Multi-linear Systems for 3D-from-2D Interpretation

Lecture 4

Dynamic \( P^k \rightarrow P^2 \) Tensors

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Material We Will Cover Today

• The general problem of reconstruction from “line of sights”

• Two Applications for $P^4 \rightarrow P^2$

• An Application for $P^6 \rightarrow P^2$

• The geometry of $P^4 \rightarrow P^2$

• Indexing into “action” (an example).

Wolf, Shashua ICCV01, IJCV’02 (Honorable Mention, Marr Prize)
Levin, Wolf, Shashua CVPR’01

Wolf, Shashua ICCV’01: (translating planes)
Wolf, Shashua CVPR’01: (segmentation tensors)

not discussed here
Dynamic configurations:

- Constant Velocity
- Points in 3D
- Straight-line Motion (of points)
- General Camera Motion
- Trajectories Span a 3D, 2D or 1D space
Dynamic configurations:

- Recover Camera motion from image observations
- How many points/views?

As Seen in the Image Space
Dynamic configurations:
(multi-body)

• 2 Rigid Configurations
• Bodies moving in relative translation to each other
Dynamic configurations:
(multi-body)

• Recover Camera motion from image observations
• How many points/views?
• How many “segmented” points are needed?

As Seen in the Image Space
A General Scheme for Constructing Multi-view Tensors of Dynamic Scenes

\[ P^2 \rightarrow P^2 \] projections of (static) planar configuration

\[ P^3 \rightarrow P^2 \] projections of (static) 3D configuration

\[ P^k \rightarrow P^2 \] projections of dynamic and multi-body scenes
\[ k = 3,4,5,6 \]

- We describe the multi-view constraints from these projection matrices.

- We show how to extract the structure and motion for each application.
When Camera Trajectory is known
Triangulation
Triangulation?
When Camera Trajectory is known
How Many Views for a Unique Line?

- 4 views: finite number of solutions (nonlinear)
- 5 views: unique solution
Notations

$L$

$L$

$p_i$

$l_i$

$M_i$

U. of Milano, 8.7.04
Plucker Coordinates

Given two 3D points

\[ P_1 = (X_1, Y_1, Z_1, 1) \]
\[ P_2 = (X_2, Y_2, Z_2, 1) \]

The 3D line is:

\[ L = P_1 \wedge P_2 \]
\[ = \begin{pmatrix} L^{12} & L^{13} & L^{14} & L^{23} & L^{24} & L^{34} \end{pmatrix} \]

For example:

\[ L^{12} = \det \begin{pmatrix} 1 & X_1 \\ 1 & X_2 \end{pmatrix} \]
\[ = \begin{pmatrix} 1 & X_1 & Y_1 & Z_1 \\ 1 & X_2 & Y_2 & Z_2 \end{pmatrix} \]
Linear Solution

\[ p^T l = 0 \]

\[ l = (MP_1) \times (MP_2) \]

\[ p^T(\tilde{M}L) = 0 \]

\[ l = \tilde{M}L \]

\[ M = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}_{3 \times 4} \]

\[ \tilde{M} = \begin{bmatrix} m_2 \land m_3 \\ m_3 \land m_1 \\ m_1 \land m_2 \land 15 \end{bmatrix}_{3 \times 6} \]
Dynamic \( P^3 \rightarrow P^2 \)

What do we have so far:

- Reconstruction using 5 views with Known Camera Motion.
- Planar scene, 3 views, unknown camera motion.

Generalization to 3D-from-2D becomes unwieldy: 3x3x3x3x3x3 tensor with 729 observations per view!

Additional Constraints are Required

\[ P_i + jV \]

Position of the feature point at frame \( j \)
General Idea

Describe the **dynamic** $P^3 \rightarrow P^2$ applications as a static $P^k \rightarrow P^2$ problem

- $P^6 \rightarrow P^2$ Multi-point, 3D, constant-velocity
- $P^5 \rightarrow P^2$ Multi-point, 3D, coplanar constant-velocity
- $P^4 \rightarrow P^2$ Multi-point, 3D, collinear constant-velocity
- $P^3 \rightarrow P^2$ Multi-point, 2D, constant-velocity
- $P^3 \rightarrow P^2$ Two-body, 3D, Segmentation (pure trans)
- $P^2 \rightarrow P^2$ Two-body, 2D, Segmentation (pure trans)
$P^4 \rightarrow P^2 \quad 3D \text{ segmentation tensor}$

\[ p_i \equiv M_{3 \times 4} P_i \quad \text{if point } P_i \text{ belongs to object 1} \]

\[ \hat{p}_i \equiv \hat{M}_{3 \times 4} P_i \quad \text{if point } P_i \text{ belongs to object 2} \]

