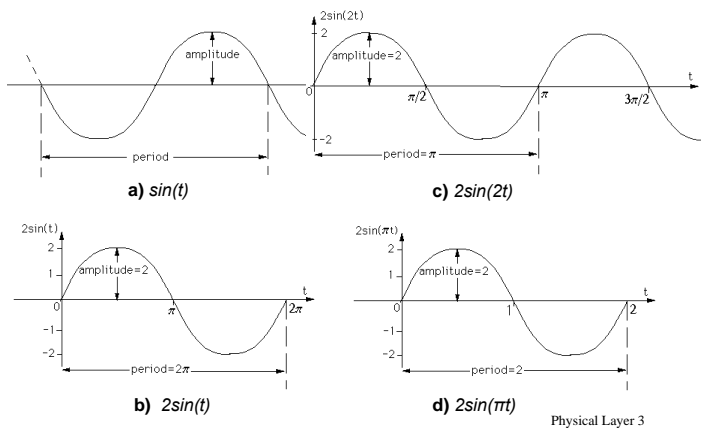
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## The Theoretical Basis for Data Communication

- Fourier Analysis
- Bandwidth-Limited Signals
- Maximum Data Rate of a Channel

Physical Layer 2

## Harmonic Analysis



Physical Layer 3

## Harmonic Analysis

In the general case:

$$A_n \cdot \sin\left(n \cdot \frac{2\pi \cdot t}{T}\right)$$

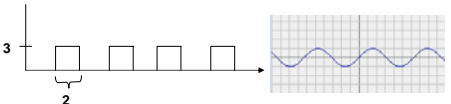
$n$  - the harmonic number

$A_n$  - the amplitude of the  $n^{\text{th}}$  harmonic

$2\pi$  - normalizes the function so that for every unit  $t$  it completes one period

$1/T$  - the base frequency, normalizes the function so that for every harmonic  $n$  the frequency becomes  $n$  times the base frequency

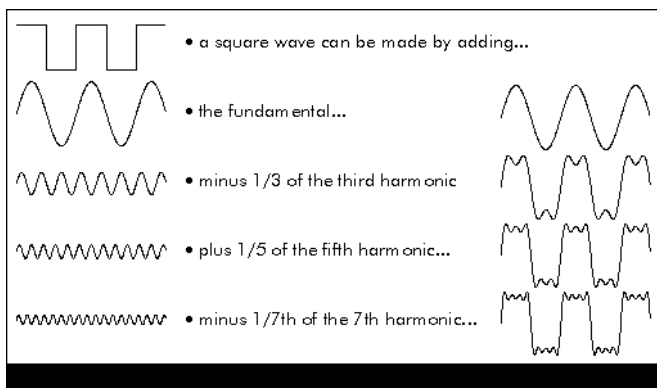
Example: the function



can be roughly approximated by  $3\sin(2\pi t/4)$ , which is called the **fundamental 1<sup>st</sup> harmonic** of our function

Physical Layer 4

## Harmonic Analysis



Physical Layer 5

## Fourier Series Theorem



- For a "sufficiently well-behaved" function of time,  $g(t)$ :

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin\left(n \cdot \frac{2\pi \cdot t}{T}\right) + \sum_{n=1}^{\infty} b_n \cos\left(n \cdot \frac{2\pi \cdot t}{T}\right)$$

- Using  $e^{it} = \cos(t) + i \sin(t)$

The Fourier Series gets the form:

$$g(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{i \cdot n \cdot \frac{2\pi}{T} \cdot t}$$

- Calculating the coefficients; denote  $f := 1/T$ :

$$a_n = \frac{2}{T} \int_0^T g(t) \sin(2\pi n f t) dt \quad b_n = \frac{2}{T} \int_0^T g(t) \cos(2\pi n f t) dt \quad c = \frac{2}{T} \int_0^T g(t) dt$$

