Highly dispersive micro-ring resonator based on one dimensional photonic crystal waveguide design and analysis

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Abstract: We propose and analyze a novel design of a hybrid micro-ring resonator and photonic crystal device. The proposed device is based on a micro-ring resonator with the addition of a series of periodic defects that are introduced to the microring. When the wavelength of operation approaches the band-gap of the periodic structure, the modal dispersion is significantly increased. The huge dispersion leads to narrowing of the spectral linewidth of the resonator. We predict an order of magnitude linewidth narrowing for a microring radius of the order of 10 μm. The proposed hybrid device is analyzed theoretically and numerically using finite-elements calculations and finite-difference-time-domain calculations. We also present as well as the design and analysis of add-drop and notch filters based on the highly dispersive ring resonator.

OCIS codes: (130.3120) Integrated optics devices, (140.4780) Optical resonators, (999.9999) slow light

References and Links
12. All the calculations in this paper are in 2D. Since we examine only channel waveguides (as opposed to 2D PhC, for example) the 2D approximation gives satisfying and qualitative and sometimes even quantitative results.
1. Introduction

Photonic Integrated Circuits (P-ICs) have been subject to extensive research over the last decades. Advances in technology enabled the realization of various exciting IC-based optical devices. Among those, a key component is the optical resonator. Various types of resonators were demonstrated, including, for example, micro-ring resonators (MRR)\(^1\), Photonic Crystals (PhCs) cavities\(^2\), Bragg-based Fabry-Perot resonators\(^3\), microdisk resonators etc. The potential applications for such resonators include modulators, filters, sensors, optical delay lines, nonlinear optics and more. An important characteristic of the optical resonators is its Quality factor ($Q$-factor) that is inversely proportional to the photon lifetime in the resonator (in time domain). In most cases (including the cases discussed in this paper) the $Q$-factor is also inversely proportional to the resonator linewidth (in spectral domain), although one can also design a high $Q$ resonator with broader linewidth. Different applications require different $Q$-factor values; however, obtaining higher $Q$-factors for a resonator is typically of high interest. More precisely, one typically seeks to maximize the $Q/V$ ratio where $V$ is the modal volume of the optical field inside the resonator.

The overall $Q$ factor of a resonator is given by \[ Q = Q_{abs} + Q_{rad} + Q_c \] where $Q_{abs}$, $Q_{rad}$, and $Q_c$ are the absorption, radiation and coupling quality factors respectively. It has been shown that $Q_c$ can become strongly wavelength dependent if the coupling is controlled by interference [4]. Moreover, it has been shown before that each of the $Q$-factor terms can be increased by using a highly dispersive material inside the resonator [5, 6]. However, such materials are not common in nature and are difficult for integration with standard PIC substrates. One possible approach to overcome this obstacle is by embedding sub-wavelength structures into ordinary photonic materials giving rise to so called “artificial materials”. These artificial materials can tailor the dispersion properties of the device\(^7\), \(^8\). For example, it is known that PhC waveguides can be used to generate slow light, i.e., low group velocity when approaching their band gap\(^9\). Thus, if properly designed, PhCs can be used instead of natural highly dispersive materials for achieving high $Q$-factors in optical resonators.

In this paper we propose and analyze a hybrid resonator realized by incorporating PhC structures into standard MRRs. We analyze Bragg-like structures embedded inside rectangular channel waveguides and examine the effect of using these channels to form a modified MRR. We provide a precise recipe for the design of these modified resonators, and find the proper conditions for achieving high $Q$-factors. MRRs are widely used for different practical applications therefore the presented analysis has the potential of impacting and improving many works in the field of P-ICs, micro- and nano-photonics. It should be mentioned that the dispersion in the proposed device is controlled by adding periodic structures into the MRR, thus it is inherently different from works where random structures where used\(^10\). It is also very different from the annular Bragg resonator geometry\(^11\), where the subwavelength structures are positioned outside the waveguide, and the periodicity is perpendicular to the propagation direction.