Because the motion between the two objects is pure translation:

\[ M = [A \quad v] \]

\[ \hat{M} = [A \quad \hat{v}] \]
3D segmentation tensor

\[ p_i \equiv [A \ v \ \hat{v}]_{3 \times 5} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

\[ \hat{p}_i \equiv [A \ v \ \hat{v}]_{3 \times 5} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]


3D segmentation tensor

Let \( \tilde{M}_j \) be the j’th projection matrix from \( P^4 \rightarrow P^2 \)

\[
p_i^j = \begin{pmatrix} x_i^j \\ y_i^j \\ 1 \end{pmatrix}
\]

be the projection of \( \tilde{P}_i \in P^4 \) in the j’th view

Note:

\[
\begin{pmatrix} 1 \\ 0 \\ -x_i^j \end{pmatrix}^T \begin{pmatrix} x_i^j \\ y_i^j \\ 1 \end{pmatrix} = 0
\]

\[
\begin{pmatrix} 0 \\ 1 \\ -y_i^j \end{pmatrix}^T \begin{pmatrix} x_i^j \\ y_i^j \\ 1 \end{pmatrix} = 0
\]
3D segmentation tensor

3x3x3 tensor (27 coefs)

Each triplet of matching Points p, p', p''
Provides 2 constraints

We need 13 points
(from both objects!)

Once the tensor is recovered, 4 segmented points are sufficient to segment the entire scene
Another Application of $P^4 \to P^2$: 2D Dynamic Motion under const. velocity

Recall from Homography Tensors lecture:
26 triplets are required for general motion along straight line paths!

Let:

\[ H_j, \ j = 0, 1, 2 \]

three homography matrices

\[ p_j = (x_j, y_j, 1)^\top, \ j = 0, 1, 2 \]

projections of

\[ (X, Y, 1) \]

Points moving under const. velocity

Note: because of the const. Velocity constraint, the reconstruction could be up to 3D affine
So we are allowed to set the 3rd coordinate to be 1.

\[ p_j \approx H_j \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} + jH_j \begin{pmatrix} dX \\ dY \\ 0 \end{pmatrix} = \tilde{H}_j \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} \]

\[ \tilde{H}_j = [H_j, \ jH_j^*]_{3\times5} \]

13 matching triplets would be sufficient!
Multi-point, 3D, Constant Velocity

Let $P_i = (X_i, Y_i, Z_i, 1)$ for $i = 1, \ldots, n$ be a 3D point configuration where each point moving independently along a direction $V_i = (dX_i, dY_i, dZ_i, 0)$

At time $j = 0, \ldots, m$ the position of each point is $P_i + jV_i$

Let $M_j$ be the $3 \times 4$ projection matrix

\[
p_i^j \equiv M_j (P_i + jV_i)
\]

\[
\begin{pmatrix}
    x_i^j \\
    y_i^j \\
    1
\end{pmatrix}
\equiv
\begin{bmatrix}
    M_j & jM_j^*
\end{bmatrix}
\begin{pmatrix}
    X_i \\
    Y_i \\
    Z_i \\
    1
\end{pmatrix}
= \tilde{M}_j
\begin{pmatrix}
    X_i \\
    Y_i \\
    Z_i \\
    1
\end{pmatrix}
\]

\[
\begin{pmatrix}
    X_i \\
    dX_i \\
    dY_i \\
    dZ_i
\end{pmatrix}
\]

\[
\begin{pmatrix}
    X_i \\
    Y_i \\
    Z_i
\end{pmatrix}
\]

$P^6 \rightarrow P^2$
Multi-point, 3D, constant velocity

3x3x3x3 tensor (81 coefs)

