Digital Communication in the Modern World

Physical Layer: Fourier Series; Physical Media

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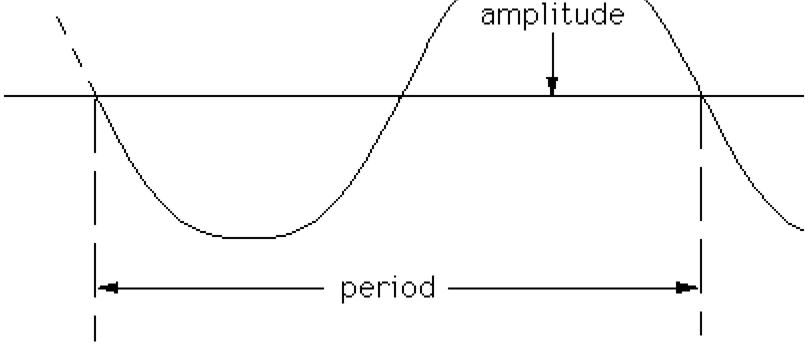
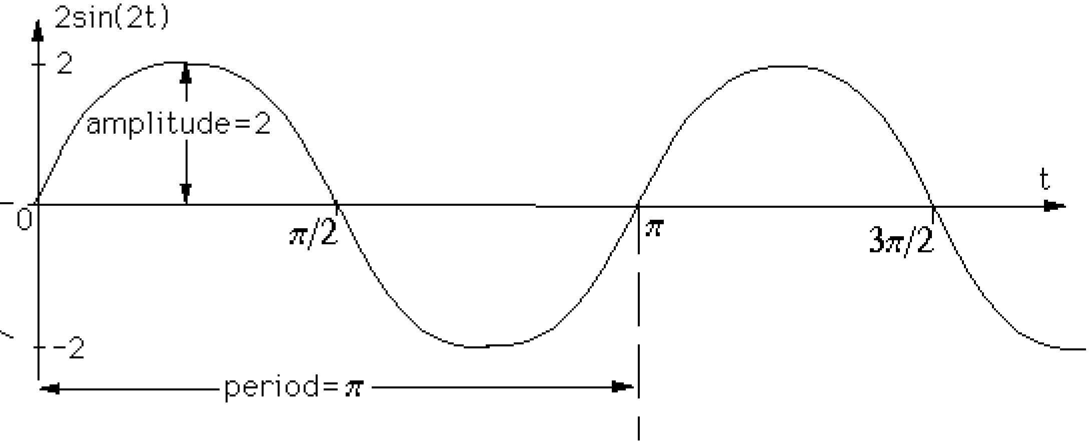
The Theoretical Basis for Data Communication

