Effect of radial polarization and apodization on spot size under tight focusing conditions

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Abstract: We study the effect of polarization and aperture geometry on the focal spot size of a high numerical aperture (NA) aplanatic lens. We show that for a clear aperture geometry, illuminating the lens by linear or circular polarization is preferable over radial polarization for spot size reduction applications. For annular aperture and objective lenses of 0.85 NA and above we give the sizes of the inner annulus which constitute the transition points to a state where the radial polarization illumination gives smaller spot size. We analyze the evolution, the profile and the effect of transverse and longitudinal field components in the focal plane, and show that they play an opposite role on the spot size in the cases of circular and radial polarization illumination. We show that in the limit of a very thin annulus the radial polarization approaches the prediction of the scalar theory at high NA, whereas the linear and circular polarizations deviate from it. We verify that the longitudinal component generated by radially polarized illumination produces the narrowest spot size for wide range of geometries. Finally, we discuss the effects of tight focusing on a dielectric interface and provide some ideas for circumventing the effects of the interface and even utilize them for spot size reduction.

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References and links


1. Introduction

The radially polarized light is gaining growing attention in the last few years mainly due to the theoretical and experimental results that demonstrated its advantages over the well known linear and circular polarizations [1-12]. Specifically, it was shown that when using an annular aperture the radially polarized beam can be focused to a tighter spot compared with a linearly or circularly polarized beam [1, 2]. This is mainly attributed to the strong longitudinal component of the field that is generated in the focal plane. Based on the unique characters of strong longitudinal component and small spot size, the radially polarized beams are considered as the preferable choice for a variety of applications such as beam trapping [13], material processing [14] and optical memories.

While the superiority of radially polarized light for some specific choice of annular illumination and high NA was demonstrated numerically and experimentally, it turns out that radially polarized light is not always the best choice. Indeed, for spot size reduction applications the advantage of radially polarized light over the linearly/circularly polarized light becomes evident only for specific combinations of aperture illumination and NA. For other cases, it seems that radially polarized light is actually inferior in terms of spot size. Therefore, the major goal of this paper is to systematically investigate tight focusing under various combinations of apodizations and NA for radially and linearly/circularly polarized incident light and to provide guidelines for choosing the optimized polarization, apodization and NA of the objective lens. The paper analyzes the various field components and provides insight to their role in controlling the final energy density distribution at the focal plane,
where the energy density is proportional to $\varepsilon |E|^2$, where $\varepsilon$ being the dielectric constant of the medium and $E$ being the electric field. Throughout the paper we will use the definition of the term “spot size” as it appears in Ref. [2], as the area that is enclosed by the line having an energy density value which is half the maximum energy density, and it will be given in units of $\lambda^2$ where $\lambda$ is the wavelength of the light. For the circular symmetrical cases of radial and circular polarizations, this definition is equivalent to $\pi \cdot r_{1/2}^2$, where $r_{1/2}$ is the full width at half max (FWHM) of the focal spot divided by 2.

Section 2 explains qualitatively the role of the transverse and longitudinal components of the field and their contribution to the spot size for the cases of circularly and radially polarized incident beams. In section 3, we show that for a clear aperture and plane wave illumination the spot size obtained for linearly/circularly polarized incident beams is smaller than that obtained with radially polarized incident beam. In section 4 we find (for a given NA lens) the specific annular apertures for which the radially polarized beam gives smaller spot size than the circular/linear polarization. The trends that rule the behavior of the different polarizations are discussed in details. We calculate the variations in spot size when increasing the NA for a very thin annular aperture, and compare it with the scalar theory prediction. We also compare the criterion of spot size to that of a two-point resolution. The longitudinal component produces by the radially polarized incident beam is discussed in more details in section 5 and is shown to be the best choice for spot size reduction in wide range of NA and annular aperture geometries, assuming that the detection system is capable of distinguishing between the transverse and longitudinal field components. The real life scenario of light focused onto an interface between different refractive index media is discussed in section 6. Summary and conclusions are given in section 7.

