

Multi-linear Systems and Invariant Theory
in the Context of Computer Vision and Graphics

Class 6: Duality and Shape Tensors

CS329
Stanford University

Amnon Shashua

Material We Will Cover Today

- The Point-View Duality Principle
- Multi-point Constraints of a single view
- Single-View Tensors
- Properties of Single View Tensors
- Single View Tensors under a stabilized reference plane

Point-View Duality Principle

$$P_i = \begin{pmatrix} X_i \\ Y_i \\ Z_i \\ W_i \end{pmatrix}$$

points in 3D projective space

$$p_i \cong MP_i$$

points in 2D projective space

Approach: with enough points, we can “eliminate” M

Point-View Duality Principle

We are free to choose a projective basis for the 3D space, and we fix 12 degrees of freedom (out of 16):

$$p_i \cong MP_i = MT^{-1}TP_i$$

Let us choose a representation of 3D space such that the first 4 points have the “standard” coordinates:

$$P_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad P_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad P_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Point-View Duality Principle

We can also choose a basis for the 2D (image) projective space:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cong M \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cong M \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cong M \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cong M \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Point-View Duality Principle

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cong M \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow M_1 = \begin{pmatrix} \alpha \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cong M \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \longrightarrow M_2 = \begin{pmatrix} 0 \\ \beta \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cong M \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \longrightarrow M_3 = \begin{pmatrix} 0 \\ 0 \\ \gamma \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cong M \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \longrightarrow M_4 = \begin{pmatrix} \delta \\ \delta \\ \delta \end{pmatrix}$$

$$M = \begin{bmatrix} \alpha & 0 & 0 & \delta \\ 0 & \beta & 0 & \delta \\ 0 & 0 & \gamma & \delta \end{bmatrix}$$

Point-View Duality Principle

$$M = \begin{bmatrix} \alpha & 0 & 0 & \delta \\ 0 & \beta & 0 & \delta \\ 0 & 0 & \gamma & \delta \end{bmatrix}$$

Note: in the new basis, M is represented by 4 numbers (!)

Note: in the new basis we cannot assume that $p \cong \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$
(third coordinate may vanish), thus

$$p \cong \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Point-View Duality Principle

for $i = 5, 6, 7, \dots$

$$p_i \cong \begin{bmatrix} \alpha & 0 & 0 & \delta \\ 0 & \beta & 0 & \delta \\ 0 & 0 & \gamma & \delta \end{bmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \\ W_i \end{pmatrix} = \begin{bmatrix} X_i & 0 & 0 & W_i \\ 0 & Y_i & 0 & W_i \\ 0 & 0 & Z_i & W_i \end{bmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

$$p_i \cong MP_i = \hat{P}_i \hat{M}$$

$$\hat{M} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

$$\text{null}(M) = \begin{pmatrix} 1/\alpha \\ 1/\beta \\ 1/\gamma \\ -1/\delta \end{pmatrix}$$

Single-view Multilinear Constraints

Let l_i, l'_i be two lines coincident with p_i

$$l_i^T p_i = 0 \quad l_i'^T p_i = 0$$

$$p_i \cong \hat{P}_i \hat{M} \quad \longrightarrow \quad \begin{aligned} l_i^T \hat{P}_i \hat{M} &= 0 \\ l_i'^T \hat{P}_i \hat{M} &= 0 \end{aligned}$$

Single-view Multilinear Constraints

for $i = 5, 6, 7, 8$

$$\begin{bmatrix} l_5^T \hat{P}_5 \\ \cdot \\ l_8^T \hat{P}_8 \\ l_5^T \hat{P}_5 \\ \cdot \\ l_8^T \hat{P}_8 \end{bmatrix}_{8 \times 4} \hat{M} = 0 \quad \text{Every 4x4 minor vanishes!}$$

The choice of 4 rows can include 2,3,4 points
(in addition to the 4 basis points)

Single-view Bilinear Constraints

Choose the 4 rows such that they include only 2 points:

$$\det \begin{bmatrix} l_5^T & \hat{P}_5 \\ l_5'^T & \hat{P}_5 \\ l_6^T & \hat{P}_6 \\ l_6'^T & \hat{P}_6 \end{bmatrix} = 0$$

Single-view Bilinear Constraints

$$\det \begin{bmatrix} l_5^T \hat{P}_5 \\ l_5'^T \hat{P}_5 \\ l_6^T \hat{P}_6 \\ l_6'^T \hat{P}_6 \end{bmatrix} = 0$$