Each quadruple of matching Points p,p',p'',p''' Provides 2 constraints

We need 40 points

\[
\begin{pmatrix}
1 & 0 & -x_i^1 \\
0 & 1 & -y_i^1 \\
1 & 0 & -x_i^2 \\
0 & 1 & -y_i^2 \\
1 & 0 & -x_i^3 \\
0 & 1 & -y_i^3 \\
l_4^T \tilde{M}_4
\end{pmatrix}
\begin{pmatrix}
X_i \\
Y_i \\
Z_i \\
dX_i \\
dY_i \\
dZ_i
\end{pmatrix}
= 0
\]
# List of tensors

<table>
<thead>
<tr>
<th>$\mathcal{P}^k$</th>
<th>Tensor Name</th>
<th>Size</th>
<th># points.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{P}^3$</td>
<td>2D segmentation tensor</td>
<td>$3^2$</td>
<td>8</td>
</tr>
<tr>
<td>$\mathcal{P}^4$</td>
<td>2D constant velocity tensor</td>
<td>$3^3$</td>
<td>13</td>
</tr>
<tr>
<td>$\mathcal{P}^4$</td>
<td>3D segmentation tensor</td>
<td>$3^3$</td>
<td>13</td>
</tr>
<tr>
<td>$\mathcal{P}^4$</td>
<td>3D constant collinear velocity</td>
<td>$3^3$</td>
<td>13</td>
</tr>
<tr>
<td>$\mathcal{P}^5$</td>
<td>3D constant coplanar velocity</td>
<td>$3^3$</td>
<td>26</td>
</tr>
<tr>
<td>$\mathcal{P}^6$</td>
<td>3D constant velocity tensor</td>
<td>$3^4$</td>
<td>40</td>
</tr>
</tbody>
</table>
The term **extensor** (cf. Barnabei, Brini, Rota 1985) is used to describe the linear space spanned by a collection of points.

Extensor of step 1 is a point  
Extensor of step 2 is a line  
Extensor of step 3 is a plane  
Extensor of step k in $P^k$ is a hyperplane

In $P^n$, the union ("join") of extensors of step $k_1$ and $k_2$ where $k_1 + k_2 \leq n + 1$ is an extensor of step $k_1 + k_2$

The intersection ("meet") of extensors of step $k_1$ and $k_2$ is an extensor of step $k_1 + k_2 - (n + 1)$
In $P^n$, the union ("join") of extensors of step $k_1$ and $k_2$ where $k_1 + k_2 \leq n + 1$ is an extensor of step $k_1 + k_2$

The intersection ("meet") of extensors of step $k_1$ and $k_2$ is an extensor of step $k_1 + k_2 - (n + 1)$

The following statements follow immediately:

1. The center of projection of a $P^k \rightarrow P^2$ projection is an extensor of step $k - 2$
   Recall, the center of projection is the null space of the $3 \times (k + 1)$ projection matrix.

2. The line of sight (image ray) joins the COP and a point (on the image plane):
   - $P^3 \rightarrow P^2$  image ray is a line, extensor of step 1+1=2
   - $P^4 \rightarrow P^2$  image ray is a plane, extensor of step 2+1=3
   - $P^6 \rightarrow P^2$  image ray is an extensor of step 4+1=5
Mathematical Background

In $P^n$, the union ("join") of extensors of step $k_1$ and $k_2$ where $k_1 + k_2 \leq n + 1$ is an extensor of step $k_1 + k_2$.

The intersection ("meet") of extensors of step $k_1$ and $k_2$ is an extensor of step $k_1 + k_2 - (n + 1)$.

The following statements follow immediately:

3. The intersection of two lines of sight (triangulation) is the meet of the two extensors.

- $P^3 \to P^2$ the intersection is generally empty $2+2-4=0$
- $P^4 \to P^2$ intersection is a point $3+3-5=1$ (no constraint from two views!)
- $P^6 \to P^2$ intersection is a plane $5+5-7=3$

4 views are required for a constraint.
(intersection of 3 rays: $3+5-7=1$)
Mathematical Background

In $P^n$, the union ("join") of extensors of step $k_1$ and $k_2$ where $k_1 + k_2 \leq n + 1$ is an extensor of step $k_1 + k_2$.

The intersection ("meet") of extensors of step $k_1$ and $k_2$ is an extensor of step $k_1 + k_2 - (n + 1)$.

