Rematch and Forward for Parallel Relay Networks

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Abstract— **The Gaussian parallel relay network problem consists of transmitting a message from a single source node to a single destination node, through a layer of parallel relay nodes. The source is connected to the relays by a Gaussian broadcast channel, while the relays are connected to the destination by a Gaussian multiple access channel. When the channels are all white with the same bandwidth, and the relays cannot decode the message, the best known strategy is "amplify and forward", which achieves the coherence gain of multiple relays. We propose a strategy which achieves this gain even when the noises are colored or the channels have different bandwidths. To that end we use analog modulo-lattice modulation of the codewords in the BC, and then forward the estimated codeword by each of the relays to the MAC. This modulation allows the relays to re-match the signal to the optimal spectrum of the MAC, thus demonstrating how a channel problem can gain from a joint source/channel approach. We show that this strategy is asymptotically optimal in some limiting cases, and that it outperforms the known alternatives in most other cases, where the optimum is unknown. We also demonstrate how to improve the achievable rate in the original white problem, for some signal to noise ratio values.**

I. INTRODUCTION

The Gaussian parallel relay network was first introduced by Schein and Gallager [15]. It differs from the relay channel [8] by having more than one relay, and not having a direct connection between the source and the destination.

In the network model of interest, depicted in Fig. 1, a message W needs to be recovered at the decoder. We assume a discrete time model, where the encoder and each relay transmit an n-dimensional block, and such blocks are denoted in bold. The additive mutually-independent noise sequences $\mathbf{Z}_1, \cdots, \mathbf{Z}_M, \mathbf{Z}_{\text{MAC}}$ are stationary Gaussian of spectra $\mathcal{S}_1(z), \cdots, \mathcal{S}_M(z), \mathcal{S}_{\text{MAC}}(z)$, respectively. The encoder and the relays are subject to average power constraints:

$$\frac{1}{n}E\|\mathbf{X}_{\text{BC}}\|^2 \leq P_{\text{BC}}$$
$$\frac{1}{n}E\|\mathbf{X}_m\|^2 \leq P_m \quad, m = 1, \cdots, M \quad,$$

where $\|\cdot\|$ denotes the Euclidean norm. We will mostly concentrate on the *symmetric* case:

$$\mathcal{S}_1(z) = \cdots = \mathcal{S}_M(z) \triangleq \mathcal{S}_{\text{BC}}(z)$$
$$P_1 = \cdots = P_M \triangleq P_{\text{MAC}} \quad. \tag{1}$$

This network can be conveniently viewed as the concatenation of a broadcast (BC) channel from the source to the relays,
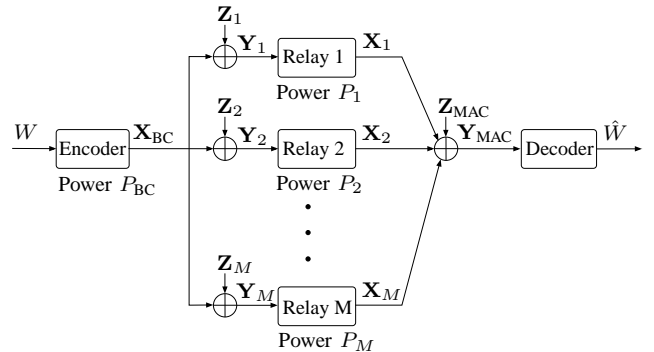
Figure 1: The Parallel Relay Network

with a multiple access channel (MAC) from the relays to the destination. A rate R is admissible if a coding scheme exists where each of $\lceil 2^{nR} \rceil$ equiprobable messages can be reconstructed with arbitrarily low probability of error, and the network capacity C is the supremum of all such rates.

The presentation in [15] concentrates on the 2-relay scenario with white noise sequences, and emphasizes the symmetric case. Two strategies are suggested there: The first is the *decode and forward* (DF) strategy, in which the relays reliably decode the message and forward it coherently to the destination. The second is *amplify and forward* (AF), where the relays forward their inputs, applying only power adjustment. Again the transmission is coherent, up to the the BC noises carried on to the MAC. The coherence improves the signal to noise ratio (SNR) experienced by the decoder[1]. It is shown that DF is optimal when the BC SNR is high compared to the MAC SNR, while AF is optimal in the limit of high MAC SNR (for any fixed BC SNR). In [4] the discussion is extended to any number of relays M, and it is shown that AF is optimal in the limit of large M. In general, if the relays cannot decode the message, AF (or time sharing between AF and DF, see [14]) is the best known strategy.