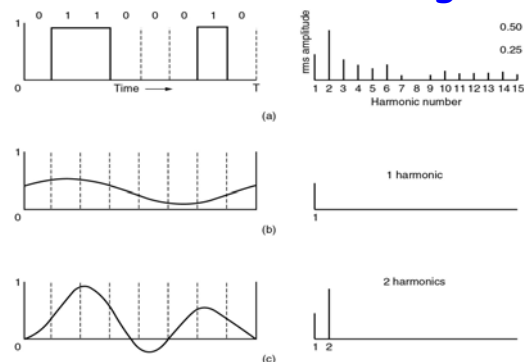
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## Bandwidth-Limited Signals

- Higher harmonic Fourier components attenuate faster
- Range of frequencies transmitted without being "strongly" attenuated is denoted the **bandwidth** of the media
- Often defined from 0 to the frequency at which half the power (amplitude) gets through per length (1 km)
- Bandwidth is a physical property of the transmission medium

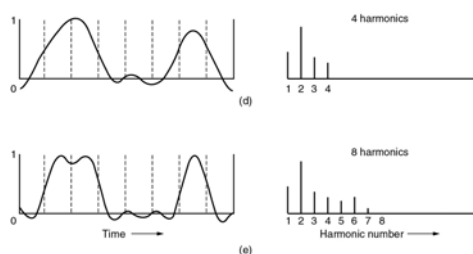
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## Bandwidth-Limited Signals



A binary signal and its root-mean-square Fourier amplitudes.  
(b) - (c) Successive approximations to the original signal.

## Bandwidth-Limited Signals (2)



(d) - (e) Successive approximations to the original signal.

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## Bandwidth-Limited Signals (3)

- Example: Assume you want to send 8 bits at 9600 bps over an ordinary phone line
- The time to send 8 bits is  $8/9600 = 0.83$  msec.
- The frequency of the first harmonic is  $9600/8 = 1200$  Hz (periods per second)
- Ordinary phone lines have an artificial cut-off bandwidth of 3000Hz.
- Thus the highest harmonic passed through is  $3000/1200 = 2.5 \Rightarrow$  highest harmonic is 2!
- The signal received would be tricky to reconstruct
- $\Rightarrow$  limiting the bandwidth limits the data rate

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## Bandwidth-Limited Signals (4)

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Relation between data rate and harmonics for send a constant of 8 bits over a 3KHz channel.

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## Nyquist's Sampling Theorem

- In 1924 Henry Nyquist derived the equation for the maximum data rate for a finite bandwidth noiseless channel:

$$\text{Max data rate} = 2H \log_2 V \text{ bps}$$

$H$  = low-pass filtered bandwidth

$V$  = number of signal levels, for binary data  $V=2$

E.g. a 3kHz noiseless filtered channel cannot transmit binary signals at a rate exceeding 6000 bps

The theorem states that by making  $2H$  samples per sec the signal can be completely reconstructed.

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## Shannon's Capacity Theorem

- In 1948 Claude Shannon fine-tuned and generalized Nyquist's result for noisy channels:

$$\text{Max data rate} = H \log_2(1 + S/N) \text{ bps}$$

$H$  = bandwidth

$S/N$  = signal to noise ratio. Given in  $10 \log_{10}$  units called **decibels** dB.

E.g. a 3kHz filtered channel with thermal noise ratio of 30 dB cannot transmit binary signals at a rate exceeding 30000 bps

Physical Layer 13

## Guided Transmission Data

- Magnetic Media
- Twisted Pair
- Coaxial Cable
- Fiber Optics

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## (unshielded) Twisted Pair



(a)



(b)

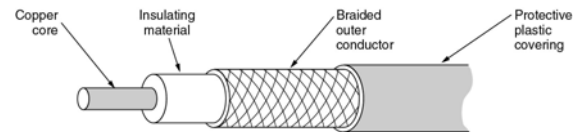
(a) Category 3 UTP - up to 16 MHz bandwidth.

(b) Category 5 UTP - up to 100 MHz bandwidth.