2. Transverse and longitudinal polarizations contribution to spot size.

When a light beam is tightly focused to a small spot, a longitudinal field component is generated at the focal plane. The existence of the longitudinal field component can be intuitively explained by assuming bending of light rays (representing tilted plane waves) as they propagate through the high NA lens. Because of the tilt, the field represented by these plane waves can now be decomposed into non-zero transverse and longitudinal components, where the strength of the longitudinal field component increases with the increase of the bend angle. Therefore, for low NA lenses the longitudinal component is negligible comparing to the transverse component but it becomes dominant as the NA is increased. Another factor affecting the strength and the distribution of the longitudinal polarization is the polarization of the incident beam. For a linear polarization the longitudinal component at the focal plane is canceled on the optical axis and appears as two separate lobes [15] symmetrically located off the optic axis. For a circularly polarized incident beam the longitudinal component is shaped as a ring centered on the optic axis. For a radially polarized incident beam the longitudinal field component that is generated at the focal plane is very different from that produced by linearly/circularly polarized field. It is much stronger due to the on axis constructive interference of plane waves propagating towards the focal plane from opposite directions of the optical axis, and, its contribution is concentrated within a small spot centered on the optic axis. Figures 1 and 2 shows the energy density distribution of the longitudinal and transverse components and the total energy density distribution at the focal plane for circularly and radially polarized beam respectively, assuming 0.95 NA aplanatic objective lens and 0.9 NA of inner annulus.
It can be seen that for spot size reduction applications the longitudinal component is desirable for radial polarization illumination, whereas the transverse component is shaped as a donut and thus is undesired. For circular polarization illumination the trend seems to be the opposite: the longitudinal component is shaped as a donut, thus it increases the obtainable spot size, whereas the transverse component is localized. We thus conclude that for radially polarized illumination one should try to enhance the contribution of the longitudinal polarization component in order to shrink the spot size, whereas for linearly/circularly polarized illumination this component should be diminished. Further discussion on these trends is given in section 4.

3. Effect of polarization for a clear aperture

Based on Richards and Wolf integrals [16], Brown and Youngworth [17] developed mathematical expressions for the optical field distribution at the focal plane of an aplanatic lens for radially polarized incident field. Using these expressions, we calculated the spot size at the focal plane for linearly, circularly and radially incident polarized beams and NA ranging from 0.1 to 0.95. For now we assume plane wave illumination and clear aperture, i.e. the center of the aperture is not blocked. Figure 3 shows the calculated spot size (in units of $\lambda^2$, where $\lambda$ is the wavelength of light) vs. the NA of the lens for the radially, and circularly polarized incident beams. The spot size of the longitudinal component of the radially polarized beam ($|E_z|^2$) is shown as well. It is obvious from the results, that for a clear aperture the circularly polarized beam can be focused to a tighter spot than the radially polarized beam, therefore, when using a clear aperture, the circularly polarized beam is preferred over the radial polarization for the purpose of spot size reduction. The linear polarization behaves very similar to the circular polarization, but it looses its circular

![Fig. 1. Energy density distribution of longitudinal (left) and transverse (center) components, and total energy density (right) for circular polarization illumination and an annular aperture with 0.95 NA objective lens and 0.9 NA of inner annulus. Axes are in wavelength units. Units in color bars are arbitrary.](image1)

![Fig. 2. Energy density distribution of longitudinal (left) and transverse (center) components, and total energy density (right) for radial polarization illumination and an annular aperture with 0.95 NA objective lens and 0.9 NA of inner annulus. Axes are in wavelength units. Units in color bars are arbitrary.](image2)
symmetry if high NA is used). Considering the conclusion of the previous section, this result is of no surprise. For clear aperture illumination, there is a significant contribution from plane waves that are barely tilted, which have a small longitudinal field component compared with their transverse field component. As a result, focusing of radially polarized beam using clear aperture will always include a non-localized donut component at the focal plane, resulting from the transverse polarization, which prevents the spot size from being reduced beyond that obtained with circularly polarized incident field. Therefore, further reduction in spot size for radially polarized incident field requires the diminishing of plane waves that contain large portion of transverse field component. This can be achieved using annular aperture illumination. Alternatively, one can either use a non-aplanatic lens, e.g. the parabolic mirror or Fresnel zone plate offering better apodization factor [18], or a nonuniform illumination function, e.g. higher order radially polarized mode beam [19].

