Computing the Communication Costs of Item Allocation

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ABSTRACT

Multiagent systems require techniques for effectively allocating resources or tasks among agents in a group. Auctions are one method for structuring communication of agents' private values for the resource or task to a central decision maker. Different auction methods vary in their communication requirements. This work makes three contributions to the understanding the types of group decision making for which auctions are appropriate methods. First, it shows that entropy is the best measure of communication bandwidth used by an auction in messages bidders send *and* receive. Second, it presents a method for measuring bandwidth usage; the dialogue trees used for this computation are a new and compact representation of the probability distribution of every possible dialogue between two agents. Third, it presents new guidelines for choosing the best auction, guidelines which differ significantly from recommendations in prior work. The new guidelines are based on detailed analysis of the communication requirements of Sealed-bid, Dutch, Staged, Japanese, and Bisection auctions. In contradistinction to previous work, the guidelines show that the auction that minimizes bandwidth depends on both the number of bidders and the sample space from which bidders' valuations are drawn.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

General Terms

Design

Keywords

auctions, communication, dialogue

1. INTRODUCTION

Multiagent system designers can achieve significant cost savings by making the correct choice of algorithm for team decision making. We show that no single auction type minimizes bandwidth usage for all team sizes or for all possible valuations for the resource.

A Sealed-bid auction requires each bidder and the auctioneer to exchange 5 bits of information in a system with 60 agents where each agent's valuation is drawn independently and uniformly from one of 32 bins (labeled $1 to $32). A Dutch auction requires an exchange of approximately one bit on average under the same assumptions. While sacrificing no team decision quality, a system designer could save over 80 percent of its communication bandwidth just by implementing a different set of auction rules.

We make three main contributions to the understanding of communication for decision making in multiagent systems. First, we argue for entropy as the metric of communication bandwidth used by all messages exchanged. Second, we provide details of a three-step method for measuring bandwidth used by an algorithm. Third, we apply the analysis to Sealed-bid, Dutch (descending), Japanese (ascending), Staged (ascending), and Bisection auctions and provide recommendations for system designers that wish to choose the auction that minimizes communication bandwidth. Rauenbusch [3] provides further details of the auctions listed and our approach.

Our recommendations differ from those of economists and computer scientists. Economic analysis typically ignores communication costs entirely. Some computer scientists [5] have focused on preference revelation, which concerns the willingness to disclose information. They consider only those messages sent from a bidder to an auctioneer and ignore message sent in the opposite direction. Some researchers [2] have used communication complexity or other metrics that assume a particular message encoding. Their results may be misleading for measuring bandwidth requirements in systems that employ more efficient encodings—our results are coding-independent.

2. ITEM ALLOCATION AND AUCTIONS

A single-item allocation problem is characterized by a group of n bidder agents and an auctioneer that possesses a single, atomic item. Each bidder has a value for the item that is private and drawn independently and uniformly from 2^R bins labeled 0 to $2^R - 1$ inclusive. The distribution from which each bidder's value is drawn is common knowledge. Bidder i's value is denoted by x_i. The auctioneer's goal is to allocate the item to the bidder with the highest value. If there is a tie for the highest value, the item may be allocated to any of the bidders with the highest value. A *solution* to a single-item allocation problem is the index i, where x_i is the maximum value among all n bidders.

3. ENTROPY: METRIC FOR MEASURING COMMUNICATION

A metric for measuring communication is required to compare auction rules by their communication cost. In each auction, information is exchanged between the auctioneer and each bidder by sending and receiving messages. In any implementation of an auction, the center and the bidders must agree to an encoding of messages. Measuring information required by a multiagent algorithm using a particular encoding for messages may produce misleading results. Work in Information Theory [4, 1] has shown that the entropy of a random variable describing a message is a lower bound on the average size of the encoding for that message. Rather than evaluate an algorithm using a particular encoding, we therefore use entropy to measure expected information communicated.

