vantage is that documents with mixed graphic forms can be communicated economically to receivers in a current time frame. Partitioning also enables symbol substitution and communication of messages with high effectiveness to a broad class of distributed user terminals. This combination of 1) low cost, 2) current and correct information transport, and 3) distributive capability offered by a layered terminal architecture, increases the probability of the process achieving architectural status.

The principal parameter, in addition to cost, that controls the consumer acceptance of new communication processes, is operability. With highly operable and friendly systems and terminals; new devices, methods, or systems can soon achieve architectural status. The HIPOUS chart is a powerful design tool in examining the operability of systems since each step in the process is critically examined on two scales. The quality scale assists in overcoming the problem of cost associated with redundancy in information, it and the quality scale column provide continual visual feedback. The second scale column, time, reminds the designer of the time of processing and machine responsiveness to users. A totaling of the time column for all user steps in system behavioral studies is also a potent message to designers. Bottlenecks in communication processes are quickly identified with the new design tool, the HIPOUS chart.

Partitioning of information and services and new design tools are key to enabling the design of cost effective and operable systems surrounding integrated communications. All of this assumes 1) an appropriate architecture for data structures in logical and physical form, 2) appropriate semantics for symbol designation and attributes describing a record, and 3) enforcement principles. Without enforcement, information hiding, and definitions criteria in these areas, the elements of control and correctness are lost. With control and correctness in language, data structures, protocols, and design tools, the following goals are achieved with electrical signal communications:

- current, correct, and controlled information,
- cost effective communications, and
- highly operable systems.

With major goals within view, the missing ingredient to achieving physical forms using the new process is optimum partitioning of man–machine tasks. Optimization of new system performance and operability, using new subsystems, design tools, and command instructions requires extensive conceptual studies. Since experience is resident only in man and not in new machines, over-involvement of the user is initially required. Testing and evaluation using the HIPOUS chart is required in major application environments. There is no alternative to conceptual explorations and studies if architectural status is to be achieved with these new systems and processes.

VIII. Concluding Remarks

A new communications process has been described to overcome the problems and barriers encountered in human communication of information in media form. The impact of partitioning a message into basic graphic forms is an immediate reduction in cost for communicating highly effective graphic messages between strangers. A series of secondary advantages which accrue due to partitioning and graphic symbol processing is an enabling of new and broader distributive forms of message delivery.

The sharing of the intelligence of both man and machine in an interactive dialog creates an optimum information package, low cost and current, correct, and controlled information communications result. With a flexible and adaptable interface via software, experimentation will also lead to highly optimized methods and services for communicant groups with beliefs in common. Advancements in technology and increased learning through experimentation will also lead to a minimum of user involvement in machine operations. Habits once established and remembered by machines, will only require a user–machine dialog in creation and addressing of messages.

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A Note on the Evaluation of Probabilistic Labelings

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***Abstract*— In a probabilistic labeling of a graph a probability vector over the possible labels is assigned to every node. Many algorithms, among them probabilistic relaxation, attempt to improve a probabilistic labeling based on some statistical model. In evaluating the new probabilistic labelings produced by such algorithms, we can use several criteria: entropy, which represents the ambiguity of the labelings; consistency with the model; and distance from the original labeling. It is shown that probabilistic labelings that maximize any of these criteria individually are not desirable. The use of linear combinations of two criteria is also discussed.**

I. Introduction

In many identification problems, the initial identification is ambiguous. From the measurements that can be initially used it may be hard to determine the identification of each object exactly; instead, probabilities are assigned to the different possible classifications that an object can have.

Graph labeling provides a convenient representation for such problems. The nodes in the graph are the objects, arcs represent relations among objects, and the labels represent classes of objects. The probabilistic identification of the objects is represented by probability vectors assigned to the nodes. These probability vectors associate a probability with each possible label that the node can have.

Given a probabilistic labeling, it is of interest to find an ordinary unambiguous labeling, where only one label is assigned to each node, that is most strongly supported by the probabilistic labeling and by some probabilistic model for the labeled graph. When the model is a probabilistic finite state grammar, and the graph is a string, methods have been developed to find such a labeling [10]. But in many cases, such as those handled by

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relaxation [1]–[9], finding such a maximal labeling directly does not seem to be feasible. The approach taken in this case is to try to use the given model to get a probabilistic labeling which is an improvement over the initial labeling. Relaxation attempts to increase the consistency of the labeling.

