Interference Alignment at Finite SNR for Time-Invariant channels

Or Ordentlich
Joint work with Uri Erez
EE-Systems, Tel Aviv University

ITW 2011, Paraty

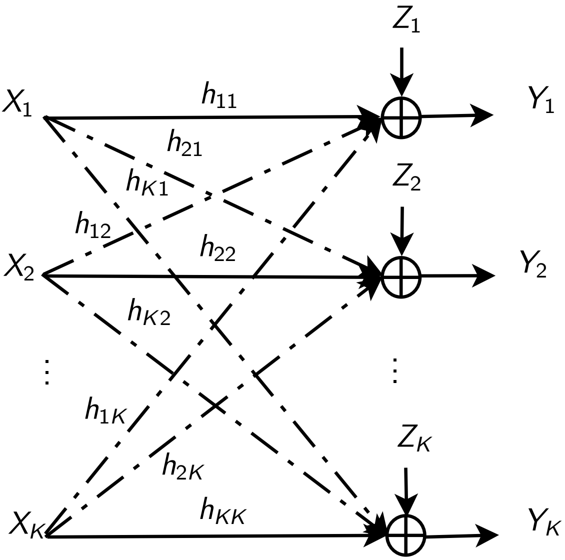
Background and previous work

- The 2-user Gaussian interference channel was recently nearly solved by Etkin et al. (IT-2008).
- For the 2-user case time-sharing is a good approach for a wide regime, and in particular achieves the maximal number of DoF.

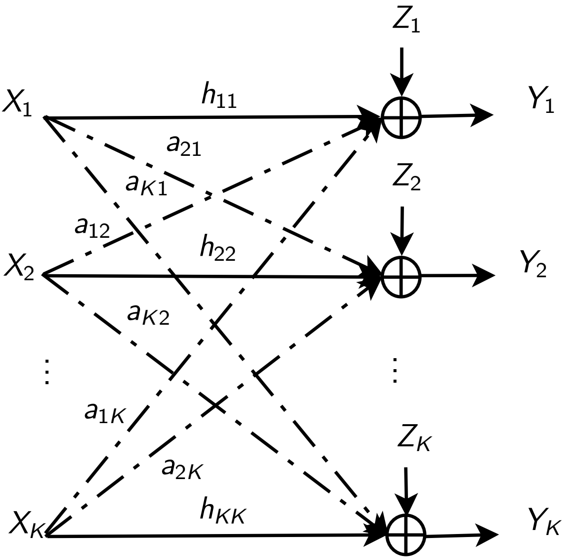
Breakthrough achieved by changing the channel model to time-varying

- Interference alignment was introduced by Maddah-Ali et al. for the MIMO X channel (IT-2008).
- Cadambe and Jafar (IT-2008) used interference alignment for the time varying K-user interference channel and showed that the DoF is $K/2$.
- Nazer et al. (ISIT-2009) showed that for finite SNR about half of the interference free ergodic capacity is achievable.
- The upper bound on the number of DoF (Host-Madsen et al. ISIT-2005) is met.

Time-invariant (constant) K-user interference channel



Special case: integer-interference channel



Background and previous work: time-invariant IC

Interference alignment is useful for the K-user integer-interference channel as well

- Etkin and Ordentlich (Arxiv-2009) showed that the DoF of an integer-interference channel is $K/2$ for irrational algebraic direct channel gains.
- The achievable scheme used an (uncoded) linear PAM constellation, in order to align all interferences to the integer lattice.
- They also gave a converse - for rational channel gains the number of DoF is strictly smaller than $K/2$.

- Motahari et al. (Arxiv-2009) showed that the DoF of almost every (general) K-user interference channel is $K/2$.

- All results above are very asymptotic in nature.
- Need to replace linear constellations (PAM) with linear codes (lattices).

Background and previous work: lattices

- Lattice codes have proven useful for many problems in network information theory (dirty MAC, 2-way relay, compute-and-forward...).
- First used for the interference channel by Bresler et al. (IT-2010) for approximating the capacity of the many-to-one interference channel.
- Sridharan et al. (Globecom-2008) used lattice codes in order to derive a very strong interference condition for the symmetric interference channel.
- Sridharan et al. (Allerton-2008) used a layered coding scheme in order to apply lattice interference alignment to a wider (but still very limited) range of channel parameters.
- The above results use a successive decoding procedure, which limits their applicability to a smaller range of channel parameters.