- Fourier Analysis
- Bandwidth-Limited Signals
- Maximum Data Rate of a Channel

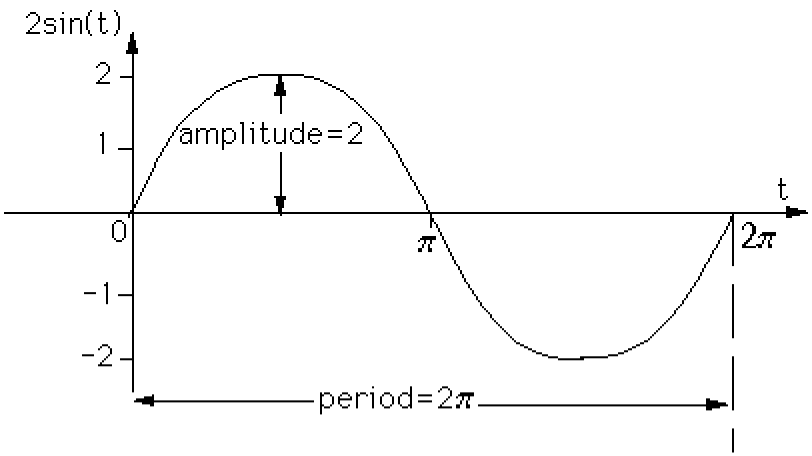
Harmonic Analysis



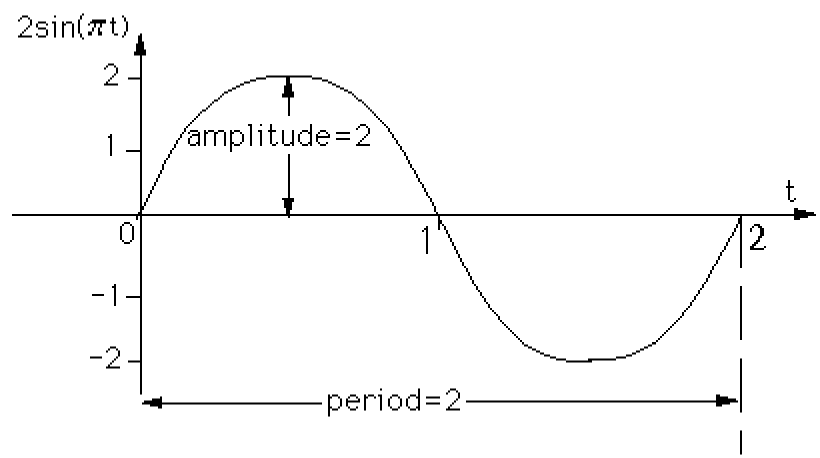
a) *sin(t)*



c) *2sin(2t)*



b) *2sin(t)*



d) *2sin(πt)*

Harmonic Analysis

In the general case:

$$A_n \cdot \sin(n \cdot \frac{2\pi \cdot t}{T})$$

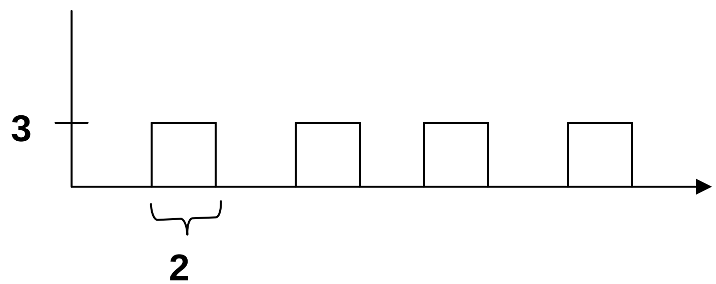
n – the harmonic number

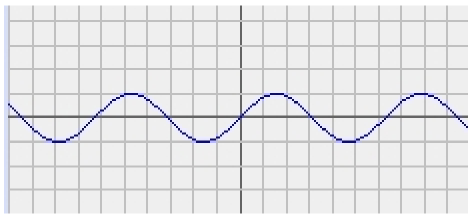
A_n – the amplitude of the n^{th} harmonic

2π – normalizes the function so that for every unit *t* it completes one period

1/T – the base frequency, normalizes the function so that for every harmonic *n* the frequency becomes *n* times the base frequency