We still have 4 d.o.f left, so we can set the coordinates of

$$P_5 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\text{Let } P_6 = \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix}$$

$$p \cong \begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow l \cong \begin{pmatrix} -z \\ 0 \\ x \end{pmatrix} \quad l' \cong \begin{pmatrix} 0 \\ -z \\ y \end{pmatrix} \quad 12$$

Single-view Bilinear Constraints

$$\det \begin{bmatrix} l_5^T \hat{P}_5 \\ l_5'^T \hat{P}_5 \\ l_6^T \hat{P}_6 \\ l_6'^T \hat{P}_6 \end{bmatrix} = 0 \quad \hat{P}_5 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \hat{P}_6 = \begin{bmatrix} X & 0 & 0 & W \\ 0 & Y & 0 & W \\ 0 & 0 & Z & W \end{bmatrix}$$

$$l \cong \begin{pmatrix} -z \\ 0 \\ x \end{pmatrix} \quad l' \cong \begin{pmatrix} 0 \\ -z \\ y \end{pmatrix}$$



$$\det \begin{bmatrix} -z_5 & 0 & x_5 & x_5 - z_5 \\ 0 & -z_5 & y_5 & y_5 - z_5 \\ -z_6 X & 0 & x_6 Z & x_6 W - z_6 W \\ 0 & -z_6 Y & y_6 Z & y_6 W - z_6 W \end{bmatrix} = 0$$

Single-view Bilinear Constraints

$$\det \begin{bmatrix} -z_5 & 0 & x_5 & x_5 - z_5 \\ 0 & -z_5 & y_5 & y_5 - z_5 \\ -z_6 X & 0 & x_6 Z & x_6 W - z_6 W \\ 0 & -z_6 Y & y_6 Z & y_6 W - z_6 W \end{bmatrix} = 0$$

$$p_6^T G p_5 = 0$$

for all viewing positions !

p_5, p_6 change with viewing position

$$G = \begin{bmatrix} 0 & WY - YW & YZ - WZ \\ XZ - XW & 0 & -XZ + WZ \\ -XY + WX & XY - WY & 0 \end{bmatrix}$$

Single-view Bilinear Constraints

$$p_6^T G p_5 = 0 \quad G = \begin{bmatrix} 0 & WY - YW & YZ - WZ \\ XZ - XW & 0 & -XZ + WZ \\ -XY + WX & XY - WY & 0 \end{bmatrix}$$

$$G_{11} = G_{22} = G_{33} = 0 \quad \sum_{ij} G_{ij} = 0$$

$$\det(G) = 0$$

The dual fundamental matrix has **additional** 4 linear constraints!
(compared to the fundamental matrix).

The Dual Fundamental Matrix

Why $\det(G) = 0$?

$p_6^T G p_5 = 0 \implies G p_5$ is a line coincident with p_6

Since $p_6^T G p_5 = 0$ holds for all camera matrices,

choose the camera projection matrix such that $MP_6 = 0$

that is, the center of projection is at P_6

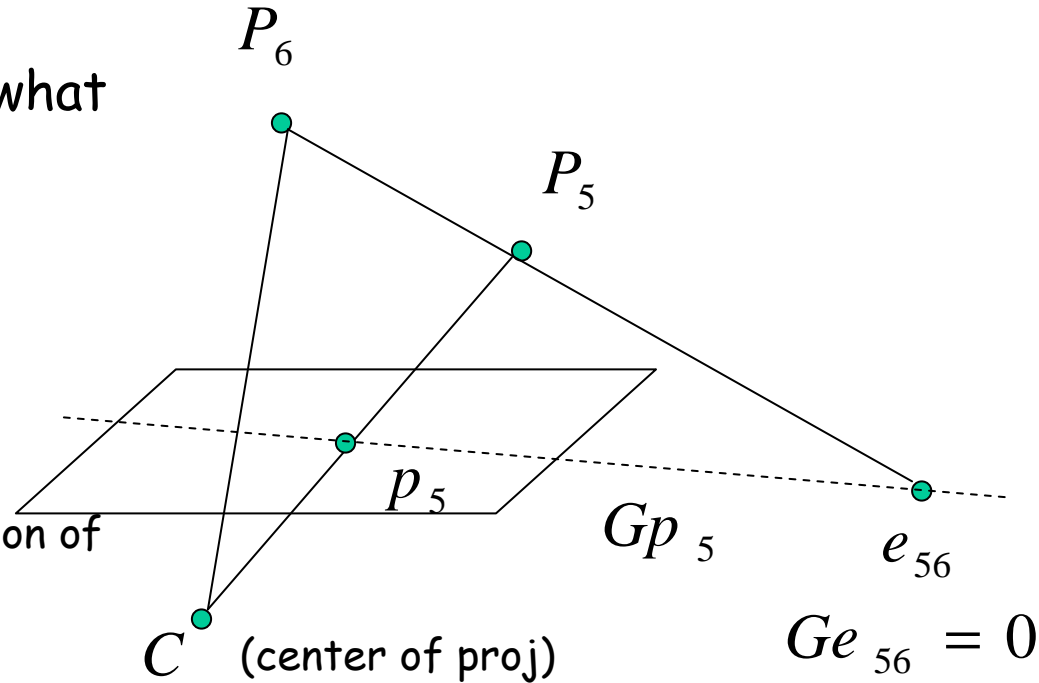
$$\implies l^T MP_6 = 0 \quad \forall l \implies l^T \hat{P}_6 \hat{M} = 0$$

