Mathematical	**Methods**	**in**	**Computer**	**Science**

Extra 1 - Do not submit

Lecturer: Nati Linial

1. Let $A, B \in M_n$ be diagonalizable. Then $AB = BA$ iff there is a matrix S such that $S^{-1}AS$ and $S^{-1}BS$ are both diagonal.
2. A is called a normal matrix if $A^{\star}A = AA^{\star}$. Show that A is normal iff there is a unitary matrix U such that $A = U^{\star}DU$ for a diagonal matrix D.
3. A is called hermitian if $A = A^{\star}$. Show that the eigenvalues of a Hermitian matrix A are real.
4. **Perron-Frobenius implementation:** Let G be a d-regular graph on n vertices. Let A_G be its adjacency matrix: $(a_{i,j}) = 1$ iff (i, j) is an edge in G.
   (a) Show that d is an eigenvalue of A_G. Give an eigenvector with eigenvalue d.
   (b) Show that the multiplicity of d is 1 iff G is connected.
   (c) For a connectd graph G: Show that $-d$ is an eigenvalue iff G is bipartite.