The following statements follow immediately:

4. The epipole is the intersection between the join of the two COPs and an image plane.

- $P^3 \rightarrow P^2$ the epipole is a point $(1+1) + 3 - 4 = 1$
- $P^4 \rightarrow P^2$ the epipole is a line $(2+2) + 3 - 5 = 2$
- $P^6 \rightarrow P^2$ the epipole is not defined because $4+4 > 7$ (the join of the two COPs is larger than the space dimension).
The Geometry of $P^4 \rightarrow P^2$ Projection Matrices

$P^3 \rightarrow P^2$

- Center of Projection is a Line $null(\tilde{M}_{3\times5})$
- Line of sight is a plane
- Two planes intersect (always) at a point
  
  \[3 + 3 - 5 = 1\]

Three views are necessary for a multilinear constraint

- A plane is the intersection of two hyperplanes
  
  \[4 + 4 - 5 = 3\]

\[
C_\pi = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}_{2\times5} \quad C_\pi P = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \forall P \in \pi
\]
The Geometry of \( P^6 \rightarrow P^2 \) Projection Matrices

\[ P^3 \rightarrow P^2 \]

- Center of Projection is a 4D-space \( \text{null}(\tilde{M}_{3 \times 7}) \)
- Line of sight is a 5D-space
- Two line-of-sights intersect in a plane
  \[ 5 + 5 - 7 = 3 \]
- A line-of-sight and a plane intersect at a point
  \[ 5 + 3 - 7 = 1 \]

Four views are necessary for a multilinear constraint

\[ \text{null}(M_{3 \times 4}) \]
The Geometry of $P^4 \rightarrow P^2$

The coefficients of the determinant expansion live in a mixed tensor of the type

$$A_{ij}^k$$

The constraint: $p^i p'^j l''_k A_{ij}^k = 0$

A triplet of matching points provides 2 constraints

$$p^i p'^j A_{ij}^k \cong p'^k$$

plane plane intersection of two planes: 3+3-5=1 is a point (projected onto $p''$)

Goal: given $A_{ij}^k$ (recovered from 13 matching triplets) we wish to find the three 3x5 projection matrices, and then to recover the 3D-2D underlying matrices.

The constraint:

$$
\begin{pmatrix}
1 & 0 & -x^1_i \\
0 & 1 & -y^1_i \\
1 & 0 & -x^2_i \\
0 & 1 & -y^2_i \\
l^T_3 \tilde{M}_3
\end{pmatrix}
\tilde{P}_i = 0
$$

U. of Milano, 8.7.04 P^k to P^2
The Geometry of $P^4 \rightarrow P^2$

Homography slices:

$$\delta^j A_{ij}^k \cong H_i^k$$

a homography matrix $1 \rightarrow 3$

induced by the plane $C_2 \vee \delta$

$$\delta^i A_{ij}^k \cong E_j^k$$

a homography matrix $2 \rightarrow 3$

induced by the plane $C_1 \vee \delta$
The Geometry of $P^4 \rightarrow P^2$

Recovering Epipoles from Homography slices:

$e_{ij} = \tilde{M}_i (\text{null}(\tilde{M}_j))$

$H_\pi e_{13} \equiv e_{31}$

$\Rightarrow (H_{\pi_1} - \lambda H_{\pi_2}) e_{13} \equiv 0$

$\Rightarrow e_{13}$ is a generalized eigen-vector of $(H_{\pi_1}, H_{\pi_2})$

$H^{-T}_\pi e_{13} \equiv e_{31}$

$\Rightarrow e_{13}$ is a generalized eigen-vector of $(H^{-T}_{\pi_1}, H^{-T}_{\pi_2})$

(slices of the form: $\delta^j A_{ij}^k$)
The Geometry of $P^4 \rightarrow P^2$

The canonical form of projection matrices:

Claim: There exist a canonical projective frame such that:

$$p \cong \begin{bmatrix} I_{3 \times 3} & 0_3 & 0_3 \end{bmatrix} \begin{pmatrix} p \\ \eta \\ \mu \end{pmatrix} \quad p' = \begin{bmatrix} H_\pi & e_1 \quad e_2 \end{bmatrix} \begin{pmatrix} p \\ \eta \\ \mu \end{pmatrix}$$

Where $H_\pi$ is a homography matrix from views 1 to 2, and $e_1, e_2$ are two points on the epipole (a line) on view 2.