Several criteria have been used to evaluate relaxation processes by computing various "goodness" measures on the sequences of probabilistic labelings that they produce. A measure on a probabilistic labeling will be a function from the probability vectors into the real numbers. Among the criteria considered are entropy, which represents the ambiguity of the labeling, consistency with the model, and distance from the original labeling. We shall show that probabilistic labelings that maximize any of these criteria individually are not desirable. The use of linear combinations of two criteria will also be discussed.

II. Measures on Probabilistic Labelings

A. Entropy

The entropy measure is defined on a probability vector; it assigns a maximal value to ambiguous vectors and has value zero for a nonambiguous vector. Given a probability vector $P=(p_1,\cdots,p_n)$, the usual definition of entropy is

$$\epsilon(P)=\sum_{i=1}^{n} p_i\log(p_i). \tag{1}$$

$\epsilon(P)$ is zero when there exists a $1\leqslant j\leqslant n$ such that

$$p_i=\delta_{ij}, \qquad i=1,\cdots,n.$$

$\epsilon(P)$ is maximal when $p_1=\cdots=p_n=1/n$. $\epsilon(P)$ is used to measure the ambiguity of a vector; the greater $\epsilon(P)$ the more ambiguous P is.

An alternative definition of entropy is used in [2]:

$$\epsilon(P)=\sum_{i=1}^{n} p_i(1-p_i)=1-\sum_{i=1}^{n} p_i^2. \tag{1a}$$

This alternative form, which has very similar properties to (1), is more easily analyzed since the "log" is eliminated. To obtain the entropy of an entire labeling, the individual entropies for each probability vector are usually averaged.

For a probabilistic vector, it seems advantageous to be less ambiguous, since for a less ambiguous vector the certainty in choosing a final label is increased. Thus it might be desirable to obtain probability vectors with low entropy. Experiments with relaxation [8] show that the entropy usually does decrease, often rapidly at first. But lowering the entropy itself is of no value. When a probabilistic labeling consists only of probability vectors such that $p_1=1$ and $p_i=0$ for $1<i\leqslant n$, it has a trivially zero entropy. But such a labeling does not take into account any knowledge we might have about the model or the initial probability labeling. For this reason the entropy measure is usually used in conjunction with other measures [2].

B. Distance from Original Labeling

The distance from the original labeling takes into account the initial probabilistic labeling. Low distance indicates similarity to the initial labeling. The distance measure is generally computed using a norm

$$D(P)=\|P-P^{(0)}\|^2=\Sigma\left(p_i-p_i^{(0)}\right)^2. \tag{2}$$

This is computed separately for each vector in the labeling. A distance for all the vectors can be obtained by adding all the individual distances. This is reasonable when (2) involves no square root (as in Euclidean distance).

The labeling which minimizes the distance is, of course, the initial labeling, which we want to improve. As in the case of the entropy, the distance measure is thus used only in conjunction with other measures to evaluate labelings.

C. Consistency

The consistency measure takes into account the probabilistic model. This model gives a probability to every labeling or sublabeling, and increasing the consistency will increase the probability under the model. A model can be as complicated as a probabilistic grammar [10] or as simple as a list of individual probabilities of labels. The model we will use here is the one most used in relaxation [1], [2]; it consists of individual probabilities of labels and joint probabilities of pairs of labels at neighboring nodes.

Let $\Pr(l_i=\alpha)$ be the probability that the label l_i (at node v_i) will be α, and let $\Pr(l_i=\alpha,\ l_j=\beta)$ be the joint probability that label l_i will be α and label l_j will be β. Also let P_i and P_j be the current probability vectors at v_i and v_j. [$\Pr(l_i=\alpha)$ is thus the *a priori* probability for l_i to be α, while $P_i(l_i=\alpha)$ is the probability in the current labeling for l_i to be α]. It was suggested in [2] that a labeling will be consistent if

$$P_i(\lambda_k)=\sum_l P_{ij}(\lambda_k|\lambda_l)P_j(\lambda_l). \tag{3}$$

The above expression looks intuitively reasonable, but the mixing of *a priori* probabilities (the P_{ij}) and the current probabilistic labeling (P_i) is not justified.