In this paper we extend these results to arbitrary noise spectra. Specifically, we treat the case where the BC and MAC sections have different noise colors, and then for the sake of simplicity we assume that each of them is still symmetric (1). In such a case, it is clear that AF is suboptimal, as the transmission cannot be simultaneously matched to channel inputs of different optimal water-filling spectrum by linear means. An important special case, is where the BC and MAC sections are both white but differ in their bandwidths (BWs).

Treating the signals in an analog manner by the relays, results in using only the minimum of these two BWs. In this paper we show how to enjoy the full BW of each channel, as well as the MAC coherence gain, even when the relays cannot decode the message. Since the relays re-match the signal to the desired spectrum, we term this strategy "rematch and forward" (RF).

Since the relays cannot decode the message, the transmission over the BC section has a *joint source/channel* flavor. We apply a variant of the Analog Matching joint source/channel colored broadcasting scheme [6], [7], based on analog modulo-lattice modulation [12]. In the sequel we show the advantage of this specific technique over other joint source/channel schemes in this setting. Regarding the MAC section, the advantage of joint source/channel methods in the presence of BW mismatch was shown in [9]. However, the situation we have is different, since we treat noisy estimations of the same source rather than uncorrelated sources.

II. PRELIMINARIES: MODULO-LATTICE MODULATION

Let Λ be a K-dimensional lattice. The basic Voronoi cell of Λ is $\mathcal{V}_0 = \{\mathbf{x} : Q(\mathbf{x}) = \mathbf{0}\}$, where $Q(\mathbf{X})$ is the the nearest neighbor quantizer associated with Λ. The second moment of a lattice per dimension is defined as:

$$\sigma^2(\Lambda) = \frac{1}{K} \int_{\mathcal{V}_0} \|\mathbf{x}\|^2 d\mathbf{x} \ ,$$

while the modulo-lattice operation is defined by:

$$\mathbf{x} \mod \Lambda = \mathbf{x} - Q(\mathbf{x}) \ .$$

For a dither vector $\mathbf{d}$, the dithered modulo-lattice operation is:

$$\mathbf{y} = [\mathbf{x} + \mathbf{d}] \mod \Lambda \ .$$

If the dither vector $\mathbf{d}$ is independent of $\mathbf{x}$ and uniformly distributed over the basic Voronoi cell $\mathcal{V}_0$, then $\mathbf{y}$ is uniformly distributed over $\mathcal{V}_0$ as well, and independent of $\mathbf{x}$.

We shall use lattices which are simultaneously good for source coding and for channel coding [3]. The combination of these properties needed for our purposes is formally stated in the following proposition (see [7]).

Proposition 1: **(Existence of good Lattices)** There exists a sequence of K-dimensional lattices Λ_K of (fixed) second moment per dimension σ^2, satisfying that if $\mathbf{Z}$ is Gaussian i.i.d. with $E\{Z_k^2\} < \sigma^2$, then

$$\lim_{K \to \infty} \Pr\{\mathbf{Z} \in \mathcal{V}_0\} = 1 \ .$$

III. THE BANDWIDTH MISMATCH CASE

Suppose that both the BC and MAC sections of the network are white, but of different BWs, so that we are allowed ρ uses of the BC section per each use of the MAC section, for some $\rho > 0$. We can treat this case within the general framework presented, by working at a sampling rate according to the wider section, and then taking that section to be white, while in the narrower section the outband noise spectrum is infinite. We denote the (inband) SNRs of the BC and MAC sections by S_{BC} and S_{MAC} respectively, where the MAC SNR is defined w.r.t. the total power of all the relays. Let

$$C(S) \triangleq \frac{1}{2} \log(1 + S) \ . \tag{2}$$

Under these definitions, we have the following upper bounds.

