Giant resonance absorption in ultra-thin metamaterial periodic structures

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Abstract: We study the interaction of an incident plane wave with a metamaterial periodic structure consisting of alternating layers of positive and negative refractive index with average zero refractive index. We show that the existence of very narrow resonance peaks for which giant absorption - 50% at layer thickness of 1% of the incident wavelength - is exhibited. Maximum absorption is obtained at a specific layer thickness satisfying the critical coupling condition. This phenomenon is explained by the Rayleigh anomaly and by the excitation of Fabry Perot modes in the periodic layer. In addition, we investigate the modes supported by the structures for several limiting cases, and show that zero phase accumulation in the periodic metamaterial is obtained at resonance.

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References

1. Introduction

Following the introduction of negative index materials (NIM) [1] by Veselago [2] in 1968 and the proposal of perfect imaging using a NIM slab by Pendry [3] in 2000, there is an ongoing interest in NIM, including concept of operation, realization of NIM based devices, and their applications [4]. In [5,6] it has been shown that periodic structures with alternating doubly positive and doubly negative materials (DPS and DNG, respectively), may have a band gap that is not the result of the conventional Bragg condition but instead is due to the zero average refractive index of the structure. It was also proven that these structures have propagating modes at discrete frequencies satisfying a Fabry Perot condition for each period. These alternating DPS/DNG structures can also support a variety of additional modes, denoted as gating modes at discrete frequencies satisfying a Fabry Perot condition for each period. These

In [8,9] it was shown that periodic structures with alternating layers of negative \( \varepsilon \) and negative \( \mu \) materials (ENG and MNG) can behave effectively as structures with alternating DPS and DNG layers. Experimental measurements of the zero refractive index band gap were also reported [10, 11]. Additionally, metamaterials have been also proposed as ultra-thin absorbers with small intrinsic losses [12]. Periodic structures with alternating DPS and DNG layers can be designed to obtain narrow transmission peaks with low-amplitude sidelobes [7,13] for incident plane waves propagating in the direction of the periodicity. While for the conventional
DPS periodic structures the Fabry-Perot interference dominates over the Bragg reflection also for frequencies slightly shifted from the resonance frequency $\omega_0$, for alternating DPS-DNG structures the Bragg reflection dominates, resulting in very narrow transmission peaks. Hereby, we use the Rigorous Coupled Wave Analysis (RCWA) method [14–17] and the transfer matrix method (TMM) [18] to study the interaction of an incident plane wave with periodic structures having alternating DPS and DNG materials. Unlike previous works, here the incident angle is normal to the direction of periodicity. For such a configuration we show that very strong effects on light propagation can be obtained even if the structure’s thickness is much smaller than the incident wavelength. As a result, a giant and narrow band of absorption is obtained, with a sharp resonance peak even for a very thin layer, $\sim$two orders of magnitude thinner than the vacuum wavelength. We thus coin this structure as “super absorber”. In Section 2 we analyze the modes in an infinitely thick DPS-DNG grating, using both the TMM and RCWA methods. In Section 3 we consider a grating with thickness far below the incident wavelength.

2. Modes of a DPS-DNG structure with infinite thickness

We start by analyzing a structure with zero average refractive index $\bar{n}$, i.e. a structure which satisfies $\bar{n} \triangleq \frac{1}{L} \int_0^L n(x) dx = 0$, where $L$ is period of the structure and $n(x)$ is the piecewise transverse index of refraction along $x$ (the transverse coordinate), as shown schematically in Fig. 1. For now we assume an infinite slab thickness, i.e. $h = \infty$. Using the TMM, the dispersion relation for TM ($H_y$) modes can be written as (with nearly the same notation as used in Ref. [7]):

$$\cos(k_{x1}L_1) \cos(\tilde{n}k_{x1}L_2) - \frac{n^2 + \varepsilon^2}{2\tilde{n}\varepsilon} \sin(k_{x1}L_1) \sin(\tilde{n}k_{x1}L_2) = \cos(K_B L)$$  \hspace{1cm} (1)