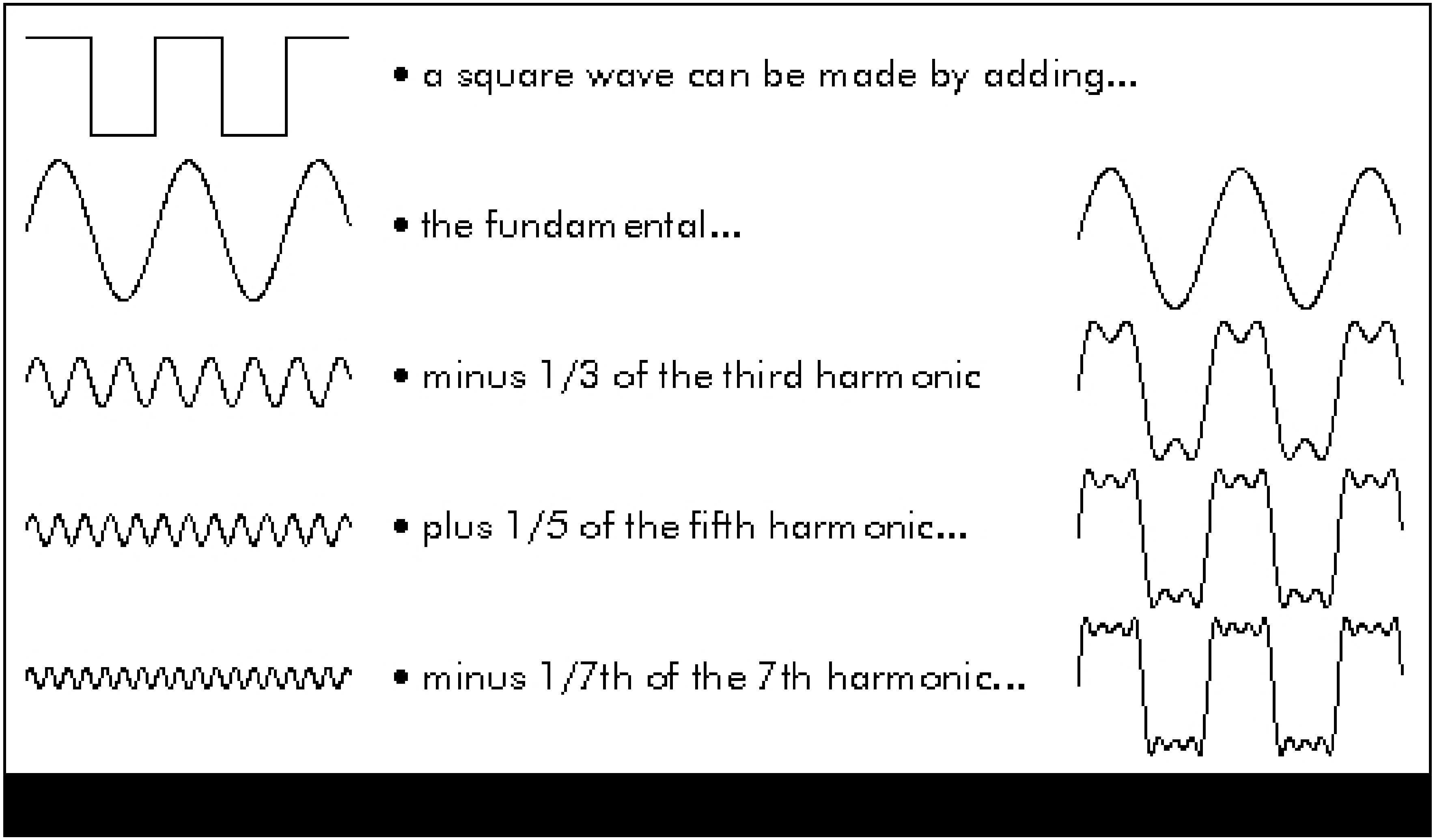
Example: the function





can be *<u>roughly</u>* approximated by *3·sin(2πt/4)*, which is called the **fundamental 1st harmonic** of our function

Harmonic Analysis



Fourier Series Theorem

- For a "sufficiently well-behaved" function of time, *g(t)*:

$$g(t) = \tfrac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(n \cdot \frac{2\pi \cdot t}{T}) + \sum_{n=1}^{\infty} b_n \cos(n \cdot \frac{2\pi \cdot t}{T})$$

- Using

$$e^{it} = \cos(t) + i\sin(t)$$

The Fourier Series gets the form:

$$g(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{i \cdot n \frac{2\pi t}{T}}$$

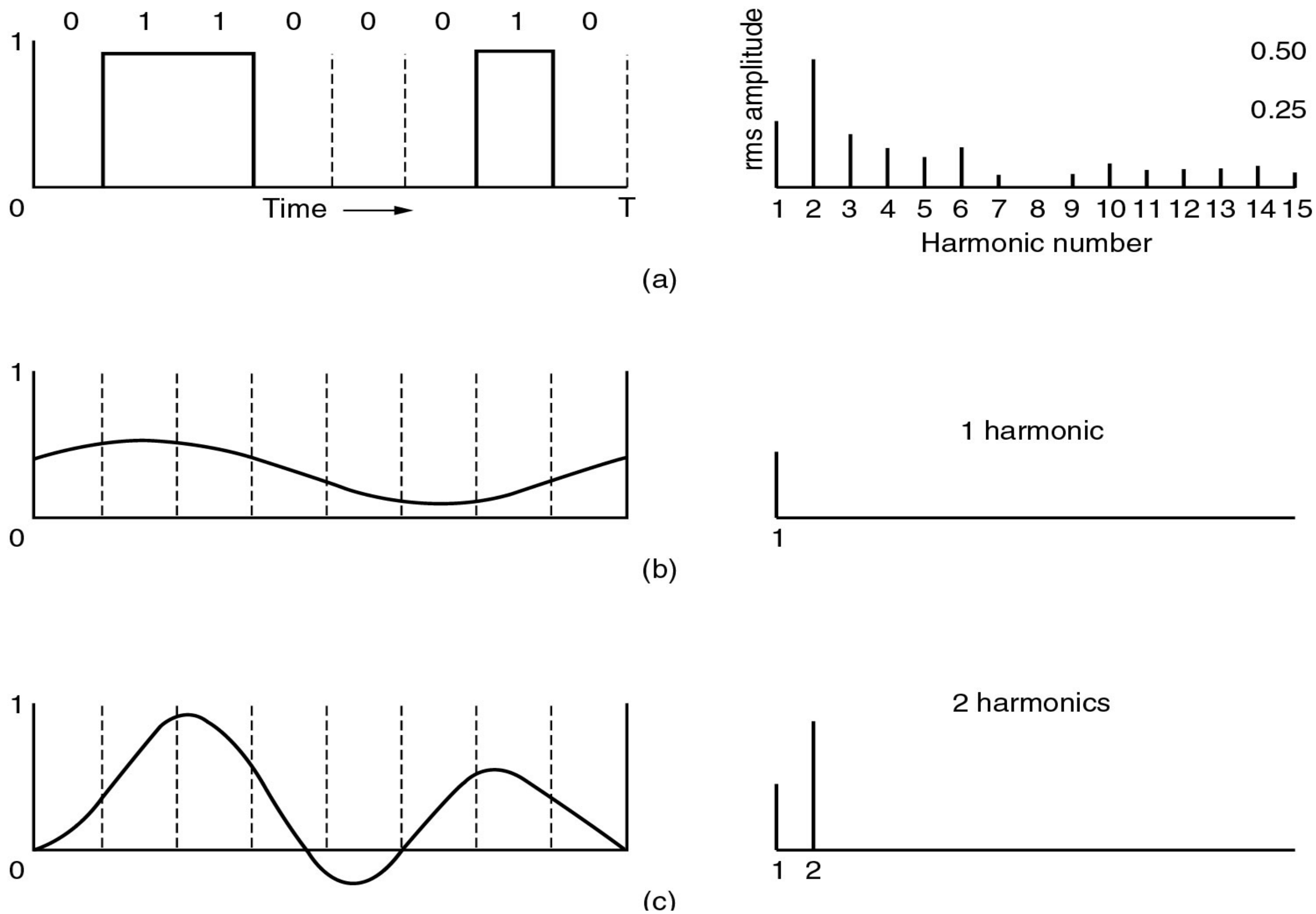
- Calculating the coefficients; denote *f:=1/T*:

$$a_n = \frac{2}{T}\int_0^T g(t)\sin(2\pi n f t)\,dt \qquad b_n = \frac{2}{T}\int_0^T g(t)\cos(2\pi n f t)\,dt \qquad c = \frac{2}{T}\int_0^T g(t)\,dt$$

Bandwidth-Limited Signals

- Higher harmonic Fourier components attenuate faster
- Range of frequencies transmitted without being "strongly" attenuated is denoted the **bandwidth** of the media
- Often defined from 0 to the frequency at which half the power (amplitude) gets through per length (1 km)
- Bandwidth is a physical property of the transmission medium

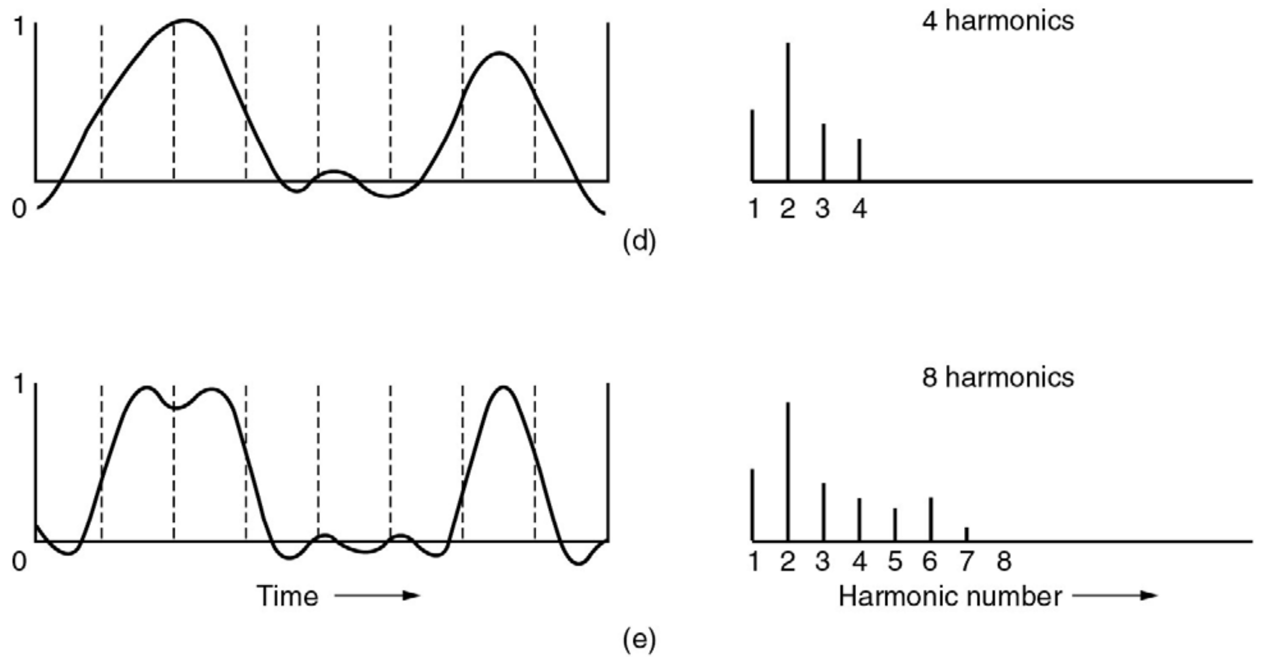
Bandwidth-Limited Signals



A binary signal and its root-mean-square Fourier amplitudes.