Proposition 2: The network capacity C satisfies:

$$C \le \rho C (M S_{BC}) \tag{3a}$$
$$C \le C (M S_{MAC}) \ . \tag{3b}$$

proof outline: For both bounds, assume cooperation between the relays, and use the data processing inequality. (3a) follows immediately, taking into account the BW ratio ρ. For (3b), consider the AWGN between $\sum_{m=1}^{M} X_m$ and Y_{MAC}. For a given power, the mutual information is maximized by a Gaussian input, and the maximal input power $M^2 P_{MAC}$ is achieved when all the relays transmit the same signal.

Assume the use of a white Gaussian codebook, designed for the MAC BW, with power P_{MAC}. If we had $\rho = 1$, we could use the amplify and forward (AF) strategy. In this strategy, the decoder receives:

$$\mathbf{Y}_{MAC} = \sum_{m=1}^{M} X_m + Z_{MAC} = \gamma(M\mathbf{X}_{BC} + \sum_{m=1}^{M} Z_m) + Z_{MAC}$$

where γ is the relay amplification factor, resulting in network capacity:

$$C_{AF} = C \left(\frac{M S_{MAC} S_{BC}}{S_{MAC} + S_{BC} + 1} \right) \ . \tag{4}$$

This capacity reflects the coherence gain of summing the M identical signal components X_{BC} from all relays, while the independent noise components Z_m are added non-coherently. In the limit $S_{BC} \gg S_{MAC} + 1$, C_{AF} approaches[2] (3b), while in the limit $S_{MAC} \gg S_{BC} + 1$ it approaches (3a).

We extend this property to $\rho \ne 1$ by turning the BC channel into an equivalent mutually independent AWGN (of the MAC BW) channels from the encoder to each of the relays:

$$\hat{\mathbf{V}}_m = \mathbf{V} + \tilde{\mathbf{Z}}_m \tag{5}$$

with $\{\tilde{\mathbf{Z}}_m\}$ white and independent of $\mathbf{V}$. This task can be achieved by treating the codeword as a Gaussian source, and applying an optimal joint source/channel scheme for the BW expansion ($\rho > 1$) or compression ($\rho < 1$) problem. Many such joint source/channel schemes exist, and an optimal $R(D) = C$ achieving scheme will result in minimum equivalent noise power. However, schemes containing digital information components, such as separation-based transmission, or hybrid digital/analog (HDA) transmission (see e.g. [11]), would fail to produce *mutually independent* equivalent noises, since the reconstruction of the digital component would have the *same* realization of estimation error in each relay. We turn, then, to the Analog Matching scheme [6], which does not include any digital information component[3]. Specifically, we use an unbiased suboptimal version of the scheme, which achieves mutual independence at the cost of some rate loss. This loss becomes negligible at high SNR, namely $S_{BC} \gg 1$.

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[2]This may still happen even in the AF region, i.e. when the relays cannot decode the message, for large enough M.

[3]Alternatively, in the special case of integer ρ, the modulo-lattice scheme of [12] may also be used as a basis for the relay strategy.
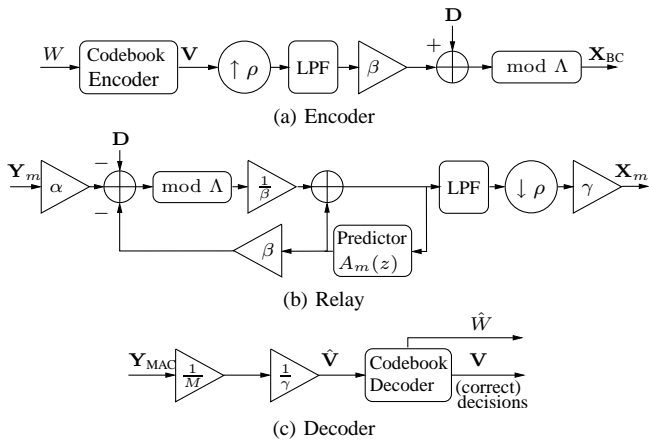
Figure 2: Components of the Scheme for $\rho > 1$

When $\rho > 1$, the error sequences $\{\mathbf{Z}_m\}$ are fully independent. When $\rho < 1$, they are *ρ-mutual independent*: If $\tilde{\mathbf{Z}}_m^{(\mathrm{LPF})}$ is $\tilde{\mathbf{Z}}_m$ passed through an ideal low pass filter of cutoff frequency $\rho/2$, then $\{\tilde{\mathbf{Z}}_m^{(\mathrm{LPF})}\}$ are mutually independent.

