Topological Methods - Exercise 2

March 25, 2004

1. Prove that a simplex is homeomorphic to its barycentric subdivision.

2. Prove the following claim. Let K_1 and K_2 be simplicial complexes. Consider an arbitrary mapping f that assigns to each simplex $F \in K_1$ a simplex $f(F) \in K_2$, and suppose that if $F' \subset F$, then also $F(F') \subset f(F)$. Then there exists a continuous map $g : \|K_1\| \to \|K_2\|$.

3. For a positive real number $\alpha < 2$, let $B(n+1, \alpha)$ be the (infinite) *Borsuk graph* with S^n as the vertex set and with two points connected by an edge iff their distance is at least α. Prove that the Borsuk-Ulam theorem is equivalent to the following statement: For every $\alpha < 2$, we have $\chi(B(n+1, \alpha)) \geq n+2$ (χ denotes the chromatic number).