where $n_i = \pm \sqrt{\varepsilon_i \mu_i}$ ($n_i$ takes a positive sign for DPS media and a negative sign for DNG media), $k_{x0}^2 = \omega^2 \varepsilon_0 \mu_0 / c^2 - k_z^2$, $\varepsilon = \varepsilon_2 / \varepsilon_1$, $n = n_2 / n_1$, $\tilde{n} = k_{x2} / k_{x1} = \pm (n^2(k_{x1}^2 + k_z^2) - k_z^2)^{0.5} / k_{x1}$ and $K_B$ is the Bloch wavevector. As shown in Fig. 1, $\varepsilon_{1,2}$ and $\mu_{1,2}$ are the permittivity and permeability of the DPS and DNG layers, respectively. It has been shown, that for an infinitely thick ($h \rightarrow \infty$) periodic stack of layers with average refractive index of $\bar{n} = 0$, a band gap is formed resulting in 100% reflection from the stack for plane waves propagating in the x-direction, except for discrete frequencies which satisfy the Fabry Perot condition ($k_{x1}L_1 = m\pi, k_{x2}L_2 = -p\pi$ where $m, p$ are nonzero integers) for which the periodic stack has full transmission. We consider
Figure 2. The log(Re($K_B$)) as function of the real and imaginary parts of $k_z/k_0$, calculated using TMM. (b) Magnetic power $|H_y|^2$ in a unit cell (media boundary at $x/\lambda_0 = 0$), for the modes with $k_z/k_0 \cong \pm 0.83 - 0.46i$. (c) Phase of $H_y$ in a unit cell for the same mode as (b). (d) Eigenmodes of the same structure as (a), calculated using RCWA.

this structure, but for transverse illumination (light propagating perpendicular to the periodicity of the structure, namely in z direction of Fig. 1) in contrast to the configurations discussed in the various references presented in the introduction. The normally incident plane-wave has a vacuum wavelength $\lambda_0$ and a transverse magnetic (TM) polarization. The modes of the structure are studied by observing the diffraction of the incident plane wave from the periodic structure which is semi-infinite in the z direction ($h = \infty$). The incidence medium (labeled as region I in Fig. 1(a) and the outgoing medium (region III) are assumed to be vacuum, while the grating (region II) is defined by the parameters $\varepsilon_1 = 1$, $\varepsilon_2 = -1$, $\mu_1 = 1$, $\mu_2 = -1$, with $L_1 = L_2 = \frac{1}{2} \lambda_0$. Here we neglect material dispersion for simplicity. This is an ideal lossless and dispersion-less case, which satisfies the $\bar{n} = 0$ condition. The effect of loss will be included starting from the next paragraph, and dispersion will also be considered towards the last section of this manuscript. The zeroth diffraction order in region II has $k_x = 0$, and $k_z = \pm k_0$ yielding no power flow (zero Poynting vector $P_z$) as a result of the anti-parallel and equal amplitude Poynting vectors at the two media i.e. $P_{z1} = -P_{z2}$ where $P_{zi} = \frac{1}{2} \int_{L_i} E_z H^*_y dx$ and E and H are the electric and magnetic fields, respectively. All other diffraction orders (modes) satisfy $k_x = \pm mK$, where K is the grating vector given by $K = 2\pi/L$. In the specific configuration we study, the periodicity is equal to the vacuum wavelength, i.e. $K = k_0$, thus all the diffraction orders satisfy the Fabry-Perot condition along the x axis and thus are propagating along the structure interface (x directed surface modes), with either zero $k_z$ for the $\pm 1$ diffraction order, or purely imaginary $k_z$ for higher diffraction orders. It therefore follows that the grating will not allow any power propagation in the z direction.
Next we add small loss to the structure. The considered material parameters are now \( \varepsilon_1 = 1 - 0.001i \), \( \varepsilon_2 = -1 - 0.001i \), \( \mu_1 = 1 \), \( \mu_2 = -1 \). For these parameters, the real part of the average refractive index is still zero (i.e. \( \text{Re}(\bar{n} = 0) \)), and because the imaginary term is small compared to the real term, we may expect to have discrete modes propagating in the x-direction satisfying the Fabry Perot condition, who are closely related to the modes discussed for the lossless \( \bar{n} = 0 \) case. The “perturbed” modes however, will experience some loss upon propagation, in the Z direction [19]. In addition, by introducing loss into the structure we now find solutions with complex values of \( k_z \) and \( k_y \). These modes also satisfy the Fabry Perot condition, since they accumulate zero phase along the unit cell, (due to the opposite phase velocity in each media). In [9] these modes were defined as photon tunneling modes. In Fig. 2(a) the logarithm of \( \text{Re}(K_B) \), calculated using Eq. (1) is plotted in the complex normalized \( k_z/k_0 \) plane, for the material and structural parameters listed above. The darker spots correspond to modes for which the real part of \( K_B \) is closest to zero. In Figs. 2(b,c) the amplitude and the phase of the magnetic field component \( H_z \) inside a single unit cell are plotted, for the “dark spots” in Fig. 2(a) corresponding to \( k_z/k_0 \simeq \pm 0.83 - 0.46i \) (denoted by black circles). As expected, the field in each half of the unit cell (DPS and DNG) satisfies the Fabry Perot condition (Fig. 2(c)). This is the result of a standing wave pattern, with mirror symmetry with respect to the layer boundaries and also to the center of each layer.

