Digital Communication in the Modern World
Physical Layer: Fourier Series; Physical Media
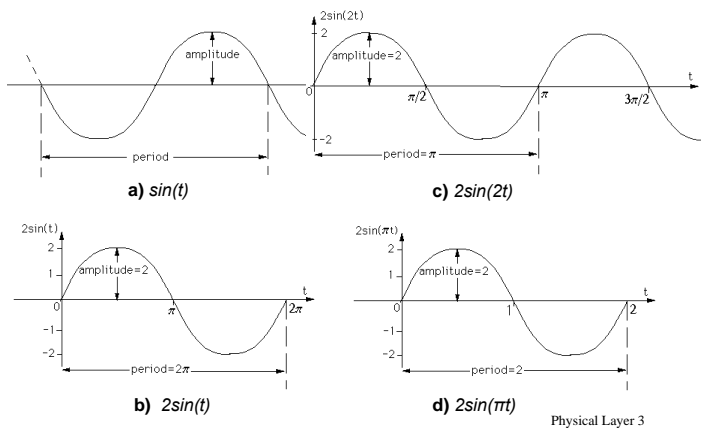
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The Theoretical Basis for Data Communication

- Fourier Analysis
- Bandwidth-Limited Signals
- Maximum Data Rate of a Channel

Harmonic Analysis

a) sin(t) **c)** 2sin(2t) **b)** 2sin(t) **d)** 2sin(πt)

Harmonic Analysis

In the general case:

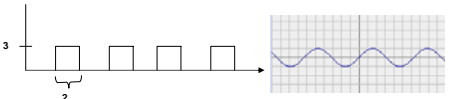$$A_n \cdot \sin(n \cdot \frac{2\pi \cdot t}{T})$$

n – the harmonic number
A_n – the amplitude of the nth harmonic
2π – normalizes the function so that for every unit t it completes one period
1/T – the base frequency, normalizes the function so that for every harmonic n the frequency becomes n times the base frequency
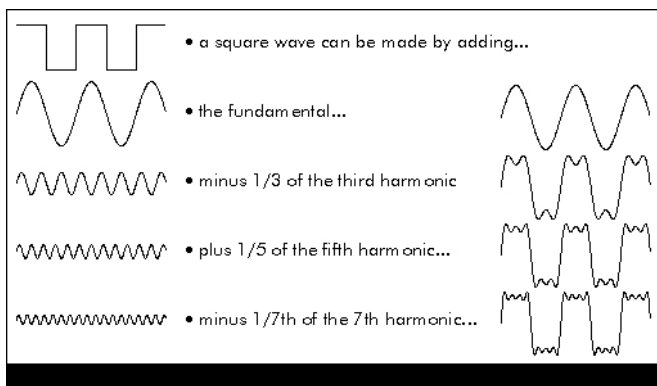
Example: the function

can be _roughly_ approximated by 3·sin(2πt/4), which is called the **fundamental 1ˢᵗ harmonic** of our function

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Harmonic Analysis

- a square wave can be made by adding...
- the fundamental...
- minus 1/3 of the third harmonic
- plus 1/5 of the fifth harmonic...
- minus 1/7th of the 7th harmonic...

Fourier Series Theorem

· For a "sufficiently well-behaved" function of time, g(t):

$$g(t) = \tfrac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(n \cdot \frac{2\pi \cdot t}{T}) + \sum_{n=1}^{\infty} b_n \cos(n \cdot \frac{2\pi \cdot t}{T})$$

· Using

$$e^{it} = \cos(t) + i\sin(t)$$

The Fourier Series gets the form:

$$g(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{i \cdot n \frac{2\pi t}{T}}$$

· Calculating the coefficients; denote f:=1/T:

$$a_n = \frac{2}{T}\int_0^T g(t)\sin(2\pi nft)\,dt \qquad b_n = \frac{2}{T}\int_0^T g(t)\cos(2\pi nft)\,dt \qquad c = \frac{2}{T}\int_0^T g(t)\,dt$$
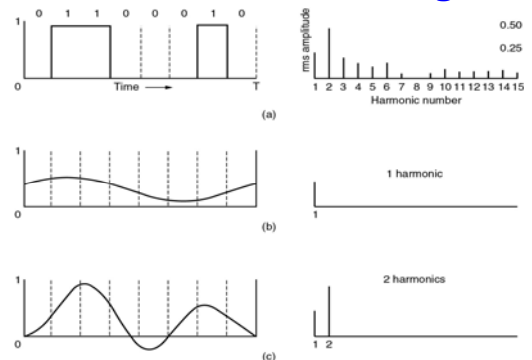
Bandwidth-Limited Signals

- Higher harmonic Fourier components attenuate faster
- Range of frequencies transmitted without being "strongly" attenuated is denoted the **bandwidth** of the media
- Often defined from 0 to the frequency at which half the power (amplitude) gets through
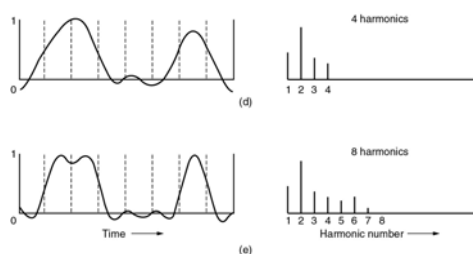- Bandwidth is a physical property of the transmission medium

Bandwidth-Limited Signals

A binary signal and its root-mean-square Fourier amplitudes.
(b) – (c) Successive approximations to the original signal.

Bandwidth-Limited Signals (2)

(d) – (e) Successive approximations to the original signal.

