Digital Communication in the Modern World Physical Layer: Fourier Series; Physical Media

> <u>http://www.cs.huji.ac.il/~com1</u> <u>com1@cs.huji.ac.il</u>

#### The Theoretical Basis for Data Communication

- Fourier Analysis
- Bandwidth-Limited Signals
- Maximum Data Rate of a Channel

#### Harmonic Analysis



## Harmonic Analysis

In the general case:

$$A_n \cdot \sin(n \cdot \frac{2\pi \cdot t}{T})$$

*n* - the harmonic number

 $A_n$  - the amplitude of the  $n^{th}$  harmonic

 $2\pi$  - normalizes the function so that for every unit *t* it completes one period

1/T - the base frequency, normalizes the function so that for every harmonic *n* the frequency becomes *n* times the base frequency



can be <u>roughly</u> approximated by  $3 \sin(2\pi t/4)$ , which is called the **fundamental** 1<sup>st</sup> harmonic of our function

## Harmonic Analysis



#### Fourier Series Theorem

• For a "sufficiently well-behaved" function of time, g(t).

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(n \cdot \frac{2\pi \cdot t}{T}) + \sum_{n=1}^{\infty} b_n \cos(n \cdot \frac{2\pi \cdot t}{T})$$

• Using 
$$e^{it} = \cos(t) + i\sin(t)$$

The Fourier Series gets the form:

$$g(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{i \cdot n \frac{2\pi t}{T}}$$

• Calculating the coefficients; denote f:=1/T:

$$a_n = \frac{2}{T} \int_0^T g(t) \sin(2\pi n ft) dt \qquad b_n = \frac{2}{T} \int_0^T g(t) \cos(2\pi n ft) dt \qquad c = \frac{2}{T} \int_0^T g(t) dt$$

#### Bandwidth-Limited Signals

- Higher harmonic Fourier components attenuate faster
- Range of frequencies transmitted without being "strongly" attenuated is denoted the <u>bandwidth</u> of the media
- Often defined from 0 to the frequency at which half the power (amplitude) gets through
- Bandwidth is a physical property of the transmission medium



A binary signal and its root-mean-square Fourier amplitudes. (b) - (c) Successive approximations to the original signal.

## Bandwidth-Limited Signals (2)



(d) - (e) Successive approximations to the original signal.

## Bandwidth-Limited Signals (3)

- Example: Assume you want to send 8 bits at 9600 bps over an ordinary phone line
- The time to send 8 bits is 8/9600 = 0.83 msec.
- The frequency of the first harmonic is 9600/8 = 1200 Hz (periods per second)
- Ordinary phone lines have an artificial cut-off bandwidth of 3000Hz.
- Thus the highest harmonic passed through is 3000/ 1200 = 2.5 => highest harmonic is 2!
- The signal received would be tricky to reconstruct

#### => limiting the bandwidth limits the data rate

## Bandwidth-Limited Signals (4)

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Relation between data rate and harmonics for send a constant of 8 bits over a 3KHz channel.

## Nyquist's Sampling Theorem

 In 1924 Henry Nyquist derived the equation for the maximum data rate for a finite bandwidth noiseless channel:

Max data rate = 2Hlog<sub>2</sub>V bps

H= low-pass filtered bandwidth

V = number of signal levels, for binary data V=2

E.g. a 3khz noiseless filtered channel cannot transmit binary signals at a rate exceeding 6000 bps

The theorem states that by making 2H samples per sec the signal can be completely reconstructed.

## Shannon's Capacity Theorem

 In 1948 Claude Shannon fine-tuned and generalized Nyquist's result for noisy channels:

H= bandwidth

S/N = signal to noise ratio. Given in  $10log_{10}$  units called **decibels** dB.

E.g. a 3khz filtered channel with thermal noise ratio of 30 dB cannot transmit binary signals at a rate exceeding 30000 bps

#### **Guided Transmission Data**

- Magnetic Media
- Twisted Pair
- Coaxial Cable
- Fiber Optics



(a) Category 3 UTP - up to 16 MHz bandwidth.(b) Category 5 UTP - up to 100 MHz bandwidth.

Upcoming are category 6 and 7 with bandwidths of 250 MHz and 600 MHz respectively.

#### Coaxial Cable



A coaxial cable.

- Bandwidth of 1GHz
- Today used for cable TV and MANs

## Fiber Optics

- In the last 20 years computing speed has increased by a factor of 20 for each decade (IBM PC in 81 ran at 4.77 MHz => 2GHz in 2001)
- Semiconductors are close to their physical limit
- Data communication has gone from 56 kbps in ARPANET to 1 Gbps in 2001 and the "sky" is still the limit. That's a 125 fold increase per decade.
- Moreover the error rate has gone from 10<sup>-5</sup> per bit to ~zero in optical networks.
- With current fiber technology, the achievable bandwidth is much in excess of 50,000 Gbps and better materials are being found!
- Modern fibers are as clear as unpolluted air!

#### Fiber Optics



(a) Three examples of a light ray from inside a silica fiber impinging on the air/silica boundary at different angles
(b) Light trapped by total internal reflection

#### Transmission of Light through Fiber



Attenuation of light through fiber in the infrared region.

#### Fiber Cables



# (a) Side view of a single fiber.(b) End view of a sheath with three fibers.

#### Fiber Optic Networks



A fiber optic ring with active repeaters.

## Fiber Optic vs. Copper

Pros of fiber optics as opposed to copper wire:

- MUCH higher bandwidth
- Low attenuation, repeaters needed every 50 Km vs.
   5 Km for copper
- Not affected by power surges/failures, electromagnetic interference
- Not affected by chemicals in the air
- Thin and lightweight; 1000 twisted pair of 1 Km of copper weighs 8 ton; 2 fibers have more capacity and weigh 100 kg
- Cheaper
- Do not leak light and are difficult to tap
- Interfaces are a problem and are expensive