![Graph showing spot size vs. NA for clear aperture and radially (triangles) and circularly (circles) input polarized fields. The z component of the radially polarized beam is also shown (rectangles). Inset: zoom in on the high NA section of the graph.]

**Fig. 3.** Spot size of energy density vs. NA for a clear aperture and radially (triangles) and circularly (circles) input polarized fields. The z component of the radially polarized beam is also shown (rectangles). Inset: zoom in on the high NA section of the graph.

4. Polarization effects for annular aperture illumination

As previously discussed, the main contribution to the longitudinal field component in the focal plane is coming from the outermost rays of the beam. These rays are strongly bent towards the focus so that the longitudinal component is increased and the transverse component is decreased. As the NA of the lens is increased, the bend angle of the outermost rays is increased and so is the longitudinal component. When blocking the inner section of the aperture only rays with relatively high bent angle can reach the focal plane and the portion of the longitudinal component relative to the portion of the transverse component is increased. The larger the inner opaque annulus the stronger the longitudinal component will be, but this, of course, is on the expense of decreasing efficiency. The later restriction can be removed by using a mode converter that converts the fundamental mode into a higher mode that better overlaps the shape of an annular aperture [20].
We calculate the energy density distribution for the case of an annular aperture and an aplanatic objective lens with a given NA, and find the spot size at the focal plane. For each NA value we find the specific annular aperture from which the spot size obtained from the radial polarization illumination will be smaller than the one obtained from linear and circular polarizations. We define the NA of the inner annulus as NA$_{\text{min}}$, and the NA of the outer annulus as NA$_{\text{max}}$, where NA$_{\text{max}}$ is the NA of the objective lens. Figures 4, 5, and 6 show the calculated spot size for linearly, circularly and radially polarized beams vs. the annulus ratio for NA$_{\text{max}}$ of 0.85, 0.9 and 0.95 respectively. The spot size of the longitudinal component for the case of radially polarized incident beam is shown as well.

![Graph](image-url)

Fig. 4. Spot size of energy density vs. NA$_{\text{min}}$/NA$_{\text{max}}$ for radially (triangles), linearly (circles) and circularly (rectangles) polarized beams for the case of NA$_{\text{max}}$ of 0.85. The longitudinal component of the radially polarized beam is shown as well (stars).
By observing these curves, we find that the transition point from a state where the linear or circular polarizations are focused into a tighter spot to a state where the radial polarization
is focused into a tighter spot is shifted towards lower annulus ratios when the NA_{max} is increased. Thus, using a higher NA objective lens allows using a larger portion of the aperture and still obtaining tighter spot for radially polarized incident beam. This of course, allows more flexibility in imaging system design to improve the light collection efficiency and to suppress the side lobs. For NA of 0.8 and below we can not identify a transition point; the linear or circular polarizations are focused into a tighter spot compared with the radial polarization for all annulus ratios. This result is somewhat expected, because at moderate NA values the longitudinal polarization component is not dominant, and its contribution to spot size reduction is minor. The results for NA of 0.9 coincide with the results of reference [2], where they have used a specific annulus ratio of ~0.917 to show that the radial polarization is focused into a tighter spot than linear and circular polarizations.

