Algorithms - Exercise 9

Due Wednesday 5/1 24:00

1. (a) Compute the DFT of $(0, 1, 2, 3)$ by running the FFT algorithm. Write your intermediate steps.

   (b) Multiply the polynomials $p(x) = 9 + 7x$ and $q(x) = 5 + 4x$ using FFT. Write down all the stages.

2. You are given as input two sets $A, B \subseteq \{1, \ldots, n\}$, and you have to compute the set

   $$S = \{c : \exists a, b \text{ such that } c = a + b,\ a \in A, \text{ and } b \in B\}$$

   Show how this can be done in time $O(n \log n)$.

   Hint: Define two polynomials with coefficients in $\{0, 1\}$ and multiply them.

3. Consider the following "pattern matching" problem. You are given a long text T in binary alphabet, you are also given a shorter string S in binary alphabet and you want to find all the places in T where S appears as a substring. More formally, if $T = t_0, \ldots, t_m$ and $S = s_0, \ldots, s_n$ $(n < m)$, you have to output all the i's such that for every $0 \leq j \leq n$, $s_j = t_{i+j}$. Show how this can be done in time $O(m \log m)$.

   Hints: Change the alphabet to $\{1, -1\}$. Look at the reverse of S (i.e. its mirror image). Consider S reverse and T as polynomials.

4. Given a list $z_0, \ldots, z_{n-1}$ possibly with repetitions, show how to find in time $O(n \log^2 n)$ the polynomial of degree bounded by n that has zeros exactly at $z_0, \ldots, z_{n-1}$ (with the same multiplicity).

   Hint: The polynomial $p(x)$ has a zero at z_i if and only if $p(x)$ is a multiple of the polynomial $(x - z_i)$.

5. (Reshut) Give an $O(n \log n)$ algorithm evaluating all derivatives of a polynomial $f(n) = \sum_{k=0}^{n-1} a_k x^k$ at a given point x_0.