Image Representation Using Voronoi Tessellation: Adaptive And Secure

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Abstract

An image is represented by the Voronoi tessellation generated from selected sampling points. Using a multiresolution approach, the density of the sampling points can be adaptive to image properties: smoother regions will have fewer sampling points than more detailed regions. The adaptation property results in better image quality than non-adaptive Voronoi representations, while preserving the property that only the holder of the seed of the pseudo-random number generator can reconstruct the original image.

1. Introduction

Ahuja, et al. [1] describe a method of representing images by a Medial Axis Transform (MAT) [2] that uses irregular polygonal blocks. The blocks of the MAT are the cells of the Voronoi tessellation generated by a planar random point process. The MAT is used to achieve compression along with secure transmission of the image - an outcome of the random selection of the cells nuclei. It was argued [1] that error levels in the reconstruction of the image could be reduced by controlling individual point locations, but at the price of losing the security of the transmission.

This paper proposes a method of selecting the sampling points, i.e., the nuclei of the cells of the Voronoi tessellation, in a fashion that is adaptive to the image properties, thus achieving substantially lower error levels in the reconstruction, and yet with no loss of security in the transmission. This will be achieved by a multiresolution approach where analysis of low resolution levels guides the placement of sampling points at the higher resolution levels.

The second part of this section will describe the Voronoi tessellation, Section 2 will briefly describe Ahuja's method of deriving the MAT and Section 3 will describe our multiresolution adaptive point selection method along with experimental results and comparison of the two methods.

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1.1. The Voronoi Tessellation

Let $S=\{p_1 \cdots p_n\}$ be a finite set of points in the Euclidean plane. For each point p_i in S there exists a region R_i in the plane with the property that every point in R_i is closer to p_i than to any other point of the set S. The region R_i is called the Voronoi polygon [3] (cell) of the point p_i.

The set of all Voronoi polygons partitions the plane and forms what is called the Voronoi tessellation. An example of the Voronoi tessellation is shown in figure 1. A good description of the Voronoi tessellation as a result of a growth process can be found in [1].

The Voronoi tessellation of n points in the plane can be constructed in $O(nlogn)$ time, an algorithm is given by Shamos and Hoey [4].

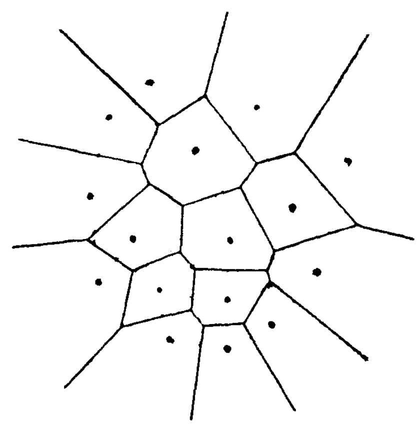

Figure 1: Voronoi tessellation constructed from a given set of points.

2. Image Representation Using Voronoi Tessellation

This section briefly describes the work of Ahuja et al. [1]. A digital Poisson point process is used to randomly select a set of sampling points in the image plane. The digital Poisson point process is making a binary decision at each pixel of the image. The probability for "success" in such a decision (the probability of the point to be selected as a sampling point) determines the total number of selected points and thus, as explained later, affects the compression and error ratios.

The Voronoi tessellation is constructed from the set of selected sampling points. In binary images each selected sampling point (a cell nucleus) is then assigned the color of the majority of pixels covered by the cell. In the case of gray level images the point is assigned the average gray level of the pixels covered by the cell. The set of sampling points (nuclei) along with the assigned colors constitute the representation of the image.

An $n \times n$ image has n^2 pixels. By selecting N sampling points and transmitting only the colors assigned to them, we get a data compression of n^2/N. Relative security of the transmission is also obtained: If the sender and the receiver use the same pseudo-random number generator, the points locations need not be sent. By having the seed of the pseudo-random number generator and receiving the list of color values, the receiver can reconstruct the Voronoi tessellation and color the cells uniformly with their color values, thus deriving an approximation of the original image.

The quality of the reproduction is a factor of the cell sizes and locations. Clearly, smaller Voronoi cells represent regions more accurately; therefore by selecting a larger number of sampling points we can achieve higher quality of reproduction, but at the price of lower compression rate.