In order to better understand the trends shown in Figs. 4-6, we need to distinguish between two different phenomena that occur when the annulus ratio is increased. We are expecting: A- a decrease in spot size of both transverse and longitudinal field components, and B - the strengthening of the longitudinal polarization component over the transverse component due to the blocking of rays with low bend angle. For circular polarization illumination the first phenomena is beneficial for spot size reduction, whereas the second is undesired. As the longitudinal polarization becomes more dominant it cancels out the reduction of the spot size and can even cause an increase of the spot size at high annulus ratios (Fig. 6). Though annular aperture is desired for spot size reduction with circular/linear polarized incident light, at high NA and high annulus ratios this can turn into a two-edged sword, by strengthening the, donut shaped, z-component so much that it broadens the achieved spot. When a radial polarization illumination is used, on the other hand, these two phenomena both assist in spot size reduction and thus the spot size is drastically reduced with the increase of the annulus ratio (Figs. 4-6). Not only the spot size of both transverse and longitudinal field components decrease, but the longitudinal component (which is always tighter than the transverse component (Fig. 2) in radial polarization illumination) becomes much more dominant. This explains the trends of the spot sizes of the different polarizations with annular aperture and the tremendous improvement of the radial polarization’s spot size due to these two complementary phenomena.

To qualitatively explain the above trends we now observe each component of the field at the focal plane. Figures 7 and 8 show the spot sizes of the transverse and longitudinal components and the spot size of the total field for circular and radial polarization illumination, respectively. When the intensity distribution has the shape of a donut (as in the transverse component of radial polarization and longitudinal component of circular polarization) we define the spot size as the area enclosed by the line having an energy density value which is half the maximum energy density of this specific polarization component, including the null area in the middle. As expected, both the transverse and longitudinal components of the field shrink when the annulus ratio is increased. The total spot size in both cases is seen to shift from the transverse component curve towards the longitudinal component curve as the annulus ratio is increased. This, as was explained above, is desired for the radial polarization but poor for the circular polarization illumination.
Next we evaluate the spot size vs. NA for a very thin annulus, i.e. for annulus ratio approaching 1. In Fig. 9 we plot the spot size vs. NA for circular, linear and radial polarized
light. For comparison, we also plot the result predicted from scalar theory (an intensity of 
$I(r) \propto J_0^2(krNA)$ \cite{21} leading to $r_{1/2} = 0.1793(\lambda / NA)$ and spot size of 
$\pi \cdot r_{1/2}^2 = 0.101(\lambda / NA)^2$). It can be seen that as the NA increase, the linear and circular 
polarizations deviates from the scalar theory prediction. Moreover, increasing the NA beyond 
0.85 results in an increase in spot size of the circularly polarized light. In contrast, the spot 
size of the radially polarized light decreases with increasing the NA, and approaches the scalar 
theory limit, but does not go beyond it. This can be understood by observing the longitudinal 
polarization term in Ref. \cite{17} (see Eq. 8). One can notice that if the annulus thickness is small, 
this term can also be approximated by $J_0(krNA)$, similar to the scalar theory limit. 
Therefore, even for the case of high NA and high annulus ratios, radially polarized light, 
although producing smaller spot compare to linear/circular light, cannot be focused beyond 
the scalar theory limit.

![Fig. 9. Spot size of energy density vs. NA for radially (triangles), linearly (circles) and 
circularly (rectangles) polarized beams. The z component of the radial polarization (stars) and 
results from scalar theory (diamonds) are shown as well. Annulus ratio of 0.99 is used.](image)

So far, we used the criterion of spot size as a figure of merit. However, reducing the spot 
size does not guaranty an imaging system with improved resolution. For example, reducing 
the spot size is often accompanied by strong side lobes which might degrade the two point 
resolution of the imaging system. To evaluate the relevancy of spot size as a criterion for 
resolution, we calculate the two-point resolution of an imaging system for circular and radial 
polarizations. As a test case we choose NA of 0.9 and 3 values of NAmIn (0.45, 0.65 and 
0.85), corresponding to cases where 1 - the circular polarization gives tighter spot, 2 - the 
circular and radial polarizations give same spot size and 3 - the radial polarization gives 
tighter spot, respectively (see Fig. 5). For each case we calculate the distance in which two 
point sources are just resolved according to Rayleigh criterion for the circular and radial 
polarizations. For each NAmIn value we find which polarization gives smaller Rayleigh 
distance, and plot the energy density for two equally bright point sources separated by this 
distance for the two polarizations (See Fig. 10).
It can be seen that for NAmin of 0.45 the circular polarization gives better two point resolution, for NAmin of 0.65 the circular and radial polarizations give similar resolution and for NAmin of 0.85 the radial polarization gives better resolution. This is in accordance with the results shown in Fig. 5.