The scheme is depicted in Fig. 2 for $\rho > 1$ and in Fig. 3 for $\rho < 1$, see the schemes in [6] for further details. For the purpose of analysis, we assume the existence of a large number K of identical and mutually independent relay networks working in parallel[4]. These networks are treated independently, except for the codebook encoding and decoding, and the dithered modulo-lattice operations, which are performed jointly across the networks. Hence, though we present the scheme for one network, the codebook and the lattice have the asymptotic properties of high dimension ($K \to \infty$): We use the AWGN channel capacity theorem for the codebook, and Prop. 1 for the modulo-lattice operation. The following Lemma states the performance of the BC section.

Lemma 1: Assume that $S_{\mathrm{BC}} > 1$ and choose $\alpha = 1$. For an appropriate choice of filters and of β in the schemes of Fig. 2 and Fig. 3, the estimation $\hat{\mathbf{V}}_m$ of $\mathbf{V}$ obtained in each of the relays, satisfies:
$$\lim_{K \to \infty} \hat{\mathbf{V}}_m = \mathbf{V} + \tilde{\mathbf{Z}}_m$$
in the MSE sense, where K is the lattice dimension, and $\{\tilde{\mathbf{Z}}_m\}$ are independent of $\mathbf{V}$, i.i.d. Gaussian with an equivalent SNR
$$\tilde{S}_{\mathrm{BC}} \triangleq \frac{P_{\mathrm{BC}}}{\sigma_{\tilde{Z}_m}^2} = S_{\mathrm{BC}}^\rho - 1 \quad . \tag{6}$$
Furthermore, for $\rho \geq 1$ the sequences $\{\tilde{\mathbf{Z}}_m\}$ are mutually independent, and for $\rho < 1$ they are ρ-mutually independent.

Proof outline: For $\rho > 1$, the BC section is similar to the BW expansion case of the Analog Matching scheme [6], where for a small enough β the modulo-lattice operations in the encoder and the decoder cancel each other by Prop. 1. If we were to use an optimal α, we could achieve the optimum performance $\tilde{S}_{\mathrm{BC}} = S_{\mathrm{BC}}^\rho$, where the error is a combination of the channel noise and of the transmitted signal $\mathbf{X}$. However, for mutual independence between the relay estimation errors,

[4]This setting can be emulated for a single network by using a large interleaving table, see [5].
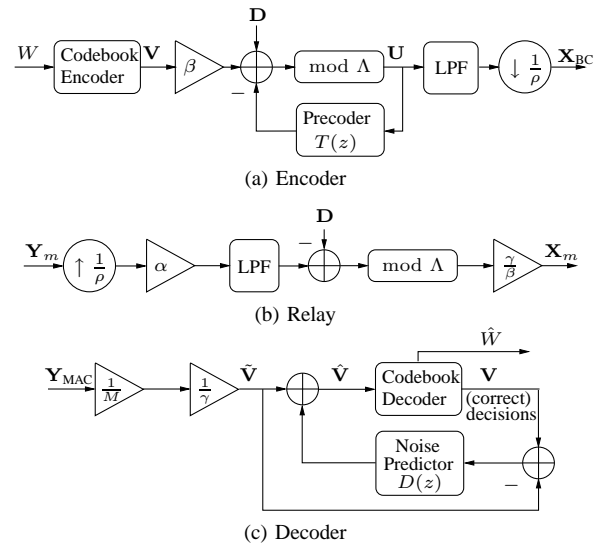

Figure 3: Components of the Scheme for $\rho < 1$

we need these errors to be independent of $\mathbf{X}_{\mathrm{BC}}$, thus we must use[5] $\alpha = 1$. The resulting MSE reflects the use of this α, and also the use of a smaller β in order to meet the condition of Prop. 1. For $\rho < 1$, the BC section is similar to the BW compression case of the Analog Matching scheme. We apply similar considerations, but since we must use LPFs, we cannot achieve independence of $\mathbf{X}_{\mathrm{BC}}$ outside the channel BW. In order to establish the ρ-independence, we use the following Lemma regarding equalization in the point-to-point setting (the proof for which is omitted in this outline):

Lemma 2: Suppose an AWGN channel of SNR S may be used $\rho < 1$ times per each i.i.d. codebook symbol. Using a FFE/DFE receiver, one may achieve unbiased slicer SNR S^ρ, where the slicer error is white and ρ-independent of the codeword and of the channel input.

