Algorithms - Exercise 5

Due Wednesday 1/12 24:00

1. (a) Show that $1 + y \leq e^y$ for any real y with strict inequality for $y \neq 0$

   (b) Show that $1 - \epsilon \leq (1 - \frac{\epsilon}{n})^n$ for $n \geq 1$ and $0 < \epsilon \leq 1$

   (c) Show that for any three sets A, B, C one has

$$A \cap (B \setminus C) = (A \setminus C) \cap (B \setminus C)$$

2. Show that the $2 - \frac{1}{k}$ bound on the approximation ratio of the algorithm for assignment distribution problem shown in class is tight. That is, show for any k there exists an instance of the problem on which the algorithm gives exactly $2 - \frac{1}{k}$-approximate solution.

3. How would you modify the approximation scheme for Subset-Sum presented in class to find a good approximation to the smallest value not less than t that is a sum of some subset of the given input list?

4. Show that the usual approximation algorithm for Vertex-Cover doesn't work in a graph with weighted vertexes, when the problem is to find a cover with the minimal weight.

5. Suppose we generalize the set-covering problem so that each set S_i in the family F has an associated weight w_i and the weight of a cover C is $\sum_{S_i \in C} w_i$. We wish to determine a minimum-weight cover. Change the algorithm given in class for Set-Cover and show that the bound on approximation ratio still holds.