$$\implies p^T G p_5 = 0 \quad \forall p \implies G p_5 = 0$$

$$\implies \det(G) = 0$$

A “sketch” – the sketch is somewhat misleading since there are two (dual) epipoles.

Note: while C changes so does the image plane!



$e_{56} = \hat{P}_5 \text{null} (\hat{P}_6)$ is the projection of P_5 when $C = P_6$

$$\underbrace{\begin{bmatrix} X_5 & 0 & 0 & W_5 \\ 0 & Y_5 & 0 & W_5 \\ 0 & 0 & Z_5 & W_5 \end{bmatrix} \begin{pmatrix} 1/X_6 \\ 1/Y_6 \\ 1/Z_6 \\ -1/W_6 \end{pmatrix}}_{\hat{P}_5 \text{null} (\hat{P}_6)} = \underbrace{\begin{bmatrix} 1/X_6 & 0 & 0 & -1/W_6 \\ 0 & 1/Y_6 & 0 & -1/W_6 \\ 0 & 0 & 1/Z_6 & -1/W_6 \end{bmatrix} \begin{pmatrix} X_5 \\ Y_5 \\ Z_5 \\ W_5 \end{pmatrix}}_{\text{projection of } P_5 \text{ when } \text{null} (M) = P_6}$$

$e_{65} = \hat{P}_6 \text{null} (\hat{P}_5)$ is the projection of P_6 when $C = P_5$

Class 6 $G^T e_{65} = 0$

The Dual Fundamental Matrix

Why $G_{11} = G_{22} = G_{33} = 0$ $\sum_{ij} G_{ij} = 0$?

Unlike the multiview tensors, the single-view point tensors have internal linear constraints (“synthetic” constraints) which are related to the fact the dual projection matrices are **sparse**.

Since $p_6^T G p_5 = 0$ holds for all camera matrices,

consider the camera matrices M_j

$$M_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad M_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad M_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad M_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

\uparrow \uparrow \uparrow \uparrow
 e_1 e_2 e_3 e_4
 Class 6 18


The Dual Fundamental Matrix

Why $G_{11} = G_{22} = G_{33} = 0$ $\sum_{ij} G_{ij} = 0$?

$$M_j P \cong \{e_j \quad 0\} \quad \forall P$$

$$\det \begin{bmatrix} l_5^T \hat{P}_5 \\ l_5'^T \hat{P}_5 \\ l_6^T \hat{P}_6 \\ l_6'^T \hat{P}_6 \end{bmatrix} = 0$$

l_5, l_5', l_6, l_6' coincident with e_j
 holds for all choices of \hat{P}_5, \hat{P}_6 !!

 $e_j^T G e_j = 0 \quad j = 1, \dots, 4$

6-point Single-view Indexing

$p_6^T G p_5 = 0$ is a function of 6 points (4 basis, 2 additional)

Each view provides one constraint for G . How many views are necessary?

G is defined up to scale and has 4 linear constraints – thus there are 4 parameters to solve for.

For a linear solution: 4 views are necessary.

For non-linear: 3 views are necessary (3-fold ambiguity).

Once G is recovered, the function $p_6^T G p_5 = 0$ is view-independent

thus provides a connection between 6 image points and their corresponding 3D points which does not depend on the point of view (indexing function).