5. Use of longitudinal component for breaking resolution limits

So far we have compared between the energy density distributions of linearly, circularly and radially polarized beams at the focal plane for various annulus ratios and various NAs. From each of the above graphs one can clearly notice that the longitudinal field component of the radially polarized beam gives a smaller spot compared with the spot size obtained by using circular/linear polarization illumination even in cases where the total energy density of the radial polarization gives bigger spot. Thus, if the longitudinal component could be separated from the transverse component of the radial polarization, it would be the best choice for applications that require spot size reduction. Beversluis, et.al. [15] used single molecule fluorescence excitation to separate the components of a radially polarized beam at the focal plane. Molecules oriented along the x, y and z axes may produce a stronger response to the x, y and z components of the field, respectively. Therefore, if it is possible to realize structures having molecules aligned along the z axis, the longitudinal component is expected to become more dominant when interacting with the sample and a smaller spot size will be obtained. Another system that can be designed to be sensitive mostly to the longitudinal field component is the near field scanning optical microscope (NSOM) tip. It was theoretically shown [22,23] that when an NSOM tip is illuminated by a longitudinally polarized field (i.e. the polarization direction is along the tip axis), the field at the apex of the NSOM tip is strongly enhanced, whereas when the same tip is illuminated by a transversely polarized beam, no field enhancement is observed. Thus, this system can use the radial polarization and detect mainly the z-polarized component, resulting in a smaller spot size and higher resolution for microscopy applications.

If the longitudinal field component can indeed be separated from the transverse field component by some experimental methods, then the constraint of very high NA and very high annulus ratio can be somewhat relaxed. For example, Fig. 11 shows the calculated spot size for circularly and radially polarized beams vs. the annulus ratio for a moderate NAmmax of 0.7. The spot size of the longitudinal component generated by radially polarized incident beam is shown as well.
Fig. 11. Spot size of energy density vs. NAmín/NAmáx for radially (triangles) and circularly (circles) polarized beams for the case of NAmáx of 0.7. The longitudinal component of the radially polarized beam is shown as well (rectangles).

It is clear that for this NA there is no annular aperture that gives tighter spot for the radial polarization over the circular one. Yet, the longitudinal component generated by the radial polarization illumination produces the tightest spot and therefore is the preferable choice even for such moderate NA value. If the NA is further decreased the advantage of using the longitudinal component of the radial polarization over using the circular polarization becomes negligible. For low NA values and low annulus ratios we can even identify cases where the spot size of the longitudinal polarization component becomes larger than that obtained with circularly polarized light. We thus conclude that if a system can be made sensitive only to the longitudinal polarization component, then using a radially polarized incident beam allows to A – achieve the smallest spot size for high NAs and high annulus ratios, and B – significantly widen the range of NAs and annulus ratios for which the radially polarized incident field is preferable over the circularly polarized incident field for spot size reduction applications.

6. Tight focus on a dielectric interface

In the previous sections we investigated free space focusing. Nevertheless, practical imaging/lithography scenarios frequently involve interfaces between media having different refractive indices. For such cases one needs to impose Maxwell’s boundary conditions and to take into account also the reflected and transmitted fields [24, 25]. For example at a dielectric interface there will be a significant difference between focused circularly polarized light and radially polarized light as calculated by Biss and Brown [26]. The effects of interfaces were also studied experimentally [27, 28].

