Tirgul 14

(and more) sample questions

DAST 2005

Kruskal's MST Algorithm (reminder?) MST-KRUSKAL(G, w) $A \leftarrow \emptyset$ for each vertex $v \Box V[G]$ 1 do MAKE-SET(v) 3 sort the edges of E into nondecreasing order by weight w for each edge $(u, v) \Box E$, taken in nondecreasing order by weight 4 5 do if FIND-SET(u) \neq FIND-SET(v) then $A \leftarrow A \Box \{(u, v)\}$ 6 UNION(u, v) 8 9 return A DAST 2005



- Q. Let *G* be a connected undirected graph with the property that every edge has a different weight. Prove that there is only one minimum spanning tree for *G*.
- A. We assume there are two such trees and show there is a contradiction:

<u>exchange lemma:</u> Let *T* and *T'* be spanning trees in G(V,E). Given any $e' \in T' - T$ there exists an edge $e \in T - T'$ such that $(T - \{e\}) \cup \{e'\}$ is also a spanning tree <u>proof (outline)</u>: adding e' to *T* results in a cycle. Removing an edge *e* along this cycle restores the spanning tree

Now suppose there are two MSTs to G, let e' be the lightest edge in T' - T, we will add e' to T, but get a contradiction when trying to remove an edge from the resulting cycle.

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