Applying Lemma 1, we obtain an equivalent white network. The relays use an amplification factor $\gamma^2 = \frac{\tilde{S}_{\mathrm{BC}}}{\tilde{S}_{\mathrm{BC}}+1}$. Utilizing the decoders depicted in Fig. 2c and Fig. 3c, and defining
$$\lambda \triangleq \min(\rho, 1) \quad , \tag{7}$$
we can lower-bound the network capacity.

Theorem 1: Assume that $S_{\mathrm{BC}} > 1$, then the capacity of the Gaussian parallel relay network with expansion factor ρ is lower-bounded by:
$$C_{\mathrm{RF}} \triangleq C\left(\frac{M S_{\mathrm{MAC}} (S_{\mathrm{BC}}^\rho - 1)}{(S_{\mathrm{BC}}^\rho + S_{\mathrm{MAC}})^\lambda (S_{\mathrm{BC}}^\rho + M S_{\mathrm{MAC}})^{1-\lambda}} \right) \quad . \tag{8}$$

Proof outline: For $\rho > 1$, it is immediate. For $\rho < 1$, $\tilde{\mathbf{V}}$ is the optimal estimator of $\mathbf{V}$ from $\mathbf{Y}_{\mathrm{MAC}}$. The error of this estimator has a two-step spectrum, since $\{\mathbf{Z}_m\}$ are added incoherently inband, but coherently outband. Then the noise predictor $D(z)$ uses past decisions to exploit the memory of this error.

[5]A slightly lower value of α, achieving a lower $\tilde{S}_{\mathrm{BC}}$ with some dependence between the relays, will result in a somewhat better overall performance of the scheme, but its computation is cumbersome.

IV. Comparison of Bounds

In this section we present more inner bounds by previously known relay strategies (see e.g. [2]), and compare these bounds, as well as the outer bounds of Prop. 2, with the new bound in different high-SNR limiting cases. Throughout this section we use λ of (7).

Amplify and Forward: This strategy was mentioned already in the previous section for $\rho = 1$. It may be extended to other ρ values, using filtering in the relays, yet filtering cannot allocate energy between bands, thus we only enjoy the narrower BW, resulting in:

$$C_{\text{AF}} = \lambda C\left(\frac{MS_{\text{MAC}}S_{\text{BC}}}{S_{\text{MAC}} + S_{\text{BC}} + 1}\right) \quad . \tag{9}$$

Decode and Forward: In this strategy, we use a low enough rate such that each relay can reliably decode the codeword. In the second stage all the relays use the same transmission, enjoying the coherence gain. Consequently:

$$C_{\text{DF}} = \min\Big(C\left(MS_{\text{MAC}}\right), \rho C\left(S_{\text{BC}}\right)\Big) \quad . \tag{10}$$

Compress and Forward: The relays digitally compress their estimations of the codeword, and subsequently send the digital data over the MAC section. The performance is given by comparing the minimal rate of the symmetric quadratic Gaussian CEO problem [10] with the Gaussian MAC capacity. This combination is suboptimal, since by using source/channel separation it fails to achieve the coherence gain (see [4]). Using this strategy, the achievable rate is $C_{\text{CF}} = \rho C(S_{\text{CF}})$, where S_{CF} is given by the solution of:

$$(1 + S_{\text{MAC}})^{\frac{1}{\rho}} = 1 + S_{\text{CF}}\left(\frac{MS_{\text{BC}}}{MS_{\text{BC}} - S_{\text{CF}} + 1}\right)^M \quad . \tag{11}$$

In order to compare these inner bounds, we consider the high SNR limit $S_{\text{BC}} \gg 1$. Within this limit, we further consider three limiting cases:

> Decodable: $S_{\text{BC}} \gg MS_{\text{MAC}}$
> MAC-limited: $MS_{\text{BC}} \gg MS_{\text{MAC}} \gg S_{\text{BC}}$
> BC-limited: $S_{\text{MAC}} \gg S_{\text{BC}}$.