Direction of Communication. It is convenient to distinguish between *coordination messages*, those sent by the auctioneer to a bidder, from *revelation messages*, those sent by the bidder to the auctioneer. In our work, the communication costs associated with coordination and with revelation are considered when calculating the expected information transmitted in an auction. In particular, the results provided are for the sum of coordination and revelation costs. This assumption is supported by Internet-like computer networks in which increased bandwidth requires costs associated with increased infrastructure for both directions of communication.

In a Sealed-bid auction, each bidder always reveals its value. Therefore, Sealed-bid auctions have the highest bandwidth requirements for revelation messages. Our results show that it would be misleading to rely on revelation messages alone when choosing an auction. Even though Sealed-bid auctions require more information transmitted in revelation than any other auction, they require no coordination. For that reason, they have low communication requirements in settings with small groups of bidders and small bins from which bidders' values are drawn.

Dialogue Trees. We developed a three-step process that an analyst may use to measure communication in an auction using a data structure named dialogue trees. In the first step, the analyst builds a dialogue tree that represents all possible sequences of messages exchanged between the auctioneer and each bidder. In the second step, the edges of the dialogue tree are labeled with the probability associated with each message. Finally, in the third step, the expected information in the dialogue is calculated using the tree representation.

A *dialogue tree* is a tree data structure with labeled edges. Each node in the tree represents a message. A label on an edge between a parent and child node indicates the receiver's belief, prior to receiving the message, that the message represented by the child node is the one that the sender will send.

Once the dialogue tree for an auction is constructed, the expected information in the dialogue (which is a measure of the expected communication requirements of an auction) is calculated. In the calculation, edge labels serve two roles: (1) in calculating the information content of a node, and (2) in calculating the probability of the message in a node being sent. The edge labels define a probability distribution over the sample space represented by the children. The probability distribution and sample space together define a probabilistic model for messages in a dialogue.

4. RESULTS

Figure 1 indicates the algorithm that has lowest expected information transmitted for increasing numbers of bidders and for increasing numbers of bins. It clearly shows that choosing the algorithm that needs least expected information transmission is highly dependent on the two parameters of the environment. For large numbers of bidders and bins, Bisection requires the least communication. Sealed-bid, Dutch, and Staged auctions each require the least communication for particular parameter settings.

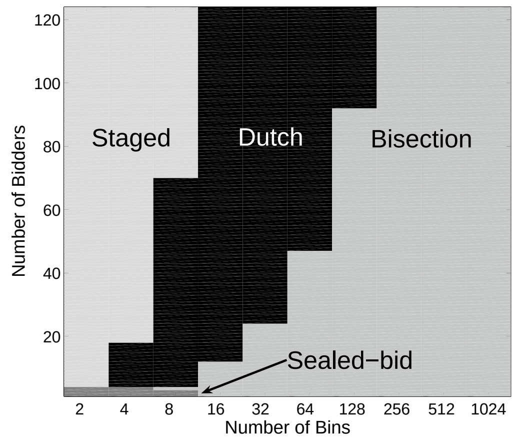

Figure 1: Algorithm with lowest expected information transmitted for varying numbers of bidders and bins

For a very small number of bidders and bins (fewer than five bidders with two or four bins, and fewer than three bidders with eight bins) the Sealed-bid auction performs best. A sealed-bid auction by definition requires maximum revelation and no coordination. Therefore, for very small problems, the savings in revelation from any other auction method are outweighed by the cost of coordination. When there are two bins, the Japanese auction has the same communication properties as the Sealed-bid auction because the first and only query in the Japanese auction is always sent and the bidder reveals its value (by its response that indicates whether its value is in the higher or lower bin).

5. FUTURE WORK

In future work, we plan to use dialogue trees to analyze algorithms for more general team decision problems than single-item assignment and for more general algorithms than auctions. Auctions are commonly suggested for item or task assignment in multiagent systems because they are a convenient method for structuring communication between agents. We plan to compare other methods for allocating a single item, such as inter-agent exchange, to auctions.

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