7-point Single-view Tensor

12 choices to choose 4 rows which include 7 points (4 basis +3).

there are 3 groups: choose 2 rows from a single point and the remaining two rows one from each remaining point.

$$0 = \det \begin{bmatrix} l_5^T \hat{P}_5 \\ l_5'^T \hat{P}_5 \\ l_6^T \hat{P}_6 \\ l_6'^T \hat{P}_6 \\ l_7^T \hat{P}_7 \\ l_7'^T \hat{P}_7 \end{bmatrix} \left. \begin{array}{l} \text{choose these two rows} \\ \text{choose 1 rows from here} \\ \text{choose 1 rows from here} \end{array} \right\}$$

Class 6



4 trilinear constraints (view independent) 21
(per group)

7-point Single-view Tensor

$$0 = \det \begin{bmatrix} l_5^T \hat{P}_5 \\ l_5^T \hat{P}_5 \\ l_6^T \hat{P}_6 \\ l_6^T \hat{P}_6 \\ l_7^T \hat{P}_7 \\ l_7^T \hat{P}_7 \end{bmatrix} \longrightarrow p_5^i l_j^6 l_k^7 T_i^{jk} = 0$$

l^6 is some line incident with P_6
 l^7 is some line incident with P_7

T_i^{jk} is a trilinear function of P_5, P_6, P_7 alone.

7-point Single-view Tensor

The “synthetic” constraints for the dual trilinear tensor:

Since $p_5^i l_j^6 l_k^7 T_i^{jk} = 0$ holds for all camera matrices,

consider the camera matrices M_j

$$M_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad M_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad M_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad M_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

\uparrow
 e_1

\uparrow
 e_2

\uparrow
 e_3

\uparrow
 e_4

$$M_j P \cong \{e_j \quad 0\} \quad \forall P$$

$$p_5^i l_j^6 l_k^7 T_i^{jk} = 0 \quad \text{where } l, l' \text{ incident with } e = e_j$$

$$\longrightarrow e^i l_j^6 l_k^7 T_i^{jk} = 0$$

4 constraints (2 choices for each line l, l')

$$\longrightarrow 16 \text{ synthetic constraints (4 for each } e = e_j)^{23}$$

7-point Single-view Tensor

The 7-point tensor is defined by 11 parameters (up to scale).

→ A linear solution will requires 3 views (each view 4 constraints)

How many Non-linear constraints?

The number of parameters needed to describe a 7-point configuration Is $3+3=6$ (because the first 5 points can be assigned the standard coordinates).

→ There should be 5 non-linear constraints (1 for the overall scale, and 4 additional).

7-point Single-view Tensor

The dual reprojection equation:

$$p_5^i l_j^6 T_i^{jk} \cong p_7^k$$

Given point p_5 and a line l^6 coincident with p_6

then the contraction $p_5^i l_j^6 T_i^{jk}$ produces the point p_7

→ 6 image points and the shape tensor determine the 7th point

Items we Skip in Class

- Homography duality (and slices of 7-point tensor), where do the non-linear constraints come from.
- 8-point dual tensor: $3 \times 3 \times 3 \times 3$ tensor, 58 synthetic constraints, 2 views are necessary to recover tensor (first view 12 constraint, second view 11 constraints).

Duality Under a Fixed Reference Plane

Assume some plane has been identified and “stabilized” throughout the sequence of images (i.e., points on the reference plane project to fixed Points throughout the sequence).



The house façade is stabilized



The road sign is stabilized

Duality Under a Fixed Reference Plane

In this case, the first 4 basis points are coplanar:

$$P_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad P_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad P_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

Duality Under a Fixed Reference Plane

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cong M \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \longrightarrow \quad M_1 \cong \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cong M \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \longrightarrow \quad M_2 \cong \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cong M \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \longrightarrow \quad M_3 \cong \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cong M \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\longrightarrow \quad M_1 + M_2 + M_3 \cong \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$M = \begin{bmatrix} \delta & 0 & 0 & \alpha \\ 0 & \delta & 0 & \beta \\ 0 & 0 & \delta & \gamma \end{bmatrix}$$

$$M_4 = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \quad \text{(unconstrained)}$$