Upcoming are category 6 and 7 with bandwidths of 250 MHz and 600 MHz respectively.

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## Coaxial Cable



A coaxial cable.

- Bandwidth of 1GHz
- Today used for cable TV and MANs

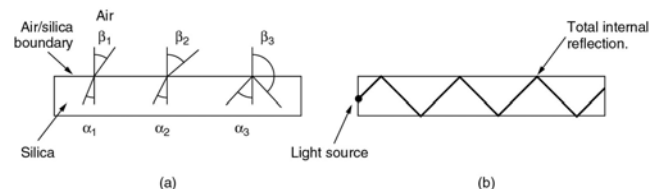
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## Fiber Optics

- In the last 20 years computing speed has increased by a factor of 20 for each decade (IBM PC in 81 ran at 4.77 MHz => 2GHz in 2001)
- Semiconductors are close to their physical limit
- Data communication has gone from 56 kbps in ARPANET to 1 Gbps in 2001 and the "sky" is still the limit. That's a 125 fold increase per decade.
- Moreover the error rate has gone from  $10^{-5}$  per bit to ~zero in optical networks.
- With current fiber technology, the achievable bandwidth is much in excess of 50,000 Gbps and better materials are being found!
- Modern fibers are as clear as unpolluted air!

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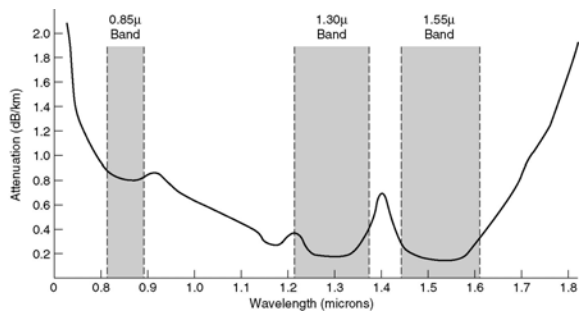
## Fiber Optics



- (a) Three examples of a light ray from inside a silica fiber impinging on the air/silica boundary at different angles
- (b) Light trapped by total internal reflection

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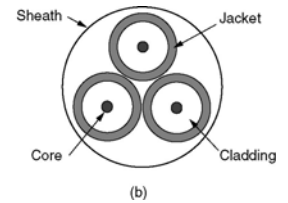
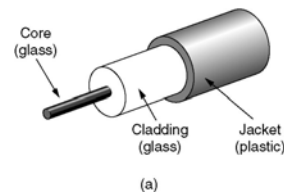
## Transmission of Light through Fiber



Attenuation of light through fiber in the infrared region.

Physical Layer 19

## Fiber Cables

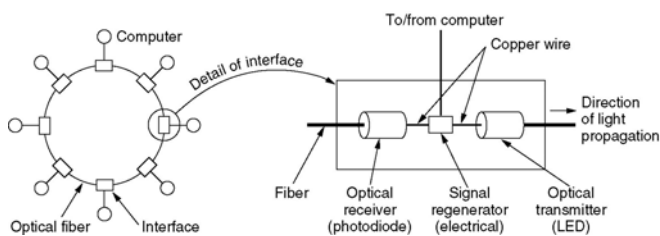


(a) Side view of a single fiber.

(b) End view of a sheath with three fibers.

Physical Layer 20

## Fiber Optic Networks



A fiber optic ring with active repeaters.

Physical Layer 21

## Fiber Optic vs. Copper

Pros of fiber optics as opposed to copper wire:

- MUCH higher bandwidth
- Low attenuation, repeaters needed every 50 Km vs. 5 Km for copper
- Not affected by power surges/failures, electromagnetic interference
- Not affected by chemicals in the air
- Thin and lightweight; 1000 twisted pair of 1 Km of copper weighs 8 ton; 2 fibers have more capacity and weigh 100 kg
- Cheaper
- Do not leak light and are difficult to tap
- Interfaces are a problem and are expensive

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