Reducing Coalition Structures via Agreement Specification

Guido Boella
Dip. di Informatica
Università di Torino
Italy
guido@di.unito.it

Luigi Sauro
Dip. di Informatica
Università di Torino
Italy
sauro@di.unito.it

Leendert van der Torre
CWI Amsterdam
and Delft Univ. of Technology
The Netherlands
torre@cwi.nl

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multiagent systems

General Terms

MAS theory

Keywords

Coalition formation, power theory

1. INTRODUCTION

The concept of power plays an important role in the social sciences and Castelfranchi [4] emphasizes the importance of this concept for multiagent systems. In [2] we build upon this work by distinguishing four viewpoints on multiagent systems: a mind structure, a power structure, a dependence structure and a coalition structure. These viewpoints are increasingly abstract conceptualizations of systems as collections of autonomous cognitive agents. In [1] we propose a way to define coalition structures from power structures. In this paper we refine this approach using task based power views, and we relate it to central notions in game theory.

2. TASK BASED POWER STRUCTURE

A power structure is a more abstract conceptualization of a multiagent system than the usual one, because it does not mention actions or capabilities of agents. It directly characterizes the power of agents as the goals they can achieve. In this paper we define task-based power structures.

A task based power structure is composed of a set of agents *Ag*, a set of all goals G, a set of all tasks T, a function *goals* that associates with each agent the subset of goals G it desires to achieve, and, finally, a function *power* that associates with a task assignment $\tau \subseteq A \times T$ the sets of goals the task assignment achieves.

In order to define game theoretical notions of on task based power structures, we also say that a task based power structure is super-additive when, given two disjoint sets of agents that separately can achieve respectively the sets of goals G_1 and G_2 by means of the task assignments τ_1 and τ_2, joined can achieve $G_1 \cup G_2$ by means of $\tau_1 \cup \tau_2$.

DEFINITION 1. *A task based power structure is a tuple*

$$PS = \langle Ag, G, T, goals\colon Ag \to 2^G, power\colon 2^{Ag\times T} \to 2^G\rangle$$

where Ag is a set of agents, G a set of goals, T a set of tasks, goals is a function that associates with each agent in Ag the subset of goals G it desires to achieve, and power is a partial function that associates with a task assignment $\tau \subseteq Ag \times T$ a set of goals that the task assignment achieves. Denoting with agents(τ) the set of agents involved in τ, i.e.

$$agents(\tau) = \{a \in Ag \mid \exists t \in T : \ (a,t) \in \tau\}$$

We say that a task based power structure is super-additive in the case that, given τ_1 and τ_2 such that agents(τ_1) $\cap$ agents(τ_2) $= \emptyset$, if $G_1 \in$ power(τ_1) and $G_2 \in$ power(τ_2), then we have $G_1 \cup G_2 \in$ power($\tau_1 \cup \tau_2$).

The more abstract power function we defined in [2] $power' : 2^{Ag} \to 2^{2^G}$ can be derived from our task based power function by $power'(A) =$

$$\{G \mid \exists\tau \subseteq Ag \times T : A = agents(\tau) \text{ and } G = power(\tau)\}$$

This more abstract power structure has been called a qualitative coalitional game by Dunne and Wooldridge [7], such that we could call our task based power structure also a task based qualitative coalitional game. In their terminology, a coalition is a set of agents, and the power function is called the characteristic function.

3. DO UT DES

There are many task assignments, but most of them will never be considered by rational agents. This raises the question when a task assignment is dominated by another task assignment, such that we can restrict our attention to task assignments which are not dominated. For example, when a task assignment contains an agent which only profits from the coalition, but which does not contribute to it, then the other agents may prefer to form the same coalition without this agent.

A general principle we introduce is the do-ut-des property, which literally says "give something to obtain something else". We consider a cost-benefit analysis in which the costs are the tasks an agent has to perform, and the benefits are the goals of an agent that will be achieved.

DEFINITION 2. *Let $PS = \langle Ag, G, T, goals, power\rangle$ be a task based power structure, and let $\tau \subseteq Ag \times T$ be a task assignment.*

The benefits and costs of the task assignment τ for agent $a \in Ag$ are respectively defined as follows.

$$\textit{benefits}(\tau, a) = \textit{goals}(a) \cap \textit{power}(\tau)$$

$$\textit{costs}(\tau, a) = \{t \in T \mid (a, t) \in \tau\}$$

A task assignment τ_1 is dominated by a contained task assignment $\tau_2 (\subset \tau_1)$ if either the benefits are larger, or the cost are less - all else being equal - for all the agents involved in τ_2 and for an agent involved in τ_1 at least one of the relations is strict.

DEFINITION 3. *Let $PS = \langle Ag, G, T, goals, power\rangle$ be a task based power structure, let $\tau_1, \tau_2 \subseteq Ag \times T$ be two task assignments, and let τ_1 be at least as good as τ_2 for agent $a \in Ag$ if $\textit{benefits}(\tau_2, a) \subseteq \textit{benefits}(\tau_1, a)$ and $\textit{costs}(\tau_1, a) \subseteq \textit{costs}(\tau_2, a)$. A task assignment τ_1 is dominated iff there exists a task assignment $\tau_2 \subset \tau_1$ such that*

1. *τ_2 is at least as good as τ_1 for all agents $a \in \textit{agents}(\tau_2)$, and*
2. *τ_2 is strictly better than τ_1 for at least one of the agents $a \in \textit{agents}(\tau_1)$.*

We call task assignments which are non dominated do-ut-des task assignments. They can be interpreted as a kind of possible agreements or contracts among the agents (though the notion of contract is much simpler than the notion of contract studied in some of our other work [3]). The power structure in which only do-ut-des task assignments occur in the function *power*, may be called a coalition structure, see [2, 1].