Next, we analyze the same problem of the propagation and diffraction of a normally incident plane wave from the periodic structure of DPS/DNG medium using the RCWA method, also known as the Fourier Modal Method (FMM). The RCWA method finds a set of eigenmodes \((k_z)_m\) of each z-invariant region, as well as its related transmission and reflection coefficients. To find these eigenmodes, the fields are expanded using a transverse wave vector \((k_z)\) basis of \(\pm N \times K + k_{z,\text{inc}}\) where \(k_{z,\text{inc}}\) is the incident plane wave transverse wave vector, and \(2N+1\) is the number of orders used for the computation. For our case of normal incidence, \(k_{z,\text{inc}} = 0\). To allow numerical computation, the diffraction problem is truncated to a finite value of \(N\), and generally converges to the “exact” solution by increasing \(N\). To cope with the variations in permeability, we modified the standard formulation of the eigen value problem matrix \(B\) used in [14], to be \(B = K_z E^{-1} K_x - M\), where \(M\) is the permeability Toeplitz matrix, constructed in the same manner as the permittivity Toeplitz matrix \(E\) (see Eq. (36) in ref. [14]). We have used \(N=150\) in our computations. It should be noted that the small inherent loss in the structure is essential for the stability of the RCWA simulation for the case of nearly zero average refractive index. In Fig. 2(d) the eigenmodes obtained by the RCWA simulation are calculated and plotted for the case of \(L = \lambda_0\), with the same material parameters as used for the TMM calculation. All eigenmodes satisfy \(-1 < \text{Re}(k_z) < 1\), but only those with \(\text{Im}(k_z) < 10\) are shown. It is seen that the obtained eigenmodes are identical to those found in Fig. 2(a) using the TMM, indicating a satisfying convergence of the RCWA simulation in the calculation of the eigenmodes.

3. Resonant modes of a DPS-DNG slab with finite thickness

After verifying the accuracy of the RCWA for calculating the eigenmodes, we use it to study the transmission, reflection and absorption of this periodic metamaterial with a finite thickness \(h\). In Fig. 3 these three parameters are plotted as a function of the incident wavelength and \(h\). The material parameters remain the same as in the previous section, and we consider the case for which \(L = \lambda_0\). As found in the previous section, all eigenmodes are complex valued, resulting in a decay of the electromagnetic field in the positive \(Z\) direction. Therefore, for a structure with semi infinite thickness no transmission is expected and the structure performs as a good mirror, reflecting the incident power with some losses. On the other hand, for a structure, thinner than the effective “skin depth” (of the order of the wavelength) of the gratings, one would expect some tunneling from region I to region III such that significant transmission is obtained.
However while this prediction is true for most wavelengths, it is not the case for wavelengths in the vicinity of $\lambda_0$, denoted as the resonance. The above expectation is supported by Fig. 3 which presents the transmission, reflection, and absorption as a function of the normalized gratings thickness and the normalized wavelength. The obtained results indicate a steep increase in the absorption followed by a steep drop in transmission around resonance. Therefore, the structure may be considered as a “super absorber” for the resonance wavelength.