2. Theoretical Analysis\(^12\)

2.1 One dimensional photonic crystal waveguides

We start by analyzing the main building block of the modified MRR i.e. channel waveguide with embedded periodic defects (Equivalent to a one dimensional PhC). The analysis is similar to that presented in Ref [13], however, focuses on the specific requirements of our work. Different types of periodic defects can be introduced (see examples in Fig. 1(a) and
although they have different distribution of the dielectric structures, they essentially yield similar dispersion curves. The dispersion curve is calculated using the Plane-Wave-Expansion (PWE) method with the super cell approach. Fig. 1(b) shows the first few modes of the circular-hole defect 1D PhC waveguide. For calculations we assumed waveguide’s core refractive index of 2.798 (equivalent to the effective refractive index of a 240nm thick Silicon Slab for in-plane polarization), clad refractive index of 1, core width 450nm and defect-hole diameter 180nm. These parameters simulate strip silicon waveguides.

As can be seen from Fig. 1(b), several band-gap regions are formed due to the periodicity and the strong dielectric contrast. More importantly, at the wavelength range close to the band-gap edge the dispersion curve is flattened and a highly dispersive mode is obtained.

![Fig. 1.](image)

(a) Two types of dielectric structures (single cell) embedded in the waveguide. Red corresponds to the waveguide core. (b) Dispersion curve of the 1D PhC waveguide with circular-hole periodic defect. The inset depicts the mode profile (Hy field – in plane polarization) of the waveguide’s lowest mode.

In this work we focus on the lowest PhC mode, in the vicinity of the band edge. Two parameters are of interest: The effective refractive index \( n_{\text{eff}} \), and the corresponding group index, \( g_{\text{eff}} = n_{\text{eff}} - \lambda \cdot (dn_{\text{eff}}/d\lambda) \). Fig. 2(a) depicts an enlarged view of the band diagram around the band-edge. Figure 2(b) shows \( n_{\text{eff}} \) and \( g_{\text{eff}} \) as a function of the normalized wavenumber (k-vector). As expected, ultra high group index values are obtained as the k-vector approaches the band-edge.

![Fig. 2.](image)

(a) Enlarged view of the dispersion diagram presented in Fig. 1 around the band edge of the first mode. (b) The corresponding calculated effective index (continuous line) and group index (dashed line).
2.2 Highly dispersive micro-ring resonator

Next we analyze a MRR composed of the waveguides described above (modified MRR). We assume that the MRR’s radius is large enough, such that the mode shift due to the bend is small, and the mode can be assumed to be identical to that of a strait waveguide. This assumption is particularly true for silicon waveguides, where the high refractive index of the core leads to strong mode confinement.

Naturally, the effects of interest occur in the wavelength range where a resonance exists. Simultaneously, we are interested in resonance frequencies within a specific range of propagation constant values, towards the band edge. To meet these two parallel requirements, we first adjust the period of the structure such that the band-gap will match the desired operation wavelength (e.g. 1.55 μm for communication applications). In addition, the MRR perimeter must meet two conditions. The first is phase matching, given by:

\[ 2\pi R \cdot K_z = m \cdot 2\pi \]

where \( R \) is the ring’s radius, \( K_z \) is the mode’s propagation constant and \( m \) is an integer. The second condition is periodicity. The existence of Bloch modes requires integer number of periods within the MRR, i.e.

\[ 2\pi R = q \cdot P \]

where \( P \) is the period and \( q \) is an integer. Equations (1) and (2) impose discrete values of \( R \) for given values of \( K_z \) and \( P \). Fig. 3 depicts the minimum possible values of \( R \) for various values of \( K_z \) (and a chosen \( P \)). Note that according to the results in Fig. 3 an MRR radius of 13 μm is sufficient to obtain a group index value of over 50. Moreover, in that radii values regime there is an increased amount of \( K_z \) values that yield minimum radius values, thus, making the device design much simpler.

2.3 Highly dispersive micro-ring resonator-based add-drop and notch filters

Once a specific device design is obtained, we turn into calculating the Q-factor enhancement. We consider both the notch filter configuration (i.e. single bus waveguide coupled to an MRR) as well as the add-drop filter configuration (i.e. two bus waveguides coupled to an MRR). The two configurations are shown schematically in Fig. 4.