As noted by Ahuja, et al. [1], error in the reproduction of the image could be reduced by carefully manipulating the position of the cells nuclei to better approximate the image, but when changing the position of the sampling points information concerning the characteristics of the image itself is revealed and thus the transmission is less secure.

In the next section we describe a multiresolution method of selecting the cells nuclei in a way that their position is chosen to reduce the error and improve the quality of the reproduction, and yet without the need to expose any information concerning the image characteristics in the transmission and therefore without reducing the security of the transmission.

3. Adaptive Selection of Sampling Points

Given an image it is reasonable to assume that, in order to obtain better quality in the reconstructed image, detailed parts of the image (edges for instance) should occupy more space in the representation than smooth parts. Therefore we would like to have more sampling points in richly detailed regions of the image than in smooth regions. By selecting more sampling points in a region it is covered by more Voronoi cells and consequently the cells are smaller and the reproduction error of the region is reduced. We therefore use a coarse Voronoi tessellation to detect the "busy" parts of the image, and select more sampling points from them to construct a more detailed Voronoi tessellation.

3.1. Multiresolution Selection

step 1 :

Using the pseudo-random number generator, randomly select a small number of points (n) in the image plane: If N is the number of pixels in the image, assign to every pixel a probability $p = \frac{n}{N}$. For every pixel use the pseudo-random number generator to choose a random number r uniformly distributed in the range [0,1], the pixel is selected if $r<p$. Due to the way p was calculated about n pixels would be selected.

step 2 :

Construct the Voronoi tessellation of the image using the selected points.

step 3 :

In the case of binary images, color each cell of the tessellation with the color of the majority of pixels in the cell, and in the case of gray level images color each cell with the average gray level of the cell's pixels.

These first three steps are actually the process described in [1] of representing and reconstructing the image.

step 4 :

Using an error measure (we used mean squared error, but it is possible to construct other measures relative to the point of interest), calculate the average error per pixel of each cell, compared with the gray level values of the original image. Assign to each sampling point its cell's average error value.

step 5 :

Select from the image a larger number of new sampling points (m) in the following way:
To every pixel i assign the probability

$$P_i = \frac{E_i}{\sum_{all\ pixels\ j} E_j} \times m$$

,where E_i is the error assigned in step 4 to the nucleus of the cell covering pixel i.

For each pixel i in the image we select a random number r_i uniformly distributed in the range [0,1]. The pixel i is selected as a sampling point if $r_i < P_i$. This process gives higher selection probability to pixels in regions with higher error rates.

step 6 :

Repeat steps 2 through 5 with the newly selected sampling points until the total number of points selected reaches the required compression rate.

3.2. Image Representation

The points selected at the last iteration of the point selecting algorithm, along with the colors assigned to them in step 3 of the algorithm, are the image representation. In order to reconstruct the location of these sampling points, the sender transmits the error values of the sampling points in all lower levels, along with the color values assigned to the points of the highest resolution sampling.

The receiver can now reconstruct an approximation of the image by repeating the process only if he possesses the seed used. He uses the seed to reconstruct the first Voronoi tessellation, then from the error values received he can reconstruct the second tessellation, and so on. The last reconstructed Voronoi tessellation has a relatively large number of cells, which are distributed according to the errors in the lower levels. By assigning to each cell its received color the receiver can get a good approximation of the original image.

The optimal number of points to select at steps 1 and 5 as well as the optimal number of iterations (step 6), are not determined yet. The question awaits further investigation and experiments. For a few examples refer to the experimental results section. Both sender and receiver should use the same number of selected points when they apply the algorithm.

3.3. Step by Step Example

Let us assume we have a gray level image of size 100×100, 1 byte per pixel, which we wish to securely transmit at a 10 to 1 compression rate. The image has 10,000 bytes therefore we want to transmit 1,000 bytes only.

First we randomly select about 50 sampling points in the image using the method described in step 1 for $n=50$ (each pixel has probability $\frac{50}{10{,}000}$ to be selected). We construct the Voronoi tessellation using the selected sampling points as the cells' nuclei, and color each cell of the tessellation with the average color of the pixels of the image covered by the cell. Using our error measure (mean squared error) we can compute the average error of each cell of the colored tessellation when compared with the original image. We save these error values.