We attribute this surprising result to two mechanisms, the Rayleigh anomaly and the guided mode resonance [20–23]. Rayleigh anomaly occurs when a propagating diffraction order in the semi-infinite incident and outgoing regions I and III is becoming an evanescent order, resulting in the redistribution of the energy within the other orders. In our case where $\epsilon = 1$ and $\mu = 1$ in both the incident and outgoing regions, the Rayleigh wavelength is equal to the period, i.e. $\lambda_R = L$. A guided mode resonance is the consequence of the matching of the diffracted orders to a leaky mode which is propagating along the x axis in region II. To gain further understanding of the physics behind the obtained transmission/absorption/reflection maps we plot (Fig. 4) the field distribution within the metamaterial structure and its vicinity. We consider a thin periodic metamaterial with thickness of $h=0.01L$. First, it can be seen from Figs. 4(a) and 4(b) that in regions I and III the diffracted wave occupies precisely a single unit cell and therefore can be identified as the Fabry Perot mode, discussed in Section 2, which is consistent with the choice of $L \approx \lambda$. The resonance in these regions is therefore in agreement with the Rayleigh anomaly.

In region II, the excited mode is resonantly guided in the periodic layer as indicated by the fact that the mode periodicity is identical to the gratings period. This is somewhat similar to the very well known case of guided mode resonance [20,24,25] where typically a waveguide mode is excited. However, here the modes satisfy the Fabry-Perot condition along the X direction rather than being conventional waveguide modes. The very narrow lineshape (of the order of $\sim 0.5\lambda_0 \times 10^{-6}$ for $h/\lambda_0 = 0.01$) is explained by the sensitivity of the Fabry Perot resonance, where small change in wavelength shifts the domination to the Bragg regime over the Fabry Perot regime, as discussed in Section 2, following references [7,13]. The high Q-factor can be also understood by observing the magnetic field distribution of the Fabry Perot mode (Fig. 4(a)) having an odd symmetry with respect to the x-axis. In contrast, the reflected and transmitted plane waves have even magnetic field distribution, resulting in a vanishing overlap integral between them and the Fabry-Perot mode, and thus giving rise to “dark modes” which typically have very large Q-factors for structures with small inherent losses (see e.g. [26]). To compare between the resonance and off resonance cases we calculated $H_y$ for a slightly off resonance case ($\lambda = 1.001L$). From the results (See Fig. 4(c)) it can be seen that the magnetic field no longer builds up coherently in the unit cell.

From Fig. 4 it is evident also that ripples with large spatial frequencies are formed at the boundaries of the DNG and DPS media. These ripples are typical for surface modes bound
Figure 4. Real part of the field profiles in the NIM grating, calculated using the RCWA method (a) $H_y$ field component at resonance ($\lambda = L$), (b) $E_z$ field component at resonance ($\lambda = L$) (saturated color scale) (c) $H_y$ field for the off-resonance case ($\lambda = 1.001L$). The black rectangles denote the region of the DNG layer.

Figure 5. Phase index (blue line, left y-axis) and absorption (green line, right y-axis) as function of $\omega - \omega_0$ normalized by $2\pi c$, calculated for $h = 0.01L$. The correspondence between the absorption peak and the zero phase index is marked by the crossing of the dashed lines.
between DPS and DNG media [27–28] when \( \varepsilon \) and \( \mu \) simultaneously change their sign over a very small region [29], as is the case in the considered structure. To gain further insight, it is also instructive to evaluate the phase index \( n_{ph} \) of light propagation in the \( z \)-direction, in the resonant frequencies. The phase index is calculated according to:

\[
    n_{ph} = \frac{c}{h} \frac{\Delta \phi}{\omega},
\]

where \( \Delta \phi \) is the phase difference between the transmitted and incident waves, and \( \omega = 2\pi c/\lambda \). In Fig. 5, we have plotted both the absorption and the phase index as function of frequency near resonance for \( h=0.01L \). It can be seen that at the peak of absorption, \( n_{ph} \) crosses zero (marked in Fig. 5 by the crossing of the dotted lines), indicating an asymptotically infinite phase velocity at this wavelength. This indicates again that at the absorption peak, the fields are evenly distributed between the DPS and DNG media. Since the phase velocity in both media is opposite, inside the grating there is full cancellation of the phase accumulation in the \( z \) direction.