Assuming single mode waveguides, a standard MRR supports two degenerate modes – clockwise traveling and counter-clockwise traveling. Typically, only one mode is excited, e.g. clockwise traveling mode. In our modified MRR, however, we excite both modes due to reflections from the periodic structure. In addition, dispersion effects need to be taken into account. To analyze our device we thus use the formalism presented in Ref. [14] to which we incorporate the waveguide dispersion effect by using the perturbation theory method.
presented in Ref. [6]. Similarly to Ref. [15] we also add reflecting structures at ports 2 and 4 (see Fig. 4). For the add-drop configuration, these reflectors ensure maximization of power transfer between the ports 1 and 3 at resonant frequencies. For the notch filter these reflectors assist in the minimization of power reflection. Throughout the analysis we assume the reflectivity of the reflectors to be one. Note that the structure period of the reflectors is different of the period inside the ring.

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Considering the two configurations shown in Fig. 4, and assuming an input signal fed through port 1, the field’s amplitudes within the resonators are given by:

$$\frac{da_{ad}}{dt} = \left( j\omega_0 - \frac{1}{\tau_0} \frac{2}{\tau_c} \right) a_{ad} + \kappa \left[ \exp(-j\beta d_1) s_{s1} + \left( s_{s2} + s_{s4} \right) \exp(-j\beta d_2) \right]$$

$$\frac{da_{nt}}{dt} = \left( j\omega_0 - \frac{1}{\tau_0} \frac{1}{\tau_c} \right) a_{nt} + \kappa \left[ \exp(-j\beta d_1) s_{s1} + \exp(-j\beta d_2) s_{s2} \right]$$

(3)

where subscripts $ad, nt$ denote the two configurations (‘$ad$’ – Add-drop, ‘$nt$’ – Notch), $\omega_0$ is the resonance frequency, $1/\tau_0$ is the decay rate due to loss, $1/\tau_c$ is the decay rate into a single coupling waveguide, $\kappa$ is the input coupling coefficient associated with the traveling modes in the waveguide (which by symmetry is equal to all ports), $\beta$ is the propagation constant of the mode propagating within the bus waveguides and $d_1, d_2$ are defined in Fig. 4.

By power conservation the outgoing waves are given by:

$$s_{s1} = \exp[-j\beta(d_1 + d_2)] \left[ s_{s2} - \kappa \exp(j\beta d_2) a \right]$$

$$s_{s2} = \exp[-j\beta(d_1 + d_2)] \left[ s_{s1} - \kappa \exp(j\beta d_1) a \right]$$

(4)

The coupling coefficient is given by:

$$\kappa = \frac{1}{\sqrt{\tau_c}}$$

(5)

The reflectors’ effect is expressed via:

$$s_{s2} = s_{s2} \cdot e^{-\Delta}$$

(6)
where $\Delta$ is the phase added by the reflector. Assuming that $s_{s_1}$ has $\exp(j\omega t)$ time dependence, the mode’s amplitude in the resonator in steady state is given by:

$$
\begin{align*}
ad_{ad} &= \sqrt{\frac{1}{\tau_e}} \exp\left(-j\beta d_{ad}\right) \left\{1+\exp\left(-j\theta\right)\right\} s_{s_1}, \\
d_{nt} &= \sqrt{\frac{1}{\tau_e}} \exp\left(-j\beta d_{nt}\right) \left\{1+\exp\left(-j\theta\right)\right\} s_{s_1},
\end{align*}
$$

where,$$
\theta = 2\beta d_{nt} + \Delta
$$

So far, we ignored dispersion within the resonator. Following Ref. [6] we now take dispersion into account by assuming that the effect of dispersion only induces a frequency dependence into the resonance frequency, $\omega_0$. The modified frequency is now defined by

$$
\tilde{\omega}_0 = \omega_0 \left(1 - \frac{n_{\text{eff}}(\omega) - n_{\text{eff}}(\omega_0)}{n_{\text{eff}}(\omega_0)}\right)
$$