An expression for consistency can be derived theoretically using the methods developed in [1]. Assuming that

$$\Pr(P_i,P_j|l_i=\alpha,l_j=\beta)=\Pr(P_i|l_i=\alpha)\Pr(P_j|l_j=\beta),$$

and given the probabilities $\Pr(l_i=\alpha)$, $\Pr(l_i=\alpha|l_j=\beta)$, and the current probability vectors P_i and P_j, the probability of l_i being α (using all the above information) is expressed as follows:

$$P_{ij}(l_i=\alpha)=P_i(l_i=\alpha)\cdot\frac{\dfrac{1}{\Pr(l_i=\alpha)}\displaystyle\sum_{\beta\in\Lambda}\Pr(l_i=\alpha|l_j=\beta)\cdot P_j(l_j=\beta)}{\displaystyle\sum_{\lambda\in\Lambda}P_i(l_i=\lambda)\cdot\frac{1}{\Pr(l_i=\lambda)}\sum_{\beta\in\Lambda}\Pr(l_i=\lambda|l_j=\beta)\cdot P_j(l_j=\beta)}. \tag{4}$$

A labeling is consistent when $P_{ij}=P_j$, which occurs when P_i is unambiguous (all probability is given to one label only), or when

$$\Pr(l_i=\lambda)=\sum_{\beta\in\Lambda}\Pr(l_i=\lambda|l_j=\beta)\cdot P_j(l_j=\beta) \tag{5}$$

for all $\lambda\in\Lambda$. Expression (5) is very similar to expression (3) but with the distinction between *a priori* and current probabilities. A measure for consistency can thus be some distance measure between the expressions and the actual values over all labels, averaged for all neighbors, and then averaged for all nodes in the graph.

It can be seen that when

$$P_i(l_i=\lambda)=\Pr(l_i=\lambda)=\sum_\beta\Pr(l_i=\lambda,l_j=\beta),$$

the labeling is consistent, since both conditions (3) and (5) hold. The above labeling, where all labels at all nodes have their *a priori* probabilities, is the most ambiguous, since no additional information to the *a priori* knowledge was used. Such a labeling is called a "no information" labeling, and it is generally undesirable to get such a labeling during relaxation.

It should be noted that another type of consistent labeling exists for (4), namely a consistent unambiguous labeling. In such a labeling, at each node the entire probability is given to one label only. To be consistent, an unambiguous labeling should have the property that $P_i(l_i=\alpha)=P_j(l_j=\beta)=1$ only when $\Pr(l_i=\alpha,\ l_j=\beta)>0$.

The relaxation scheme described in [1] has as fixed points both the "no information" and the unambiguous labelings. The more traditional relaxation of [3] does not have this property. More on the advantages of the new relaxation can be found in [6].

The measure (5), and also the very similar measure (3), give the minimum value to the "no information" labeling, while much more desirable labelings like the consistent unambiguous labeling have a high level of "inconsistency." Thus, to get a good measure, the consistency measures (3) or (5) have to be combined with other measures.

Another measure of consistency was used in [4]. Given any iterative algorithm that increases consistency (as relaxation attempts to do), the rate of change between iterations can serve as an indicator of the consistency of the labeling. Experiments with relaxation [8] have shown that the rate of change is greatest at the first iteration and often drops rapidly at later iterations. In [4] it was proposed to stop iterating the relaxation process when the rate of change becomes smaller by an order of magnitude than the rate of change at the first iteration.

III. Combining Measures

It was shown in Section II that minimizing any measure by itself does not yield a satisfactory probabilistic labeling. In this section we will study the effect of combining any two measures out of the three described in the previous section.

A. Entropy and Distance

Both the entropy measure (1) and the distance measure (2) are defined on individual nodes, so minimizing these measures at each node separately will minimize the measure for the entire graph.

