Towards a Formal Model for Task Allocation via Coalition Formation

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ABSTRACT

This paper focuses on the problem of generating *coalition structures* for task allocation via coalition formation. It provides a unified formal framework for constructing those coalitions structures. The framework takes as input a set of *coalitions* whose structures are abstract, a *conflict* relation between the coalitions, and a preference relation between the coalitions and returns the coalitions structures. Three semantics for coalitions structures will be proposed: a *basic* semantics which will return a unique coalition structure, *stable* semantics and *preferred* semantics. These two last may return several coalitions structures at the same time.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Coherence and coordination

General Terms

Human Factors, Theory

Keywords

Coalition formation

1. INTRODUCTION

Generally, to perform complex tasks in multi-agent environments, agents need to form *coalitions* which are temporary associations between agents in order to carry out joint tasks. As argued in [1, 3, 4, 5, 6], task allocation via coalition formation follows a three steps process: i) generating the *coalition structures*. The idea here is to form the coalitions such that agents within a coalition should coordinate to achieve a task (or a set of tasks), but those in different coalitions do not. ii) Discussing these structures between the agents in order to select the one which will be adopted. iii) Distributing the gain between the agents of each coalition in the coalitions structure. The way in which the coalitions structures are generated depend broadly on the studied problem. In some applications, for example, it may be required that the tasks are independent, or that a single agent should belong only to one coalition at the same time.

Inspired from work on argumentation theory, particularly the system developed in [2], this paper provides a *unified* and *general* formal framework for generating the coalitions structures. That framework is defined in terms of a set of *coalitions*, a *conflict* relation between these coalitions and finally a *preference* relation between the coalitions. The framework returns three semantics of coalition structures: the *basic* semantics which returns a unique coalitions structure, *stable* semantics and *preferred* semantics which are two different refinements of the basic one and may return several coalitions structures at the same time.

2. FORMAL FRAMEWORK

Task allocation via coalition formation can be defined as a finite set $\mathcal{N}$ of agents who should achieve a finite set $\mathcal{T}$ of tasks. Each agent aims at maximizing its own satisfaction and also the satisfaction of the whole multi-agent system in which it is a member.

DEFINITION 1 (FORMAL FRAMEWORK). *A* framework for generating coalition structures *(FGS) is a triplet* $<\mathcal{C}, \mathcal{R}, \succ>$ *where* $\mathcal{C}$ *is a set of coalitions,* $\mathcal{R}$ *is a binary relation representing a* defeat *relationship between coalitions,* $\mathcal{R} \subseteq \mathcal{C} \times \mathcal{C}$, *and* $\succ$ *is a (partial or complete) preordering on* $\mathcal{C}$.

DEFINITION 2. *Let* $C_1, C_2 \in \mathcal{C}$, *and* $S \subseteq \mathcal{C}$. C_1 attacks C_2 *iff* $C_1 \ \mathcal{R} \ C_2$ *and not* ($C_2 \succ C_1$). S *is* conflict-free *iff* $\nexists \, C_1, C_2 \in S$ *s.t* C_1 *attacks* C_2. S defends C_1 *iff for all* C_2 *such that* C_2 *attacks* C_1, *then there is* $C_3 \in S$ *s.t* C_3 *attacks* C_2.

Let's define the basic coalition structure. Intuitively, it is clear that a non-attacked coalitions will belong to the coalition structure. $\mathcal{C}_{Att}$ gathers all such coalitions. This notion is very restrictive since it discards coalitions which appear "good".

DEFINITION 3 (COALITIONS STRUCTURE). *Let* $<\mathcal{C}, \mathcal{R}, \succ>$ *be a finitary FGS. The* basic coalitions structure *is:*
$\underline{\mathcal{S}}_{\mathcal{R},\succ} = \bigcup \mathcal{F}^{i>0}(\emptyset) = \mathcal{C}_{Att} \cup [\bigcup \mathcal{F}^{i\geq 1}(\mathcal{C}_{Att})]$,
where $\mathcal{F}(S) = \{C \in \mathcal{C} \mid C$ *is defended by* $S\}$.

In some cases, the set $\underline{\mathcal{S}}_{\mathcal{R},\succ}$ may be empty. This is not always desirable in multi-agents applications. In order to palliate this problem, we define the *stable* structures and the *preferred* ones.

DEFINITION 4 (STABLE STRUCTURES). *Let* $<\mathcal{C}, \mathcal{R}, \succ>$ *be a FGS, and* $S \subseteq \mathcal{C}$. S *is a* stable structure *iff* S *is conflict-free and* S *defeats any coalition which is not in* S.

A framework FGS may have several stable structures. These stable structures correspond to different ways of achieving the tasks.

DEFINITION 5 (PREFERRED STRUCTURES). *Let* $<\mathcal{C}, \mathcal{R}, \succ>$ *be a FGS, and* $S \subseteq \mathcal{C}$. S *is a* preferred extension *iff* S *is conflict-free,* S *defends all its elements, and* S *is maximal (for* $\subseteq$*) among the sets satisfying the 2 above conditions.*

Note that each framework FGS has at least one preferred structure.