(b) - (c) Successive approximations to the original signal.

Bandwidth-Limited Signals (2)



(d) – (e) Successive approximations to the original signal.

Bandwidth-Limited Signals (3)

- Example: Assume you want to send 8 bits at 9600 bps over an ordinary phone line
- The time to send 8 bits is 8/9600 = 0.83 msec.
- The frequency of the first harmonic is 9600/8 = 1200 Hz (periods per second)
- Ordinary phone lines have an artificial cut-off bandwidth of 3000Hz.
- Thus the highest harmonic passed through is 3000/ 1200 = 2.5 => highest harmonic is 2!
- The signal received would be tricky to reconstruct

- $\Rightarrow$ limiting the bandwidth limits the data rate

Bandwidth-Limited Signals (4)

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Relation between data rate and harmonics for send a constant of 8 bits over a 3KHz channel.

Nyquist's Sampling Theorem

- In 1924 Henry Nyquist derived the equation for the maximum data rate for a finite bandwidth noiseless channel:

$$\text{Max data rate} = 2H \cdot \log_2 V \text{ bps}$$

H = low-pass filtered bandwidth

V = number of signal levels, for binary data $V=2$

E.g. a 3khz noiseless filtered channel cannot transmit binary signals at a rate exceeding 6000 bps

The theorem states that by making 2H samples per sec the signal can be completely reconstructed.

Shannon's Capacity Theorem

- In 1948 Claude Shannon fine-tuned and generalized Nyquist's result for noisy channels:

Max data rate = $H \cdot log_2(1+S/N)$ bps

H = bandwidth

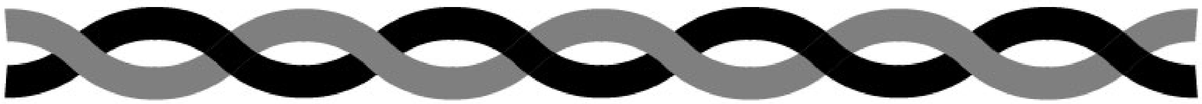
S/N = signal to noise ratio. Given in $10log_{10}$ units called **decibels** dB.

E.g. a 3khz filtered channel with thermal noise ratio of 30 dB cannot transmit binary signals at a rate exceeding 30000 bps

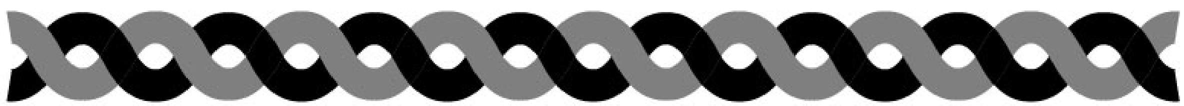
Guided Transmission Data

- Magnetic Media
- Twisted Pair
- Coaxial Cable
- Fiber Optics

(unshielded) Twisted Pair



(a)

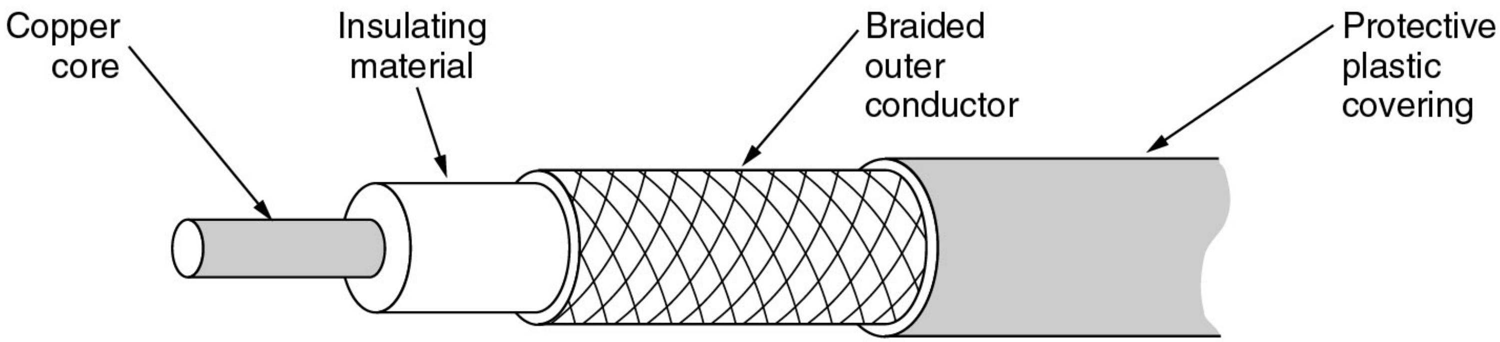


(b)

(a) Category 3 UTP - up to 16 MHz bandwidth.
(b) Category 5 UTP - up to 100 MHz bandwidth.