The family of homography matrices between views 1 to 2 has the general Form with 7 degrees of freedom:

$$\lambda H_\pi + e_1 n_1^T + e_2 n_2^T$$
The Geometry of $P^4 \rightarrow P^2$

The canonical form of projection matrices:

Proof:

Let $\tilde{M}_1, \tilde{M}_2$ be the two 3x5 projections matrices, a point $P$ in space and matching points $p, p'$ satisfying:

$$p \cong \tilde{M}_1 P \quad p' \cong \tilde{M}_2 P$$

Let $C, C'$ be two points spanning the center-of-projection of the first camera:

$$\tilde{M}_1 C = 0 \quad \tilde{M}_1 C' = 0$$

Let a 5x5 projective change of basis $W$ be defined: $W = [U, C, C']$ where $U$ is some 5x3 matrix chosen such that

$$\tilde{M}_1 U = I_{3 \times 3}$$

$$\tilde{M}_1 W = [I_{3 \times 3}; 0_{3 \times 2}]$$

$$\tilde{M}_2 W = \tilde{M}_2 [U, C, C'] = [\tilde{M}_2 U, e_1, e_2]$$

Our goal will be to create the 5x3 matrix $U$ such that $\tilde{M}_2 U$ is a homography matrix.
The Geometry of $P^4 \rightarrow P^2$

The canonical form of projection matrices:

Proof:

Consider the following construction of $U$: 
\[
U = \begin{bmatrix} \tilde{M}_1 & \cdot & \cdot \\ C_\pi \end{bmatrix}_{1-3}^{-1}
\]
(cols 1-3 from the 5x5 matrix)

Note: $\tilde{M}_1 U = I_{3\times3}$

We have:
\[
\begin{bmatrix} \tilde{M}_1 \\ C_\pi \end{bmatrix} P = \begin{bmatrix} \tilde{M}_1 P \\ C_\pi P \end{bmatrix} = \begin{bmatrix} p \\ 0 \\ 0 \end{bmatrix} \quad \forall P \in \pi
\]

We have thus shown that $\tilde{M}_2 U p \cong p'$ for all matching points arising from the plane $\pi$. 

U. of Milano, 8.7.04

P^k to P^2

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Comparison With $P^3 \rightarrow P^2$

$P^3 \rightarrow P^2 :$

$\tilde{M}_1 = [I_{3x3} \ 0_3 ]$

$\tilde{M}_2 = [H \ e ]$

$P^4 \rightarrow P^2 :$

$\tilde{M}_1 = [I_{3x3} \ 0_3 \ 0_3 ]$

$\tilde{M}_2 = [H_{12} \ e_1 \ e_2 ]$

A basis is determined by 5 points:

$H$ determines 3 points.

$C_1$ determines 1 point.


A basis is determined by 6 points:

$H_{12}$ determines 3 points.

$C_1$ determines 2 point (a line).

Scales between $H_{12}, e_1, e_2 :$ adds 1.

Note: the third camera matrix $\tilde{M}_3$ can be recovered from $\tilde{M}_1, \tilde{M}_2$ and the tensor $A_{ij}^k$. 

U. of Milano, 8.7.04

P^k to P^2

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Performing the Segmentation

- Points from each object are located on a different hyper-plane.

- Points on the first object are on a hyper-plane:

\[
\begin{bmatrix}
X_i \\
Y_i \\
Z_i \\
1 \\
0
\end{bmatrix}
\]

- Four points are sufficient to recover this hyper-plane.

Take 4 matching points \( p_i, p_i', p_i'' \) which are known to come from one of the objects and reconstruct the points \( \tilde{P}_i, i = 1, 2, 3, 4 \).

Solve for the hyperplane \([\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \tilde{P}_1] \pi = 0\).

Given a new matching triplet \( p, p', p'' \) reconstruct \( \tilde{P} \) and if \( \tilde{P}^\top \pi = 0 \) then the point belongs to object 1.
Recovery of structure

- Done in two stages:
  - Reconstruction of $3 \times (k+1)$ camera matrices per view in $\mathbb{P}^k$
  - Reconstruction of the constituent $3 \times 4$ camera matrices in 3D.
- We get Affine 3D structure
Experiments
Traditional solution

- Pick 7 points from each object.
- For each object compute the trifocal tensor separately.
- Does not require relative translation.
Computing the tensor

- We use all points to compute tensor.
segmenting

• Only 4 known points from one object are needed.
  (instead of 7)
Results
The Geometry of $P^6 \rightarrow P^2$ Projection Matrices