PROPERTY 1. *Each stable structure is also a preferred one. The reverse is not always true. Moreover, the coalition structure* $\underline{\mathcal{S}}_{\mathcal{R},\succ}$ *is included in every stable (resp. preferred) structure.*

3. ILLUSTRATION

Let's consider the problem of coalition formation described in [5]. In that work, a multi-agent system is supposed to perform a *service* that requires several *criteria* $<c_1, \ldots, c_r>$. Each agent $a_i \in \mathcal{N}$ is supposed to have non-negative *capabilities* $B^i = <b_1^i, \ldots, b_r^i>$. A capability b_j^i represents the capacity of the agent a_i regarding the criterion c_j. To each task $t \in \mathcal{T}$ a vector $B^t = <b_1^t, \ldots, b_r^t>$ of its capabilities is given. An element $b_k^{t_j}$ represents the amount of c_k necessary for its satisfaction. The function $Value$ returns the value of a given coalition. It is assumed that the tasks are independent, an agent cannot belong to more than one coalition at a time, and a coalition can work on a single task at a time.
A coalition should be *minimal* since each coalition has a cost, and the more the coalition is large, the more costly it is. Moreover, an agent cannot be in a coalition if it is not useful and it cannot help in the achievement of the task. Before giving the formal definition of a coalition, let's first define formally when a task is achievable.

DEFINITION 6. *Let* $C \subseteq \mathcal{N}$ *and* $t \in T$. C achieves *the task* t*, denoted by* $C \Vdash t$*, iff* $\forall\, 1 \leq j \leq r$, $\sum_{a_i \in C} b_j^i < b_j^t$.

The above definition says that a task is achievable by a group of agents if the capabilities of the agents taken together, are sufficient to what is required by the task.

DEFINITION 7 (COALITION). *A* coalition *is a pair* $<C, t>$ *s.t:* $C \subseteq \mathcal{N}$, $t \in T$, $C \Vdash t$, C *is minimal for (set* $\subseteq$*) among the sets satisfying the above conditions.* C *is the* support *of the coalition, and* t *its* task. $\mathcal{C}(AS)$ *is the set of all the coalitions.*

The value of a coalition may be equal to the benefit obtained from the coalition minus the cost of that coalition. For the sake of simplicity, we suppose that this value is given and it is a numerical value. The values of coalitions make it possible to compare them.

DEFINITION 8. *Let* $<C_1, t_1>, <C_2, t_2> \in \mathcal{C}(AS)$. $<C_1, t_1>$ *is more* beneficial *than* $<C_2, t_2>$*, denoted* $<C_1, t_1> \succ <C_2, t_2>$ *iff Value(*$<C_1, t_1>$*)* $>$ *Value(*$<C_2, t_2>$*).*

The coalition structures should satisfy the hypothesis already fixed when defining the problem. The first requirement is that an agent cannot belong to more than one coalition at the same time. This kind of conflict will be called here "Interfere".

DEFINITION 9 (INTERFERING COALITIONS). *Let* $<C_1, t_1>$, $<C_2, t_2> \in \mathcal{C}(AS)$. $<C_1, t_1>$ interferes *with* C_2, t_2 *iff* $C_1 \cap C_2 \neq \emptyset$.

The second requirement is that the same task cannot be affected to more than one coalition at the same time. In the coalition structure, it cannot then be the case that two coalitions achieve the same task. This requirement gives raise to another kind of conflict between coalitions. In what follows, this conflict will be called "Competition".

DEFINITION 10 (COMPETING COALITIONS). *Let* $<C_1, t_1>$, $<C_2, t_2> \in \mathcal{C}(AS)$. $<C_1, t_1>$ *is in* competition *with* $<C_2, t_2>$ *iff* $t_1 = t_2$.

The two above relations are brought together in a unique definition of defeat as follows:

DEFINITION 11. *Let* $<C_1, t_1>, <C_2, t_2> \in \mathcal{C}(AS)$. $<C_1, t_1>$ $\mathcal{R}$ $<C_2, t_2>$ *iff:* $<C_1, t_1>$ interferes *with* $<C_2, t_2>$ *or* $<C_1, t_1>$ *is in* competition *with* $<C_2, t_2>$.

The basic coalition structure of this system is: $\underline{\mathcal{S}}_{\mathcal{R},\succ} = \bigcup \mathcal{F}^{i>0}(\emptyset) = \mathcal{C}_{\mathcal{R},\succ} \cup [\bigcup \mathcal{F}^{i\geq 1}(\mathcal{C}_{\mathcal{R},\succ})]$.
The following result can be shown:

THEOREM 1. *If the agents do not misrepresent the capabilities of the others, and if they have all the same values for the different coalitions, then their respective frameworks will all return the same coalition structure. Thus, there is no need to the negotiation step.*

This result is of great importance since it shows that with such a framework, more work is done by the agents themselves, and consequently this may minimize greatly the communication which is very costly.

4. CONCLUSION

Inspired from works on argumentation theory, we have proposed a *unified*, *general* and *abstract* framework for generating coalition structures in an elegant way. The formal framework has three components: a set of coalitions, a defeasibility relation between the coalitions, and finally a preference relation between the coalitions. In this abstract framework, the notion of coalition remains an abstract entity and its exact definition depends on the studied application. Regarding the notion of defeasibility, it is induced and defined from the constraints of the application. Finally, the preference relation comes from the values that agents can assign to each coalition. We have proposed three semantics for the coalitions structures. This work is of great importance, since it allows agents to reason about the coalitions, and minimize the negotiation between agents in the second step of the coalition formation process. Moreover, this framework is general enough to capture different propositions made in the literature.

5. REFERENCES

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