The resulted reflection and transmission coefficients for configuration ‘ad’ are given by:

$$
\frac{s}{s_{s_1}} = -\frac{1}{\sqrt{\tau_e}} \left\{1 + \exp\left(-j\theta\right)\right\} \exp\left(-j\beta d_{ad}\right) \cdots
$$

$$
T \equiv \left|\frac{s}{s_{s_1}}\right|^2 = \left[\frac{2}{\tau_e} (1 + \cos \theta)\right]^2 + \left[\frac{1}{\tau_0} \left(\frac{2}{\tau_e} (1 + \cos \theta)\right)^2 + \left[\omega - \tilde{\omega}_0 + \frac{2}{\tau_e} \sin \theta\right]^2\right]
$$

$$
R \equiv \left|\frac{s}{s_{s_1}}\right|^2 = 1 + T + 2 \Re \left\{\frac{s}{s_{s_1}}\right\}
$$

The reflection coefficient for configuration ‘nt’ is given by:

$$
R \equiv \left|\frac{s}{s_{s_1}}\right|^2 = \left[\frac{1}{\tau_0} \left(\frac{1}{\tau_e} (1 + \cos \theta)\right)^2 + \left[\omega - \tilde{\omega}_0 + \frac{1}{\tau_e} \sin \theta\right]^2\right]
$$

We used the above equations and the dispersion curve shown in Fig. 2 to compare between spectral characteristics of a standard MRR and our modified MRR. We chose four values for the dispersive waveguide’s normalized propagation constant, $K_n$ ($K_n = K_Z \cdot \text{Period}/2\pi$) and calculated the corresponding transmission and reflection coefficients. To make the comparison more realistic, we included waveguide dispersion in the standard MRR calculation. For standard MRR the additional reflectors are not necessary since the field within the resonator is purely traveling. For the loss decay rate calculations we have effectively expressed all the loss components into the propagation loss constant, $\alpha$, that is given in $cm^{-1}$ units. The parameters used for the calculations are given in Table 1. The Period
was chosen to allow resonance at approximately 1.56μm wavelength and simultaneously at high $K_n$ values. Once those parameters were chosen, the minimum radii for the specific $K_n$ were calculated. Note that by more accurately choosing the $K_n$ values one can get the minimum radii values presented in Fig. 3. This also explains the non-consistent increase of the corresponding minimum radii in Table 1. We chose very high attenuation constants in Table 1 in order to take into account radiation loss resulted from coupling into leaky modes by the 1D PhC structure in a “worst case” scenario. The coupling decay rates were chosen such that high transmission efficiency (add-drop) and maximum transmission slope in (notch) can be obtained.
Table 1. Parameters for calculation of the frequency response obtained by the modified MRR filter

<table>
<thead>
<tr>
<th>Period [μm]</th>
<th>0.37</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_n$ values</td>
<td>0.48 0.49 0.495 0.497</td>
</tr>
<tr>
<td>$n_g (\alpha_g)$</td>
<td>6.88 12.29 23.82 39.46</td>
</tr>
<tr>
<td>Min. radii [μm]</td>
<td>1.47 5.88 11.77 9.83</td>
</tr>
<tr>
<td>$\alpha [dB/cm]$</td>
<td>$\frac{1}{3}$ – standard MRR $\frac{1}{60}$ – modified MRR</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>add-drop - $\tau_e = 0.1 \cdot \tau_0$ notch - $\tau_e$ - set to critical coupling</td>
</tr>
<tr>
<td>$n_{g,reg}$</td>
<td>3.35</td>
</tr>
</tbody>
</table>

- $^a$ corresponding group index at resonance
- $^b$ attenuation constant
- $^c$ regular waveguide group index