Lattice interference alignment: an example

- Assume receiver 1 sees the linear combination

$$\mathbf{y}_1 = h_1\mathbf{x}_1 + \sum_{k=2}^{K} h_k\mathbf{x}_k + \mathbf{z},$$

  where $\mathbf{z}$ is AWGN.
- If all the signals $\{\mathbf{x}_k\}_{k=2}^{K}$ are points from the same lattice Λ, and all interference gains $\{h_k\}_{k=2}^{K}$ are integers

$$\left[\sum_{k=2}^{K} h_k\mathbf{x}_k\right] = \mathbf{x}_{\text{IF}} \in \Lambda,$$

- The decoder therefore sees

$$\mathbf{y}_1 = h_1\mathbf{x}_1 + \mathbf{x}_{\text{IF}} + \mathbf{z}$$

- Two-user MAC: 1/2 goes to IF and 1/2 to the intended signal.
- For many values of h_1 successive decoding is not possible...
- MAC theorem does not hold: a new coding theorem is needed.

Interference alignment at finite SNR for time invariant channels

In this work:

- We derive an achievable symmetric rate region for the two-user MAC with a single linear code; this rate region has interesting properties.
- We use the new MAC result for deriving an achievable symmetric rate region for the integer-interference channel.
- Our rate region is valid for any SNR, and recovers the known asymptotic DoF results.
- The new rate region sheds light on the robustness of lattice interference alignment w.r.t. the direct channel gains.

MAC with one linear code: coding theorem

Theorem

For the channel $Y = X_1 + \gamma X_2 + Z$ where both users use the same linear code, the following symmetric rate is achievable

$$R_{\text{lin}} < \max_{p \in \mathcal{P}'(\gamma)} \min \left\{ -\frac{1}{2} \log \left(\frac{1}{p^2} + \sqrt{\frac{2\pi/3}{\text{SNR}}} + \frac{1}{p} e^{-\frac{3\text{SNR}}{2p^2}\delta^2(p,\gamma)} + 2e^{-\frac{3\text{SNR}}{8}} \right), \right.$$
$$\left. -\log \left(\frac{1}{p} + \sqrt{\frac{2\pi/3}{\delta^2(p,\gamma)\text{SNR}}} + 2e^{-\frac{3\text{SNR}}{8}} \right) \right\},$$

where $\delta(p,\gamma) = \min_{l \in \mathbb{Z}_p \setminus \{0\}} l \cdot \left| \gamma - \frac{\lfloor l\gamma \rceil}{l} \right|$, and

$$\mathcal{P}'(\gamma) = \left[p \in \mathcal{P} \middle| e^{-\frac{3\text{SNR}}{2p^2}\left(\gamma \bmod [-\frac{1}{4},\frac{1}{4})\right)^2} < 1 - 2p \cdot e^{-\frac{3\text{SNR}}{8}} \right].$$

- If $\gamma = \frac{m}{q}$ is a rational number, $R_{\text{lin}} < \log q$ for any value of SNR.

Efficiency of the MAC with one linear code

$$Y = X_1 + \gamma X_2 + Z$$

Random Gaussian codebooks

Recall that the symmetric capacity of the MAC is achieved using two different random Gaussian codebooks and is given by

$$R_{\text{rand}} = \min\left\{\frac{1}{2}\log\left(1+\text{SNR}\right), \frac{1}{2}\log\left(1+\gamma^2\text{SNR}\right),\right.$$
$$\left.\frac{1}{4}\log\left(1+(1+\gamma^2)\text{SNR}\right)\right\}.$$