Upcoming are category 6 and 7 with bandwidths of 250 MHz and 600 MHz respectively.

Coaxial Cable



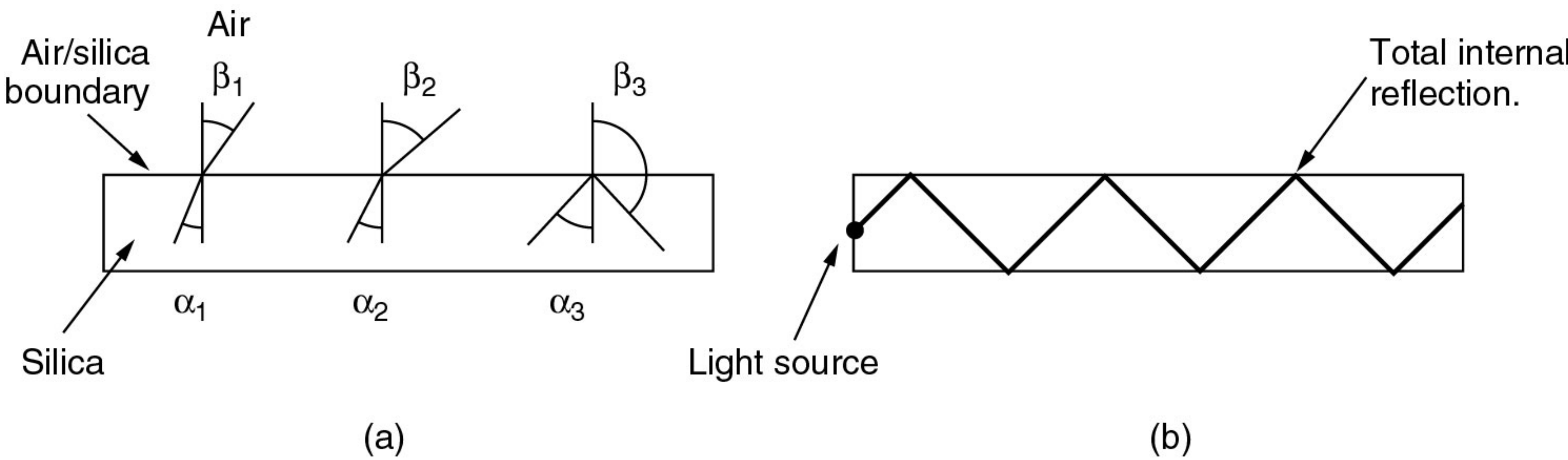
A coaxial cable.

- Bandwidth of 1GHz
- Today used for cable TV and MANs

Fiber Optics

- In the last 20 years computing speed has increased by a factor of 20 for each decade (IBM PC in 81 ran at 4.77 MHz => 2GHz in 2001)
- Semiconductors are close to their physical limit
- Data communication has gone from 56 kbps in ARPANET to 1 Gbps in 2001 and the "sky" is still the limit. That's a 125 fold increase per decade.
- Moreover the error rate has gone from 10^{-5} per bit to ~zero in optical networks.
- With current fiber technology, the achievable bandwidth is much in excess of 50,000 Gbps and better materials are being found!
- Modern fibers are as clear as unpolluted air!