For these cases, the effective SNRs according to the different strategies are summarized in the following table, along with the outer bounds of Prop. 2.

Bound	Decodable	MAC-limited	BC-limited
AF		$M^\lambda S_{\text{MAC}}^\lambda$	$M^\lambda S_{\text{BC}}^\lambda$
RF	MS_{MAC}	$\left(MS_{\text{MAC}}S_{\text{BC}}^{1-\lambda}\right)^\lambda$	$M^\lambda S_{\text{BC}}^\rho$
DF		S_{BC}^ρ	
CF		S_{MAC}	$\min\left(\log(S_{\text{MAC}}), M^\rho\right)$ $\cdot S_{\text{BC}}^\rho$
outer		MS_{MAC}	$M^\rho S_{\text{BC}}^\rho$

Figure 4: Comparison of bounds on the effective SNR. The expression for CF in the BC-limited case is an outer bound, tight for large M.

In all these limiting cases, the RF strategy is at least as good as the other strategies, except for the BC-limited case
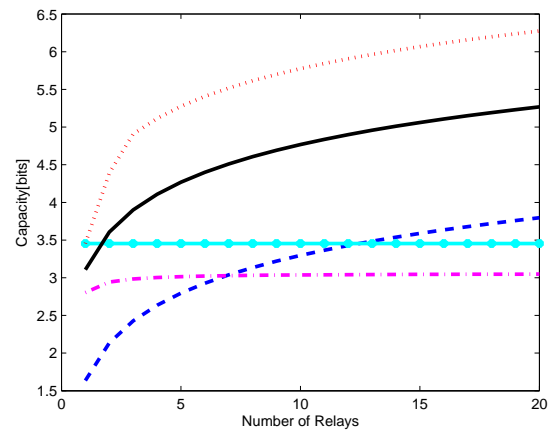
(a) $\rho = 2, S_{\text{BC}} = 10, S_{\text{MAC}} = 300$
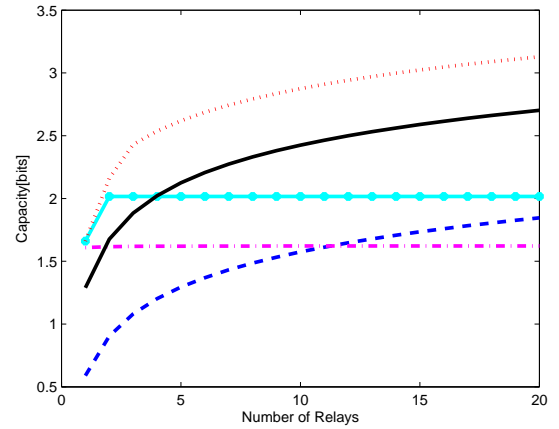
(b) $\rho = 0.5, S_{\text{BC}} = 300, S_{\text{MAC}} = 10$

Figure 5: Capacity vs. number of relays. solid = RF, dashed = AF, dash-dotted = CF, starred = DF, dotted = outer bound.

for $\rho > 1$, where CF may be better[6] if $\log(S_{\text{MAC}}) > M$. In the decodable case, RF approaches the optimum performance achieved by DF (and by AF for $\rho > 1$). In the BC-limited case with $\rho < 1$, RF approaches the optimum performance achieved by AF.

The graphs of Fig. 5 show non-asymptotic results for two cases. It is evident, that RF achieves the coherence gain of AF for any M (which the other strategies do not), while making better use of the BW. The graphs suggest that a potential benefit in combining RF and DF for a low number of relays M; This can be achieved by means of time sharing, as suggested For AF and DF in [14], see also the following section.