Bandwidth-Limited Signals (3)

- Example: Assume you want to send 8 bits at 9600 bps over an ordinary phone line
- The time to send 8 bits is 8/9600 = 0.83 msec.
- The frequency of the first harmonic is 9600/8 = 1200 Hz (periods per second)
- Ordinary phone lines have an artificial cut-off bandwidth of 3000Hz.
- Thus the highest harmonic passed through is 3000/ 1200 = 2.5 => highest harmonic is 2!
- The signal received would be tricky to reconstruct

- => limiting the bandwidth limits the data rate

Bandwidth-Limited Signals (4)

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Relation between data rate and harmonics for send a constant of 8 bits over a 3KHz channel.

Nyquist's Sampling Theorem

- In 1924 Henry Nyquist derived the equation for the maximum data rate for a finite bandwidth noiseless channel:

Max data rate = $2H log_2 V$ bps

H = low-pass filtered bandwidth
V = number of signal levels, for binary data $V=2$

E.g. a 3khz noiseless filtered channel cannot transmit binary signals at a rate exceeding 6000 bps

The theorem states that by making 2H samples per sec the signal can be completely reconstructed.

Shannon's Capacity Theorem

- In 1948 Claude Shannon fine-tuned and generalized Nyquist's result for noisy channels:

$$\text{Max data rate} = H log_2(1+S/N) \text{ bps}$$

H = bandwidth
S/N = signal to noise ratio. Given in $10log_{10}$ units called **decibels** dB.

E.g. a 3khz filtered channel with thermal noise ratio of 30 dB cannot transmit binary signals at a rate exceeding 30000 bps

Guided Transmission Data

- Magnetic Media
- Twisted Pair
- Coaxial Cable
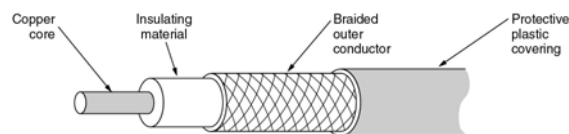- Fiber Optics

(unshielded) Twisted Pair

(a) Category 3 UTP - up to 16 MHz bandwidth.
(b) Category 5 UTP - up to 100 MHz bandwidth.

Upcoming are category 6 and 7 with bandwidths of 250 MHz and 600 MHz respectively.

Coaxial Cable

A coaxial cable.

- Bandwidth of 1GHz
- Today used for cable TV and MANs

Fiber Optics

- In the last 20 years computing speed has increased by a factor of 20 for each decade (IBM PC in 81 ran at 4.77 MHz => 2GHz in 2001)
- Semiconductors are close to their physical limit
- Data communication has gone from 56 kbps in ARPANET to 1 Gbps in 2001 and the "sky" is still the limit. That's a 125 fold increase per decade.
- Moreover the error rate has gone from 10^{-5} per bit to ~zero in optical networks.
- With current fiber technology, the achievable bandwidth is much in excess of 50,000 Gbps and better materials are being found!
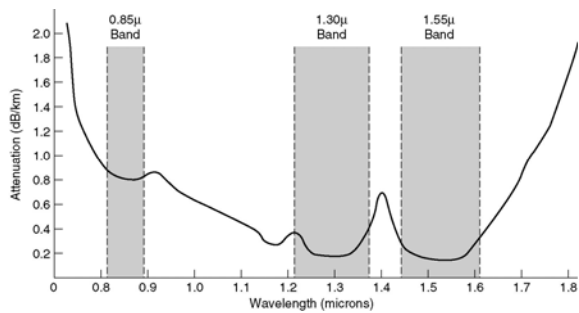- Modern fibers are as clear as unpolluted air!

Fiber Optics
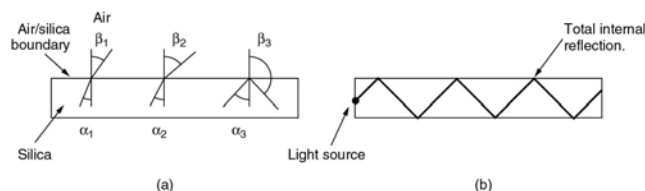
(a) Three examples of a light ray from inside a silica fiber impinging on the air/silica boundary at different angles
(b) Light trapped by total internal reflection

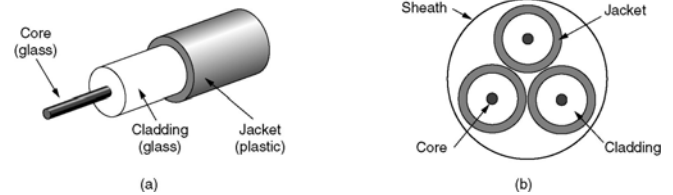# Transmission of Light through Fiber

Attenuation of light through fiber in the infrared region.

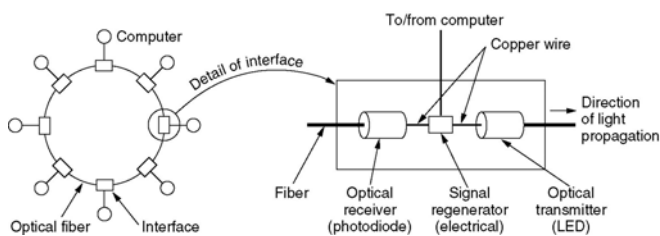# Fiber Cables

(a) Side view of a single fiber.
(b) End view of a sheath with three fibers.

Fiber Optic Networks

A fiber optic ring with active repeaters.

Fiber Optic vs. Copper

Pros of fiber optics as opposed to copper wire:
- MUCH higher bandwidth
- Low attenuation, repeaters needed every 50 Km vs. 5 Km for copper
- Not affected by power surges/failures, electromagnetic interference
- Not affected by chemicals in the air
- Thin and lightweight; 1000 twisted pair of 1 Km of copper weighs 8 ton; 2 fibers have more capacity and weigh 100 kg
- Cheaper
- Do not leak light and are difficult to tap
- Interfaces are a problem and are expensive