Figure 5(a) shows the transmission vs. the normalized wavelength deviation for several values of $k_n$ in the ‘ad’ configuration. As expected, the linewidth shrinks as $k_n$ approaches 0.5. Figure 5(b) shows the FWHM of the modified MRRs’ spectral response vs. $k_n$ as calculated from Fig. 5(a). The FWHM values were normalized with respect to the FWHM of the standard MRR. Figure 6(a) shows the reflection vs. the normalized wavelength deviation for several values of $k_n$ in the ‘nt’ configuration. Figure 6(b) shows graph of the maximum slope obtained for curves in Fig. 6(a). The slope values were normalized with respect to the maximum slope of a standard MRR. One can see that in spite of its high propagation loss, the quality factor ($Q = \frac{1}{\delta \nu}$) of the modified MRR improves by a factor of 8 with less than 10μm radius. Increasing the radius to 30μm allows to increase the $Q$-factor by a factor larger than 70. It is important to note that the spectral responses of the dispersive MRRs are significantly asymmetrical. This is because of the asymmetry in effective index as the resonance frequency is approaching the band-gap wavelength region ($k_n$ approaches 0.5).
3. Numerical simulations

To verify the theoretical model and the calculations presented above we performed several numerical simulations using the finite element approach. All simulations were two-dimensional (2D). We used in-plane (TE) polarization excitation. In order to correctly simulate silicon structures in 2D geometry we used the value of 2.798 for silicon’s effective refractive index, corresponding to a silicon slab thickness of 240nm. We also assumed waveguide width of 450nm.

The first simulated structure was similar to that depicted in Fig. 4(b). For simplicity we removed the reflector previously located on the coupling waveguide. We chose the period of the 1D PhC to be 0.37 μm and designed the modified MRR to have 100 periods, yielding MRR radius of 5.889 μm. The separation between the MRR and the bus waveguide (the MRR-waveguide gap) was 250nm. Fig. 7 shows the simulation transmission vs. wavelength. Unlike the case of a standard MRR where the resonance wavelengths are almost equally spaced, we observe a rapidly changing free-spectral-range (FSR) when approaching the band-edge wavelength and a band-gap region where no resonance exists. The shrinking FSR results from the increase of the group index inside the MRR. We should note that we achieve a fairly deep notch, however, not a perfect null in the transmission curve. The reason for that is that the MRR-waveguide gap resulted a loss decay rate, $1/\tau_g$, much smaller than the coupling decay rate, $1/\tau_c$. A perfect null in transmission can be obtained by modifying the coupling decay rate to achieve critical coupling. The dashed vertical lines in Fig. 7(a) indicate the locations of the expected resonances according to the theoretical model presented in the previous section. Although the simulation results agree qualitatively with the theoretical model, the quantitative results are somewhat different. That difference is expressed in both band edge wavelength that is shifted by approximately 20nm and in FSR values that are smaller in the simulation. We believe that the differences arise from the assumption of a non-bended 1D PhC waveguide. For this assumption to be valid a larger MRR radius is required. In practice however, we could only simulate MRRs with a radius smaller than 9μm because of limited computer memory. The FSR shrinking and the comparison between simulation and calculation are clearly depicted in Fig. 7(b) where we present the group index calculated by the theoretical model and the group index obtained from the numerical simulation. The numerical simulations still show the exponential behavior towards the band-gap; however the group index is higher than predicted by the theoretical model.
Next we examine the transmission deeps in more details. Fig. 8(a) shows a magnified view of the transmission deeps that appear in Fig. 7(a). For comparison purposes all curves were centered and normalized with respect to the resonance wavelength $\lambda_0$. We expected the transmission deeps to become narrower as the resonance wavelength approaches the region of high group index, i.e., the band-edge. Indeed, we see that Notch1 (the transmission deep closest to the band-edge) is significantly narrower than the others. Nevertheless, no major difference can be observed between all the other transmission deeps. For quantitative comparison purposes we also calculated the maximal slope for each resonance wavelength. Results are shown in Fig. 8(b). It is clear that maximal slope is obtained for Notch1, while for all other resonance wavelengths no major difference in slope values can be observed. We believe that this can be attributed to the operation conditions of the device, i.e., the coupling efficiency as well as the radiation loss. These conditions are different for each resonance wavelength. Apparently, because of the high sensitivity around the band edge, the device needs to be optimized for each resonance wavelength independently. We note that by increasing the MRR radius we could obtain higher group index and further reduction in linewidth of the filters. Unfortunately, such simulation requires large computational resources that are unavailable to us.