V. Improving the Equal-BW Rate

In this part we show how, surprisingly, our approach may be used to improve the achievable rate even in the original white equal-BW ($\rho = 1$) case of [15]. Consider the case where $MS_{\text{MAC}} > S_{\text{BC}}$, thus DF is suboptimal, yet S_{MAC} is

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[6]This limit for the CF strategy corresponds to the extreme case where the MAC capacity is high enough, so that the relay inputs may be recovered almost perfectly at the decoder.
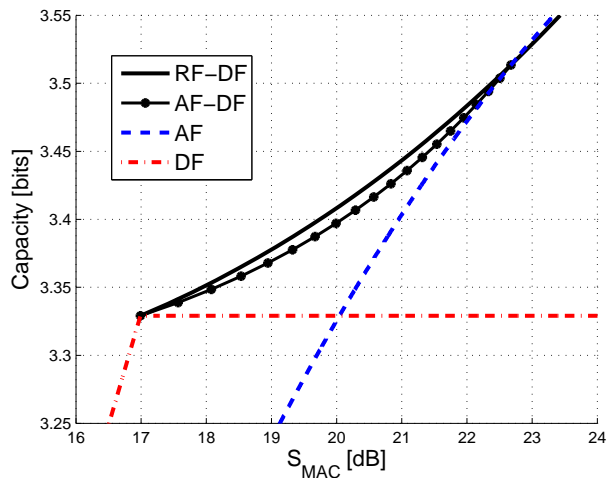
Figure 6: Achievable Rates by Time Sharing

not too high. In this region, time sharing between AF and DF is better than any of these two used alone [14, Sec. 2.3]. The sharing strategy allocates different power to the AF and DF relay transmissions, such that they effectively function with different SNRs, satisfying the total power constraint.

We note that replacing AF by RF gives another degree of freedom in the design: The BW allocated to RF may change between the transmitter and the relays. In other words, using RF we may introduce an artificial BW change to the equal-BW case. Allowing this flexibility improves the performance beyond the AF time-sharing line, as long as the SNR is high enough s.t. the losses of RF are small. Fig. 6 demonstrates this improvement for two relays and $S_{BC} = 20dB$. While clearly this strategy is not optimal, and the improvement is small, it demonstrates that the known bounds of [14] can be improved.

VI. EXTENSION TO NON-WHITE SPECTRA

We now abandon the assumption that the noise spectra are flat within the channel BW. In high SNR conditions (within the respective BW), the capacity gain over a white channel is:

$$\Delta C = \frac{1}{2}\left[\log \int_{-\frac{1}{2}}^{\frac{1}{2}} \mathcal{S}_N(e^{j2\pi f})df - \int_{-\frac{1}{2}}^{\frac{1}{2}} \log \mathcal{S}_N(e^{j2\pi f})df\right].$$

In point-to-point communications (and when the codebook bandwidth is adjusted to the channel bandwidth), this gain can be achieved by a zero-forcing (ZF) FFE/DFE receiver [1]. We can achieve this gain by combining such filtering inband, with low-pass filtering as done in the white RF scheme, adding a precoder to the encoder and a DFE to the decoder. The details of this combination are not included, for lack of space.

VII. EXTENSION TO LAYERED NETWORKS

The RF strategy for the parallel relay network can be used as a building stone for more general networks. We consider *layered networks*, which are directed acyclic graphs (DAGs) where the nodes can be divided into layers, and nodes in each layer receive the (noisy) sum of some transmissions from the adjacent preceding layer only. For such networks, examples

can be shown where using the RF strategy recursively or combining it with the CF strategy are beneficial. The general study of these networks is left for further research.

VIII. CONCLUSION

We conclude with two remarks regarding the implementation of the RF strategy in a communication network.

Encapsulation: In our presentation, we assumed that the encoder and decoder use time-domain equalizers combined with an AWGN code. While this approach is close in spirit to the RF strategy, thus making the analysis simple, it is not compulsory. Any other approach for ISI channels, such as MLSE or DMT, can be used, making RF an alternative to AF, regardless of other scheme components. As a consequence, the condition of large K stems from the lattice dimension only. For high SNR, it is possible to take $K = 1$ with some fixed capacity loss, resulting in *scalar* relays and no excess delay to the scheme.

MIMO channels: The BW mismatch framework may be thought of as a model for combining channels with a different number of antennas. For example, it may reflect relays communicating with the end-users using one antenna, while using multiple antennas for the link with the base station. For a recent work regarding parallel relays in the MIMO setting, see [13], which in contrast to the present work, assumes a digital use of the MAC section, optimized by a CF approach.

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