We now select about 250 sampling points according to step 5 ($m = 250$). From these 250 new sampling points we again construct the Voronoi tessellation, color the cells, compute the error values and save them. According to these latest error values we calculate the error based probabilities as before and select the final 700 sampling points. From these 700 sampling points we yet again construct the Voronoi tessellation and color the cells. We assign to each of the 700 points the average color in its cell.

Finally, we transmit the 50 first error values, the 250 second error values and the 700 color values, a total of 1,000 bytes as desired.

The receiver, having the same pseudo-random number generator and the same seed, generates the same first 50 points and constructs from them the first Voronoi tessellation. He then assigns the first 50 error values to the cells and computes the probabilities exactly as the sender. Using the probabilities he selects the same 250 points and constructs the Voronoi tessellation from them. From the 250 error values he again computes the probabilities and selects the final 700 points. He then constructs the final Voronoi tessellation and colors the cells with the 700 color values received, obtaining an approximation to the original image.

3.4. Compression, Reduced Error And Security

By definition the algorithm gives any desired compression rate: The number of points selected at the last step of the algorithm is determined by the the predefined compression rate.

The reproduction error is a factor of both efficient placement and the number of the final sampling points. If more iterations of the algorithm are applied, more accurate information about the image is obtained and the final points are better positioned, but more intermediate error values must be transmitted and therefore fewer sampling points could be selected. The exact optimal ratio between the two is image dependent, but experiments show that using a compromise between the two results in higher quality of reproduction compared to using either of the extremes. Ahuja's algorithm is one of the extremes: no iterations, only random selected points. Although when using no iterations there are more final cells, their locations are totally random and, as shown in the experimental results, the error is higher than when using some intermediate error values to place a smaller number of final cells.

At the first step of the algorithm the points are randomly selected. Therefore the transmission is relatively secure from an eavesdropper who does not possess the seed of the pseudo-random number generator used.

Figure 2: The images used.

3.5. Experimental Results

Figure 2 displays the two 100×100 images we used for representation. Figures 3 and 4 compare the adaptive and the non-adaptive Voronoi representations using two compression rates each. In the adaptive case two iterations (3 samplings) of the algorithm were applied. The number of points selected at each sampling is noted in parentheses.

4. Concluding Remarks

We have presented an adaptive Voronoi representation of images, that yields improved results compared to the previous non-adaptive Voronoi representation suggested in [1]. The adaptation results from a multiresolution approach in which the information derived from the coarse samplings at lower levels is used to better position the final fine sampling. The number of optimal resolution levels and the number of points at each level, are still to be considered.

A method for improving the compression rate was suggested by Ahuja et al. [1]: Delete cells which are homogeneous and surrounded by homogeneous cells. This idea could still be applied while using our algorithm, thus yielding higher compression rates.

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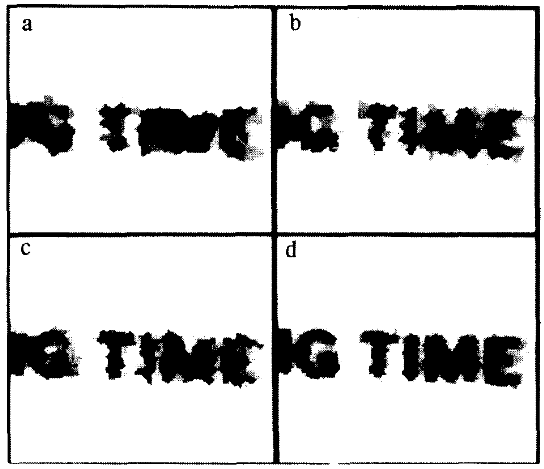

Figure 3: Non-adaptive Adaptive

a: Non-adaptive Voronoi representation - 20:1 compression rate.

b: The adaptive Voronoi representation (50,100,350) - 20:1 compression rate.

c: Non-adaptive Voronoi representation - 10:1 compression rate.

d: The adaptive Voronoi representation (50,150,800) - 10:1 compression rate.

Figure 4: Non-adaptive Adaptive

a: Non-adaptive Voronoi representation - 10:1 compression rate.

b: The adaptive Voronoi representation (50,150,800) - 10:1 compression rate.

c: Non-adaptive Voronoi representation - 4:1 compression rate.

d: The adaptive Voronoi representation (100,400,2000) - 4:1 compression rate.