$P^3 \rightarrow P^2$

- Center of Projection is a 4D-space $\text{null}(\tilde{M}_{3\times7})$
- Line of sight is a 5D-space
- Two line-of-sights intersect in a plane
  \[ 5 + 5 - 7 = 3 \]
- A line-of-sight and a plane intersect at a point
  \[ 5 + 3 - 7 = 1 \]
- Four views are necessary for a multilinear constraint

$null(M_{3\times4})$
Joint Epipole

- There is no such thing as epipole: the join of the two camera centers is larger than the whole space.
- We use joint epipole: the projection of the intersection of two camera centers.
- $C^3_{12}$: the projection of the intersection of the first two camera centers onto the third view.
Reconstruction of projection matrices from the tensor

\[
\begin{align*}
\tilde{M}_1 & \in \mathbb{R}^{3 \times 3} \\
\tilde{M}_2 & \in \mathbb{R}^{3 \times 3} \\
\tilde{M}_3 & \in \mathbb{R}^{3 \times 3} \\
\tilde{M}_4 & \text{can be determined linearly from the tensor and } \tilde{M}_1, \tilde{M}_2, \tilde{M}_3
\end{align*}
\]

where:

\(H_{13}\) is a homography \(1 \to 3\) of a plane contained in \(C_2\)

\(H_{23}\) is a homography \(2 \to 3\) of a plane contained in \(C_1\)

\(c_{21}^3\) is the joint epipole of \(C_1, C_2\) in view 3.
Constant Velocity Experiments

The scene is 3D, The velocities are coplanar:
A situations for a $P^5 \rightarrow P^2$ tensor
Computing the tensor

- We use all tracked points across three images to compute the tensor.
Results: 3-D velocity projected to first view

- Arrow’s base – point in first view
- Arrow’s tip – 3-D point in time = 2 projected to the first view
Indexing into “Action”

Training Sequence

Single View:
is this a view of a person sitting?
Algebraic Polynomials for Indexing into Action

\[ f(p_1, \ldots, p_k, j) = 0 \]

If the collection of image points at time \( j \) are a projection of the dynamic configuration.

Example: Affine Camera Model

\[
\begin{pmatrix}
  x_{ij} \\
  y_{ij}
\end{pmatrix} =
\begin{bmatrix}
  a_j \\
  b_j
\end{bmatrix}_{2 \times 4}
\begin{pmatrix}
  P_i + jV_i \\
  1
\end{pmatrix}
\]
\[
\begin{pmatrix}
    a_j \\
    b_j
\end{pmatrix}
\begin{bmatrix}
    P_i + jV_i \\ 1
\end{bmatrix}
\begin{pmatrix}
    x_{ij} \\
    y_{ij}
\end{pmatrix}
= 0
\]

\[
\begin{pmatrix}
    (P_i^T + jV_i^T) & 1 & x_{ij} \\
    (P_i^T + jV_i^T) & 1 & y_{ij}
\end{pmatrix}
\begin{pmatrix}
    a_j \\
    b_j
\end{pmatrix}
= 0
\]

\[
\begin{pmatrix}
    P_1^T + jV_1^T & 1 & 0 \\
    P_2^T + jV_2^T & 1 & 1 \\
    P_3^T + jV_3^T & 1 & 0 \\
    P_4^T + jV_4^T & 1 & x_{4j} \\
    P_5^T + jV_5^T & 1 & x_{5j}
\end{pmatrix}
= 0
\]

\[
\begin{pmatrix}
    P_1^T + jV_1^T & 1 & 0 \\
    P_2^T + jV_2^T & 1 & 0 \\
    P_3^T + jV_3^T & 1 & 1 \\
    P_4^T + jV_4^T & 1 & y_{4j} \\
    P_5^T + jV_5^T & 1 & y_{5j}
\end{pmatrix}
= 0
\]
\[ f_1(p_4, p_5, j) = 0 \quad f_2(p_4, p_5, j) = 0 \]

The coefficients of these polynomials depend on shape P1,…,P5 and action V1,…,V5

These coefficients can be recovered using 8 views.
“time” increases monotonically for motions of the same “class”
change of $j$ for a “sitting” motion  

change of $j$ for a “standing-up” motion

change of $j$ for a “walking” motion using “sitting” polynomials
\[ P_i = O_i + \cos(\lambda_j)U_i + \sin(\lambda_j)V_i \]
The End