Duality Under a Fixed Reference Plane

for $i = 5, 6, 7, \dots$

$$p_i \cong \begin{bmatrix} \delta & 0 & 0 & \alpha \\ 0 & \delta & 0 & \beta \\ 0 & 0 & \delta & \gamma \end{bmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \\ W_i \end{pmatrix} = \begin{bmatrix} W_i & 0 & 0 & X_i \\ 0 & W_i & 0 & Y_i \\ 0 & 0 & W_i & Z_i \end{bmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

$$p_i \cong MP_i = \hat{P}_i \hat{M}$$

$$\hat{M} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

$$\text{null}(M) = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ -\delta \end{pmatrix}$$

Duality Under a Fixed Reference Plane

$$\hat{M} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

$$\text{null}(M) = \begin{pmatrix} 1/\alpha \\ 1/\beta \\ 1/\gamma \\ -1/\delta \end{pmatrix}$$

$$\text{null}(M) = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ -\delta \end{pmatrix}$$

(general)

(ref plane stabilized)

When \hat{M} varies
along a linear subspace
(say a line)

$\text{null}(M)$ varies
along an algebraic surface

$\text{null}(M)$ varies
along a linear subspace
of the same dimension

(easy geometric interp)

6-point Tensor Under a Fixed Reference Plane

$$\det \begin{bmatrix} l_5^T \hat{P}_5 \\ l_5'^T \hat{P}_5 \\ l_6^T \hat{P}_6 \\ l_6'^T \hat{P}_6 \end{bmatrix} = 0 \quad \hat{P}_5 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \hat{P}_6 = \begin{bmatrix} W & 0 & 0 & X \\ 0 & W & 0 & Y \\ 0 & 0 & W & Z \end{bmatrix}$$

$$p_6^T G p_5 = 0$$

What is different?

6-point Tensor Under a Fixed Reference Plane

$$p_6^T G p_5 = 0$$

Since P_1, P_2, P_3, P_4 are coplanar

$$P_i^T n = 0 \quad \text{where} \quad n = (0, 0, 0, 1)^T$$

Since the bilinear constraint holds for all choices of M

consider: $M = un^T \longrightarrow MP \cong \begin{Bmatrix} u & 0 \end{Bmatrix} \quad \forall P$

$$\det \begin{bmatrix} l_5^T \hat{P}_5 \\ l_5'^T \hat{P}_5 \\ l_6^T \hat{P}_6 \\ l_6'^T \hat{P}_6 \end{bmatrix} = 0$$

l_5, l_5', l_6, l_6' coincident with u

holds for all choices of \hat{P}_5, \hat{P}_6 !!

$$\longrightarrow u^T G u = 0$$

$\longrightarrow G$ is skew-symmetric (only 3 parameters)

The Dual Image Ray

Recall: image ray in multi-view geometry is defined as follows:

Let l, l' be 2 lines coincident with $p \cong MP$

$$\left. \begin{array}{l} l^T MP = 0 \\ l'^T MP = 0 \end{array} \right\} P \text{ has a 1-parameter degree of freedom} \\ \text{defined by the intersection of the two planes}$$

$l^T M, l'^T M$ **this is the image ray**

(the lines passing through p and $null(M)$)

The Dual Image Ray

In Dual:

$$\left. \begin{aligned} l^T \hat{P} \hat{M} &= 0 \\ l'^T \hat{P} \hat{M} &= 0 \end{aligned} \right\} \begin{aligned} &\hat{M} \text{ has a 1-parameter degree of freedom} \\ &\text{The line passing through } p \text{ and } \textit{null}(\hat{P}) \end{aligned}$$

In general case: the camera center varies along a **non-linear curve!**

The Dual Image Ray

In ref-plane stabilized: the camera center varies along a line,
and the line is the one joining p and P

$$0 = l^T \hat{P} \hat{M} = l^T \underbrace{\begin{bmatrix} -W & 0 & 0 \\ 0 & -W & 0 \\ 0 & 0 & -W \end{bmatrix}}_{\text{null} = P} \underbrace{\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}}_{\text{null} (M)} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ -\delta \end{pmatrix}$$

 $\text{null} (M)$ varies along the line \overline{pP}

Items we Skip in Class

- Number of Synthetic Constraints for 7-point tensor is 21. There are no non-linear constraints. Only 2 views are required to solve.
- 8-point dual tensor: $3 \times 3 \times 3 \times 3$ tensor, 72 synthetic constraints, no non-linear constraints. 2 views are required (first contributes 5 constraints, the second 4 constraints).