We implement the equations given by Ref [26] to investigate a real life scenario involving tight focusing of radially polarized light through an interface. Figure 12 shows an example of the interface effect when passing from a medium with refractive index of unity to a medium with refractive index of 1.5. This scenario describes a typical case of recording a pattern in photoresist (we neglect the absorption of light in the photoresist). The cross sections of the energy density before and after the interface are very different, as the transmitted beam has two distinct separated lobes.
This phenomenon can be explained by the discontinuity of the longitudinal field at the boundary. From Maxwell’s equations one can show that the transmitted longitudinal electric field is decreased by a factor of $\frac{\varepsilon_2}{\varepsilon_1}$ ($\varepsilon_1, \varepsilon_2$ are the dielectric constant of the media before and after the interface respectively) with respect to the longitudinal field in front of the interface, whereas the transverse component is continuous over the interface. As a result, the contribution of the longitudinal field to the energy density is dropped by a factor of $\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2$ with respect to the contribution of the transverse field. This problem can be solved by using an oil immersion lens with matching refractive index which removes the effect of the interface. For such a case our previous curves are still valid with the exception of replacing $\lambda$ by $\frac{\lambda}{\sqrt{\varepsilon}}$. For example, if light is propagating through a 1.425 NA oil immersion lens in a medium of 1.5 refractive index, one can refer to the NA=0.95 curve and divide the spot size by $(1.5)^2 = 2.25$. Another solution can be the use of a solid immersion lens with higher refractive index, [29]. For such a case, the contribution of the longitudinal field to the energy density will be enhanced with respect to that of the transverse field, and the spot size will be reduced compare with the case with no interface.

Another practical scenario is the focusing of light on the interface between a thin layer of photoresist with refractive index 1.5 and a silicon wafer with refractive index 3.5. For simplicity we use index matched oil immersion lens so that there is only one interface in the described geometry. Absorption is ignored. Figure 13 shows the energy density of the field passing through the interface. Now we are interested in the energy density in front of the interface, where the exposure of the photoresist takes place. Figure 14 shows the cross section of the energy density, just in front of the interface. For comparison, we also plot the cross section of the energy density for a case where the interface is removed, i.e. the silicon is replaced with a medium having refractive index of 1.5. Surprisingly, the FWHM for the first
case is smaller by 20% compare with the latter configuration, further improving the achievable spot size for lithography applications.

Fig. 13. Energy density before and after the interface for the case of passing from refractive index of 1.5 to 3.5 media. Clear aperture and NA=0.95 lens is assumed. Scale is in arbitrary units.

Fig. 14. Cross section of the energy density at the focal plane for the case of passing from refractive index of 1.5 to 3.5 media (red). For comparison, we also plot the energy density for a case focusing through a uniform medium having refractive index of 1 (blue).
7. Summary and conclusions

In summary, we investigate the effect of polarization and aperture geometry on the spot size at the focal plane of a high NA aplanatic lens. We find that if a clear aperture is used, the radial polarization is inferior compared with the linear and circular polarization for spot size reduction applications. For an annular aperture and objective lenses of 0.85 and above there exist transition points from a state where the linear or circular polarizations are focused into a tighter spot to a state where the radial polarization is focused into a tighter spot. These transition points are shifted towards lower annulus ratios when the NA of the objective lens is increased. We also find that the spot size generated by the radially polarized incident field reduces drastically with the increase of the annulus ratio, whereas the spot size generated by the circular/linear incident polarization decreases moderately and even increases for very high NAs and annulus ratios. These findings are explained by considering the opposite spatial distribution of transverse and longitudinal field components for radial and circular polarization illumination. We show that in the limit of a very thin annulus the radial polarization approaches the prediction of the scalar theory at high NA, whereas the linear and circular polarizations deviate from it. It is shown that the longitudinal component of the radially polarized field provides the tightest spot size for all annulus ratios (including the case of clear aperture) at relatively high NAs (~0.7 and above). The isolation of the longitudinal field component requires the realization of materials and detectors that are mainly sensitive to the longitudinal component, such as z oriented molecules and NSOM tips. Finally, we discuss the effects of tight focusing on a dielectric interface and provide some ideas for circumventing the effects of the interface and even utilize them for spot size reduction.