Fig. 8. (a). Magnified view of Fig. 7(a), where all deeps are superposed on each other for comparison purposes. (b) Maximum slope calculated for each of the deeps in Fig. 7(a).
The theoretical model presented in the previous section was based on the assumption that the coupling to the clockwise and counter-clockwise modes is equal, thus a standing wave is obtained within the modified MRR. This assumption is now confirmed by finite element simulation. Fig. 9 shows the power flow of a modified MRR where an incident optical field is fed into the left side of the upper bus waveguide. One can see that in the lower waveguide the optical power propagates equally towards the left and the right.

Our last example presents a comparison between two pairs of standard and modified MRR in an add-drop configuration. The modified MRRs are based on the configuration presented in Fig. 4(a). Both have a 1D PhC with a period of 0.37μm. The first modified MRR includes 100 periods while the second includes 150 periods. Their radii are 5.89μm and 8.83μm respectively. The waveguide-MRR separation is set to 400μm in both modified MRRs. This pair of modified MRRs was compared to a second pair of MRRs with a standard add-drop configuration and parameters identical to that of the modified MRRs (Radius and waveguide-MRR separation). For the comparison to serve our purposes we located the in-waveguide reflectors of the modified MRR (see Fig. 4) at a specific distance so that the resulted phase, $\theta$ [see Eq. (8)], is equal to $\pi/2$. The latter ensures that the coupling decay rate multiplier is equal to unity and thus the multiplier does not affect the spectral width of the resonator’s response.

Fig. 10 presents the resulted transmission peaks and Fig. 11 depicts the power flow in the modified MRR based add-drop filter. It is clear that, the transmission from both modified MRRs is narrower compared with the transmission obtained by the standard MRR. By definition, this implies higher $Q$-factors for the modified MRRs. An even more interesting outcome of Fig. 10 is found when analyzing the peak width. When increasing the MRR radius several processes affect the photon decay rate in the resonator: (A) increase in coupling into the bus waveguide is increased, correspondingly increasing the photon decay rate. (B) MRR perimeter is increased, leading to a decrease in the photon decay rate. (C) Stronger dispersion due to closer proximity to the band-edge, further decreasing the decay rate (this effect is true only for the modified MRR). Thus, when increasing the MRR radius there is a competition between two (standard MRR) or three (modified MRR) factors that may lead eventually to either an increase or a decrease in spectral linewidth. From Fig. 10 it is clear that for standard MRR the width is slightly increased with the increase of the MRR radius, while a much more significant decrease in peak width is obtained for the modified MRR. We can conclude that as expected, the third factor, i.e., the increase in dispersion of the modified MRR is probably the major reason for the increase in $Q$-factor.
4. Discussion and conclusions

In this paper we proposed and analyzed a new device based on a standard micro-ring resonator that is modified by inserting periodic defects into the waveguide of the micro-ring. The modified configuration yields a device with unique optical features. We exploit the slow light phenomenon that occurs around the band-edge of PhCs to increase the $Q$-factor of the resonator and to reduce its FSR. We presented a theoretical analysis that predicted the qualitative behavior of the device and provided rough quantitative approximations. Further theoretical analysis that takes into account the curvature of the waveguide in the MRR may produce more accurate calculations, primarily for small radii. To confirm our theory we performed numerical simulations of the device using finite element tool. We demonstrated numerically the increase of the group index by observing a decrease in the FSR. We showed the increase of MRR’s quality factor using two different criteria: a) by comparing the modified MRR’s peaks as the resonance wavelength gets closer to the band edge, b) by comparing the frequency response of standard and modified MRRs with similar parameters. While both approaches showed improvement in quality factor, the improvement was moderate and did not reach an order of magnitude increase. The theoretical model predicts that MRRs
with radius larger than 10μm are needed for to produce an order of magnitude increase in the $Q$-factor. However, we could not accurately simulate such structures because of limited computational resources.

The highly dispersive MRR are expected to be useful for large variety of applications ranging from optical delay lines through various types of communications filters to enhancing nonlinear effects to ultra sensitive sensors. Also due to the circular nature of the modified MRR it might be useful for Sagnac effect applications\(^7\). We intend to experimentally demonstrate such devices in the near future.