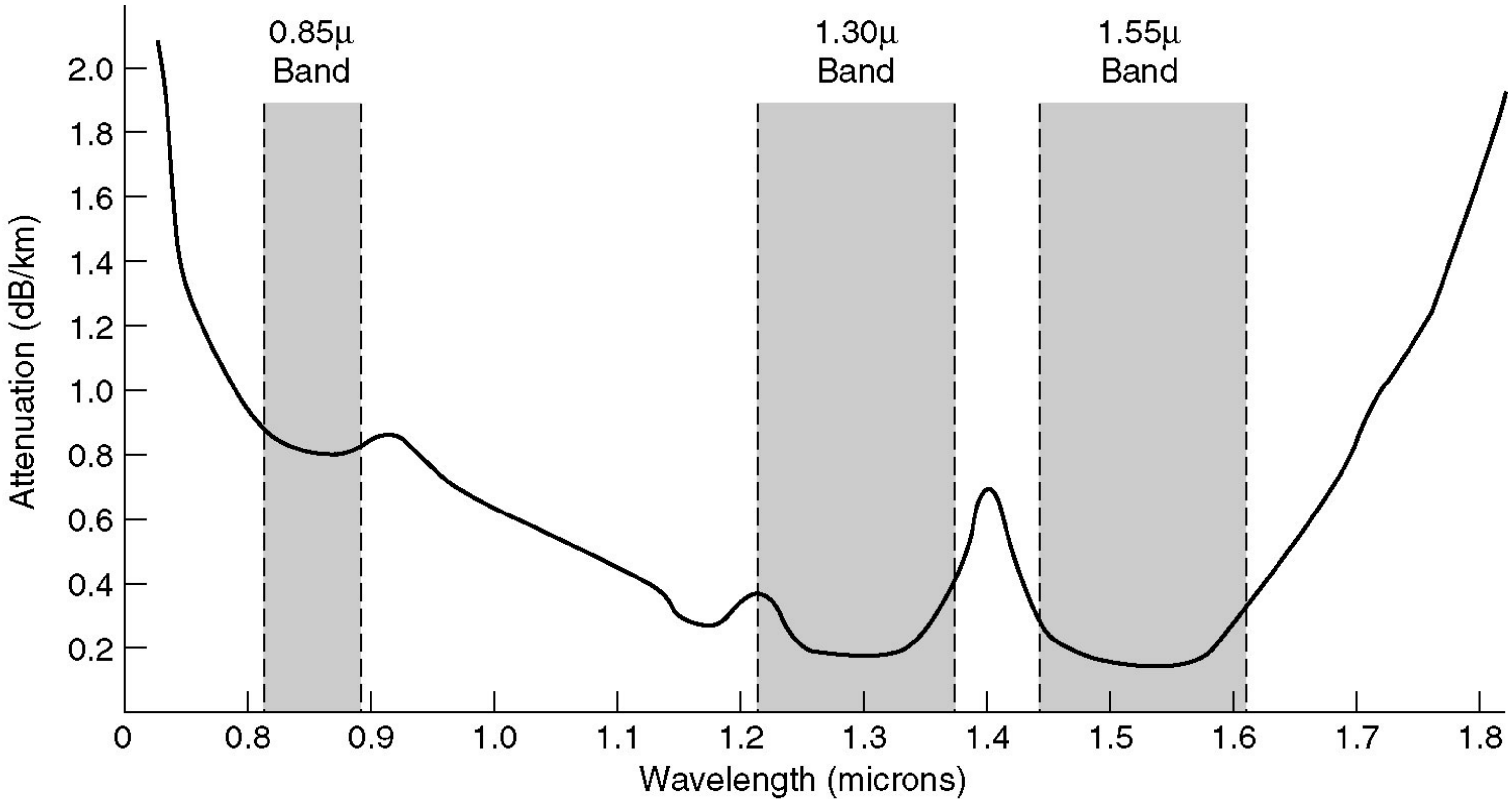
Fiber Optics



(a) Three examples of a light ray from inside a silica fiber impinging on the air/silica boundary at different angles

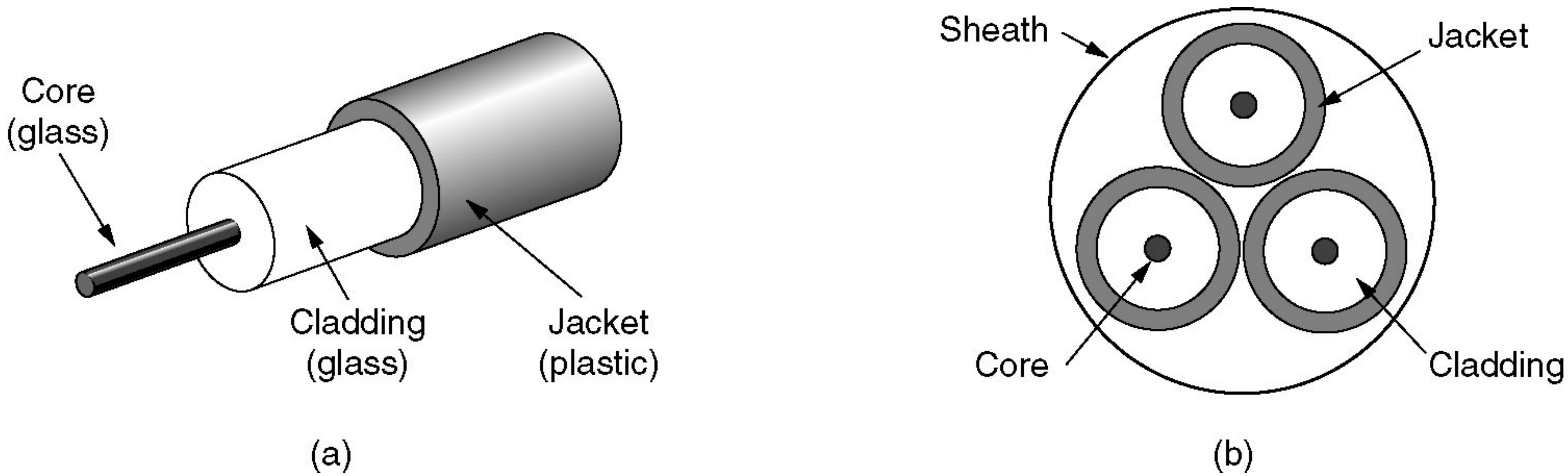
(b) Light trapped by total internal reflection