4. BALANCING GOALS

Many task assignments are incomparable. For example, one task assignment may have one goal as its benefits, and another task assignment may have another goal as its benefits. Likewise, a task assignment may have one task as its costs, and another task assignment may have another task as its costs. This is analogous to the comparison of multiple objectives in multiple criteria decision making [5].

In this section we show under which hypothesis the do-ut-des property can be related with the game theoretical notion of *core* as defined by Osborne and Rubinstein in [6] for super-additive coalitional games. Therefore in the following we consider only super-additive task based power structures and we assume that a utility function $u(a, \tau)$ associates with each agent *a* and task assignment τ a real value representing the profitability of τ for *a*. We also assume that $u(a, \tau)$ is a trade-off function $f(\textit{benefits}(a, \tau), \textit{costs}(a, \tau))$, this assumption is used in the proof of the Theorem 1, even if for space reason we do not provide it here.

DEFINITION 4. *A task based power structure with utilities PS is a tuple $\langle Ag, G, T, goals: Ag \to 2^G, power: 2^{Ag\times T} \to 2^G, u: Ag \times 2^{Ag\times T} \to I\!R\rangle$, where $\langle Ag, G, T, goals: Ag \to 2^G, power: 2^{Ag\times T} \to 2^G\rangle$ is a task based power structure and $u : Ag \times 2^{Ag\times T} \to I\!R$ is a utility function on $\textit{benefits}(a, \tau)$ and $\textit{costs}(a, \tau)$, $u(a, \tau) = f(\textit{benefits}(a, \tau), \textit{costs}(a, \tau))$.*

The core is a dominance criterion over the set of all possible task assignments $2^{Ag\times T}$. In our context we adapt its definition as follows:

DEFINITION 5. *Let $PS = \langle Ag, G, T, goals, power, u\rangle$ be a task based power structure, a task assignment τ is in the core iff there does not exist an agent a such that $u(a, \tau) < u(a, \emptyset)$ and there does not exist a $\tau' \neq \emptyset$ such that for all $a \in \textit{agents}(\tau')$, $u(a, \tau) < u(a, \tau')$.*

We define a quantitative version of the qualitative do-ut-des property, called gt-do-ut-des optimality, and we use it to relate the do-ut-des property with the notion of core. Informally, a task assignment τ is gt-do-ut-des optimal if there does not exist another task assignment τ' such that all the agents involved in τ' earns by τ' at least the same as by τ and there exists at least an agent that earns by τ' more than by τ.

DEFINITION 6. *We say that a task assignment τ is gt-do-ut-des dominated iff there exists a τ' such that*

- *for all $a \in \tau'$, $u(a, \tau) \leq u(a, \tau')$ and*
- *there exists an agent a such that $u(a, \tau) < u(a, \tau')$*

A task assignment is gt-do-ut-des optimal if it is not gt-do-ut-des dominated by any task assignment.

It can be shown that all do-ut-des optimal task assignments are in the core.

Now we define which conditions the function *u* has to satisfy in order to relate the do-ut-des property with do-ut-des optimality.

DEFINITION 7. *A utility function u is do-ut-des compatible iff τ_2 is strictly better than τ_1 for an agent a implies that $u(a, \tau_1) < u(a, \tau_2)$.*

The following theorem relates the do-ut-des task assignments with the gt-do-ut-des optimal task assignments.

THEOREM 1. *Given a do-ut-des compatible utility function u, the following properties hold*

- *if τ is gt-do-ut-des optimal, then τ is do-ut-des.*
- *if τ is not gt-do-ut-des optimal, then there exists a do-ut-des τ' that gt-do-ut-des dominates τ.*

Due to the previous theorem, in order to find gt-do-ut-des optimal task assignments, it is possible first to reduce the search space to the coalition structure of all the do-ut-des task assignments and hence only in that coalition structure to verify gt-do-ut-des optimality. This is worthwhile, because to check if a task assignment τ is do-ut-des you have to consider only the task assignments such that $\tau' \subset \tau$ and not all the possible task assignments τ' as in Definition 6. Once found a gt-do-ut-des optimal task assignment we know that it is also in the core.

5. REFERENCES

[1] G. Boella, L. Sauro, and L. van der Torre. Abstraction from Power to Coalition Structures. In *Procs. of ECAI'04*, 2004.

[2] G. Boella, L. Sauro, and L. van der Torre. Social Viewpoints on Multiagent Systems. In *Procs. of AAMAS'04*, 2004.

[3] G. Boella and L. van der Torre. A game theoretic approach to contracts in multiagent systems. *IEEE Trans. SMC, Part C*, to appear.

[4] C. Castelfranchi. The Micro-Macro Constitution of Power. *ProtoSociology*, 18-19, 2003.

[5] R. Keeney and H. Raiffa. *Decisions with Multiple Objective*. John Wiley & Sons, Inc., 1976.

[6] M. Osborne and A. Rubinstein. *A Course in Game Theory*. MIT Press, 1994.

[7] M. Wooldridge and P. Dunne. On the computational complexity of qualitative coalitional games. *Artificial Intelligence*, 158(1):27–73, 2004.