It is well known from the theory of resonators [30], e.g. guided mode resonance filters [24–25], Ring Resonators [31] and prism coupling of surface plasmon polaritons [32], that an optimal coupling condition known as the “critical coupling” exists. Under the critical coupling condition, coupling losses and internal losses of the resonator (in our case it is the periodic layer) are equal, resulting in an absorption peak and zero transmission. In Fig. 6(a), the absorption as a function of the layer thickness is plotted, calculated using the same parameters as above for the wavelength of, \( \lambda = (1 + 6 \times 10^{-7})L \). It can be seen that for this wavelength, the optimal coupling into the periodic layer is obtained around \( h/L \approx 0.01 \). For smaller or larger values of \( h/L \) the internal losses in the resonator are detuned from their optimal value due to variations in mode confinement and the absorption in the periodic layers is reduced. Furthermore, it can be seen from Fig. 6(b) that the resonant absorption peak near \( h/L \approx 0.01 \), is a sharp peak in a slowly varying envelope of the absorption. This slowly varying envelope is the consequence of interference of waves inside Region II, similar to interference effects in a conventional lossy homogeneous dielectric slab, and is not the consequence of a transverse resonant mode (e.g. the transverse Fabry–Perot mode discussed above).

In order to verify if critical coupling is indeed obtained, we fitted the transmitted and reflected intensities to a Fano profile, using the temporal coupled mode theory (TCMT) model described in [30,33]. In order to extract the coupling and internal loss coefficients, we fitted the reflected field amplitude \( r \) obtained from the RCWA simulation to the function

\[
    r = r_D + \frac{1}{j(\omega - \omega_{cen}) + (1/\tau + 1/\gamma)} (-r_D + jI_D)
\]

and the transmitted field amplitude \( t \) to the function

\[
    t = jI_D + \frac{1}{j(\omega - \omega_{cen}) + (1/\tau + 1/\gamma)} (-r_D - jI_D).
\]

In this model, \( r_D \) and \( I_D \) are the direct (off resonance)
transmission and reflection coefficients respectively, $\omega_{\text{res}}$ is the resonant frequency, $\tau$ is the decay time in the resonator due to coupling and $\gamma$ is the decay time due to internal losses in the resonator. In Fig. 7, fitting results for $h=0.01L$ are shown, with $\gamma = 1.49 \times 10^6 L/(2\pi c)$ and $\tau = 1.22 \times 10^6 L/(2\pi c)$. This indeed indicates that the resonator is near critical coupling ($\gamma$ is of the order of $\tau$) [30,33].

Until now, we considered non-dispersive and lossy DNG and DPS media. While making the analysis of the structure easier, the above assumed media violate the causality restrictions given by the Kramers-Kronig relations. To remove this deficiency, we now repeat the RCWA calculation, but assume that both the permittivity and permeability functions of the DNG medium are given by Drude dispersion models, and that the DPS media in Region II is vacuum, namely:

$$
\varepsilon_1 = 1, \mu_1 = 1, \varepsilon_2 = 1 - \frac{\omega_p^2}{(\omega^2 - i\omega \gamma_\varepsilon)} \quad \text{and} \quad \mu_2 = 1 - \frac{\omega_p^2}{(\omega^2 - i\omega \gamma_\mu)}.
$$

The material parameters are: $\omega_p = 2\sqrt{2}\pi c/L$, $\gamma_\varepsilon = 1 \times 10^{-3} \omega_p$, and $\gamma_\mu = 1 \times 10^{-2} \omega_p$. As can be observed from Fig. 8 the resonance near $\lambda_0 \approx L$ is very similar to the resonance in the dispersionless case discussed above.

Figure 8. RCWA calculation of: (a) Transmission (b) Reflection and (c) Absorption as a function of $\lambda - \lambda_0$ and the normalized slab thickness. Calculation is made assuming a Drude dispersive model of the DNG medium.
4. Conclusions

We study the interaction of light with an infinite metamaterial periodic structure consisting of alternating layers of positive and negative refractive index. The modes supported by the structure are found using a full-vectorial electromagnetic calculation. We show that very high absorption of about 50% and sharp resonance are obtained for ultrathin periodic structures, down to ~two orders of magnitude thinner than the vacuum wavelength. This finding is attributed to the excitation of a Fabry Perot mode in the structure as well as for the existence of Rayleigh anomaly.

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