Transmission of Light through Fiber



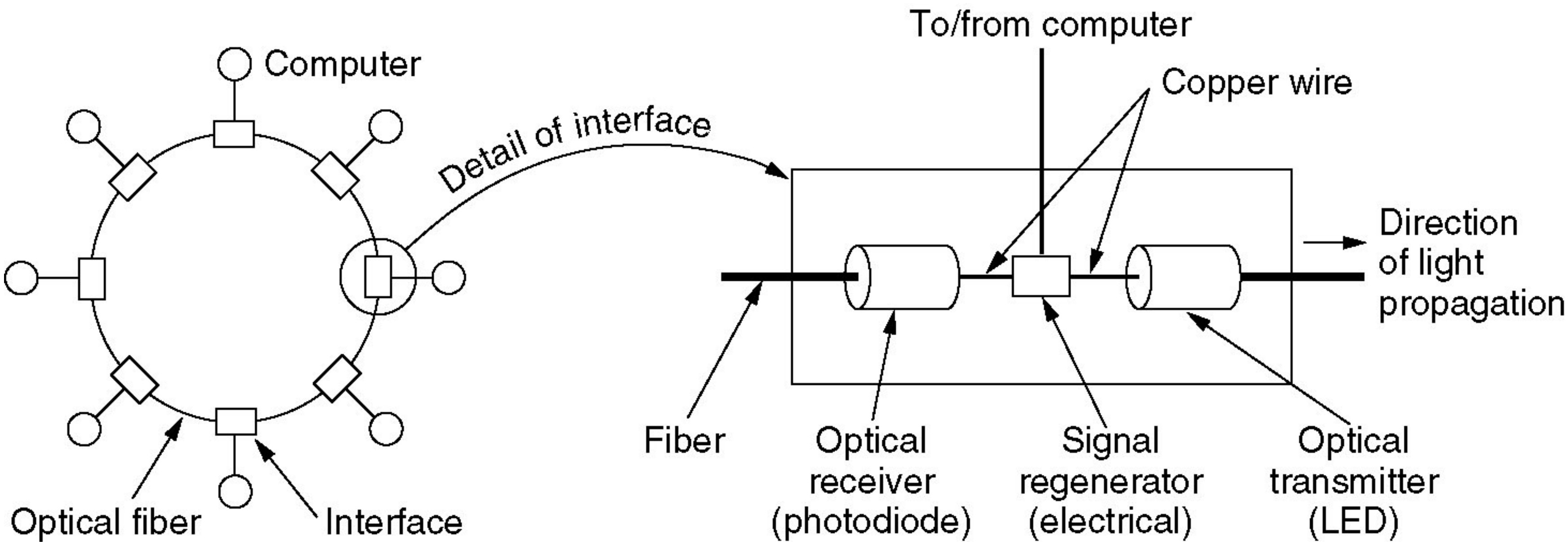
Attenuation of light through fiber in the infrared region.

Fiber Cables



(a) Side view of a single fiber.
(b) End view of a sheath with three fibers.

Fiber Optic Networks



A fiber optic ring with active repeaters.

Fiber Optic vs. Copper

Pros of fiber optics as opposed to copper wire:

- MUCH higher bandwidth
- Low attenuation, repeaters needed every 50 Km vs. 5 Km for copper
- Not affected by power surges/failures, electromagnetic interference
- Not affected by chemicals in the air
- Thin and lightweight; 1000 twisted pair of 1 Km of copper weighs 8 ton; 2 fibers have more capacity and weigh 100 kg
- Cheaper
- Do not leak light and are difficult to tap
- Interfaces are a problem and are expensive