Improving image resolution using subpixel motion

Shmuel PELEG, Danny KEREN and Limor SCHWEITZER

Department of Computer Science, The Hebrew University of Jerusalem, 91904 Jerusalem, Israel

Received 11 April 1986

Abstract: Given a low resolution camera, we would like to get an image with improved resolution using camera motion. From several low resolution images, with subpixel relative displacements, we get an improved resolution image. We find a high resolution image such that when simulating the imaging process we get low resolution images closest to the observed images. The method is tested on computer simulations, and simulated annealing is also used for the optimization process.

Key words: Image enhancement, sub-pixel resolution, sub-pixel matching.

1. Introduction

Given a sequence of images with subpixel relative displacements, a higher resolution image can be constructed. Traditionally, the imaging process has been written as equations with the pixel values of the high-resolution image as unknowns, resulting with the values of the observed low-resolution images. With some boundary conditions these equations could be solved, giving pixel values of the high-resolution image. This approach has been taken by Magee [1], who used as boundary conditions the assumption that at least one row and one column have zero values in the low-resolution images.

In this work we take a different approach: starting with an initial guess for a high-resolution image, we simulate the imaging process to compute simulated low resolution images. This guess of the high-resolution image is iteratively improved to achieve simulated low-resolution images closest to the observed ones. Using such a method we can avoid the need to use boundary conditions, as well as the instability due to round-off errors. Also, resolution can be improved from any number of overlapping low-resolution frames. The method is schematically described in Figure 1.

2. The optimization process

The optimization problem is to find a high-resolution image $P$ such that the set of low-resolution images obtained from $P$ by simulated imaging is closest to the observed set of low-resolution images. We therefore assume that the imaging process is known, and that the low-resolution images could be registered at subpixel accuracy. These problems have been treated elsewhere [2,3], and are not considered here.

We are given a set of low-resolution observed images $\{O_i(x,y)\}$. Starting with an initial guess for the high-resolution image, $G^{(0)}(x,y)$, we simulate the imaging process to get an initial set of simulated low-resolution images $\{L_i^{(0)}(x,y)\}$. The error between this set and the set of the observed low-resolution images $\{O_i(x,y)\}$ is defined as

$$E^{(0)} = \sum_i \sum_{(x,y)} |L_i^{(0)}(x,y) - O_i(x,y)|.$$  \hspace{1cm} (1)

We examine in each pixel the guess $G$, say $G(x,y)$. If the current grey level is $G(x,y) = l$, we consider the three possibilities $\{l-1, l, l+1\}$ for the value of $G(x,y)$. For each of these values we compute the change in the $L_i$ images, and the changes in the error as in (1). $G(x,y)$ is then assigned that value...
which resulted in the minimal error. The process is continued iteratively until no further improvement can be obtained in the error function, or until the maximum number of allowed iterations is reached.

3. Simulation results

The method has been tested with the following simulation: Given an image \( I(x, y) \) of side length \( l \), we assumed that the imaging process divides the image into blocks of size \( k \times k \), producing an average of each block to give a low-resolution image of side length \( l/k \). In the examples shown later we used \( l = 64 \), \( k = 4 \), and for each \( 4 \times 4 \) block the average has been computed using the weights

\[
\begin{bmatrix}
1 & 2 & 2 & 1 \\
2 & 4 & 4 & 2 \\
2 & 4 & 4 & 2 \\
1 & 2 & 2 & 1 \\
\end{bmatrix}
\]

By using different displacements of the \( 4 \times 4 \) blocks, sixteen different low-resolution images can be obtained with horizontal and vertical movements of the blocks. These sixteen low-resolution images were computed, and denoted by \( \{ O_i(x, y) \}_{i=1}^{16} \). The goal now is to create the best approximation to the high-resolution image from any given subset of \( \{ O_i(x, y) \}_{i=1}^{16} \).

For the initial guess we magnify each low-resolution image by representing each pixel with a \( 4 \times 4 \) block having same grey level. These magnified images are averaged using the right relative displacement to give the initial guess. Starting from this initial guess, each pixel is iteratively examined and updated to minimize the error (1) between low-resolution images \( \{ L_i \} \) computed from the guess, and the original observed low-resolution images \( \{ O_i \} \). Figure 2 shows the simulation result for a \( 64 \times 64 \) image in three cases: All 16 possible low-resolution images are used; 8 low-resolution pictures are used; and only 4 low-resolution images used. Even when only 4 images were used, the increase in resolution is substantial.

It should be noted that when all 16 low-resolution images are given, the problem is that of
Figure 2. Simulation results. (a) Original $64 \times 64$ image. (b) one of the $16 \times 16$ low-resolution images. (c) Initial guess with all 16 low-resolution images examined. (d) high-resolution images reconstructed from all 16 images. (e-f) Initial guess and reconstructed image when only 8 low-resolution images were examined. (g-h) Initial guess and reconstructed image when only 4 images were examined.

Figure 3. Simulated annealing. (a) Final reconstruction using all 16 images (same as Figure 2d). (b) Final reconstruction using all 16 images and simulated annealing.

regular deblurring: each low-resolution image is a sample of every fourth point at every fourth row from an image created by convolving the original image with the $4 \times 4$ matrix. The full blurred image can thus be created from all 16 low-resolution images, and conventional deblurring methods [4] can be used. These methods, however, could not be applied to cases where not all 16 possible low-resolution images are given, cases where the optimization approach is applicable.

4. Simulated annealing

The optimization process described in the last section often converges to a local minimum of the error function. We tried to use simulated an-

![Graph showing error functions for 'greedy' and simulated annealing algorithms.]
nealing [5,6] to get a better minimum. The principle of these algorithms is to modify the 'greedy' algorithm, that minimizes the target function at each step, to have some positive probability to increase the target function. The probability to 'spoil' the target function is related to 'temperature': it is high at the beginning, and gets lower as the process continues. This probability gives a chance to break out of local minima and reach better results. Specifically in our case, after each pixel finds the value which minimalizes its error, we change this value by 1 randomly, with a probability decreasing exponentially with the number of iterations. Simulated annealing seems nearly always to give better error results than the regular algorithm. Results using simulated annealing are shown in Figure 3, and the error function in these experiments is shown in Figure 4.

5. Concluding remarks

The suggested method to increase image resolution can probably be improved with some speed-up tricks in the optimization steps, and by studying the best annealing scheme for the process. As it can be applied to any number of overlapping images, it compares favorably with deblurring methods. To be applicable in real cases, some model for the imaging process should be given, and reliable sub-pixel registration performed.

References