Let $\epsilon(P)$ and $D(P)$ be the entropy and the distance measures of the probability vector P. They can be combined by

$$D\epsilon=\alpha\epsilon+\beta D \tag{6}$$

where α and β are weights. We will analyze the behavior of $D\epsilon$ by using the modified definition of entropy in (1a) and breaking it into two parts:

$$\begin{aligned}\epsilon(P)&=\sum_{i=1}^{N} p_i(1-p_i)=1-\sum_{i=1}^{N} p_i^2\\&=1-\frac{1}{N}-\sum_{i=1}^{N}\left(p_i-\frac{1}{N}\right)^2.\end{aligned}$$

The distance from the initial labeling $P^{(0)}$, as in (2), is

$$D(P)=\sum_{i=1}^{N}\left(p_i^{(0)}-p_i\right)^2.$$

Combining both gives us

$$\begin{aligned}D\epsilon(P)&=\alpha\left[1-\frac{1}{N}-\sum_{i=1}^{N}\left(p_i-\frac{1}{N}\right)^2\right]+\beta\left[\sum_{i=1}^{N}\left(p_i^{(0)}-p_i\right)^2\right]\\&=\alpha\left(1-\frac{1}{N}\right)+\beta\sum_{i=1}^{N}\left(p_i^{(0)}-p_i\right)^2-\alpha\sum_{i=1}^{N}\left(\frac{1}{N}-p_i\right)^2.\end{aligned} \tag{7}$$

The first term in (7) is constant, the second measures the distance from the initial labeling, and the third measures the distance from the labeling with all probabilities equal, $P_E=(1/N,1/N,\cdots,1/N)$. Overall to minimize (7) we should get closer to $P^{(0)}$ and further from P_E. In R^N we can go as far as we want from P_E, so (7) does not have a minimum when $\alpha \geqslant \beta$. But within the probability space, the unambiguous labelings, which are the vertices of the probability space, are the furthest from P_E. Hence the vertex closest to $P^{(0)}$ will minimize (7). Let us denote this vertex by P^*; in it, the label having maximal probability in $P^{(0)}$ gets probability one, and all other labels get probability zero.

When $\beta>\alpha$, the minimum occurs on a path between $P^{(0)}$ and P^*, depending on α and β. Any point on this path, however, is closer to P^* than to any other vertex. Since in most uses of relaxation the final step involves choosing the maximal label for each node, such optimization does not affect the final labeling (namely P^*), and there is no need for the optimization.

B. Consistency and Entropy

Since both consistency and entropy are optimized for labelings that are independent of the initial labeling, any linear combination of both is guaranteed to have the same property. In [2] experiments were conducted using such combinations. In these experiments a steepest descent algorithm was used, starting from the initial labeling. It was justified by the claim that the combination of entropy and consistency has many local optima, and the algorithm will yield an optimum which is closest to the initial labeling. Unfortunately the many strong assumptions used (the presence of many optima, convergence into the closest one, etc.) are not explained or justified in [2]. The virtue of the approach used in [2] is that it specifically designed an algorithm to minimize ambiguity and inconsistency, unlike other approaches, in which the algorithms were designed independently of such criteria.

C. Consistency and Distance

The combination of consistency and distance measures seems to be the best choice, since these really represent two desirable properties: closeness to the initial labeling and consistency. It was suggested in [4] that the rate of change of the relaxation can be used to measure the consistency. In this case the higher the inconsistency found at the first iteration, the farther away from the initial labeling the inconsistency will be minimized. The expression used was

$$CD_k=\alpha\|P^{(0)}-P^{(k-1)}\|+(1-\alpha)\|P^{(k-1)}-P^{(k)}\|,$$

where the first term measures difference from the initial labeling, and the second term measures consistency. Since convergence can occur only when $\|P^{(k-1)}-P^{(k)}\|$ goes to 0, the point was made that it is natural to use a small α.

IV. Concluding Remarks

None of the measures discussed in this report seem to be clearly superior. All three measures are reasonable, but optimizing each of them separately, or even some combinations of them, seems to yield uninteresting results.

A study of relaxation and its applications shows that in most cases a maximal unambiguous labeling is determined from the last probabilistic labeling [5]. This suggests that it might make better sense to evaluate these unambiguous labelings rather than the probabilistic labelings. This approach was taken in [10] where a single unambiguous labeling was chosen for a probabilistic string and a probabilistic grammar. A similar approach can be taken for given *a priori* probabilities and initial probabilistic labelings of general graphs. Future research is planned in this direction.

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