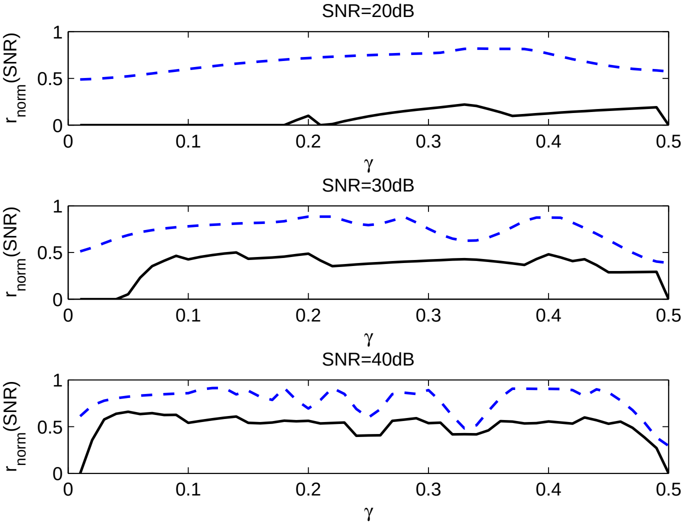
Definition

We define the efficiency of the two user MAC with the same linear code by

$$R_{\text{norm}} = \frac{R_{\text{lin}}}{R_{\text{rand}}}$$

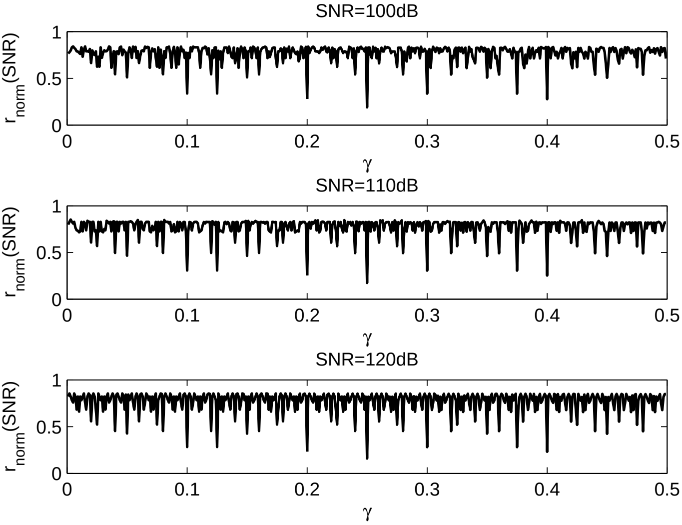
Efficiency of the MAC with one linear code

- R_{norm} vs. γ for "reasonable" SNR values



Efficiency of the MAC with one linear code

- R_{norm} vs. γ for extremely high SNR values



K-user integer-interference channel: achievable symmetric rate

- Assume all the interference gains at each receiver are integers, i.e., for all $j \neq k$, $h_{jk} = a_{jk} \in \mathbb{Z}$. The direct gains h_{jj} can take any value in $\mathbb{R}$.

Theorem

The following symmetric rate is achievable

$$R_{\text{sym}} < \max_{p \in \bigcap_{j=1}^{K} \mathcal{P}'(h_{jj})} \min_{j \in \{1,\ldots,K\}} \min \left\{ -\frac{1}{2} \log \left(\frac{1}{p^2} + \sqrt{\frac{2\pi/3}{\text{SNR}}} + \frac{1}{p} e^{-\frac{3\text{SNR}}{2p^2} \delta^2(p, h_{jj})} + 2e^{-\frac{3\text{SNR}}{8}} \right), - \log \left(\frac{1}{p} + \sqrt{\frac{2\pi/3}{\delta^2(p, h_{jj})\text{SNR}} + 2e^{-\frac{3\text{SNR}}{8}}} \right) \right\}.$$

K-user integer-interference channel: examples

Sanity check: the derived rate agrees with known DoF results

The linear code alignment scheme we use achieves $K/2$ degrees of freedom for almost every integer-interference channel.

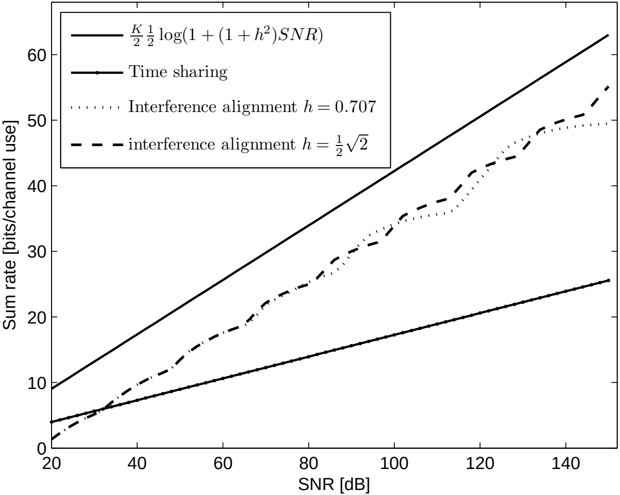
- As an example we consider the 5-user integer-interference channel

$$H = \begin{pmatrix} h & 1 & 2 & 3 & 4 \\ 5 & h & 3 & 6 & 7 \\ 2 & 11 & h & 1 & 3 \\ 3 & 7 & 6 & h & 9 \\ 11 & 2 & 6 & 4 & h \end{pmatrix}.$$

- Consider 2 different values of h: $h = 0.707$ and $h = \sqrt{2}/2$.

K-user integer-interference channel: examples

- $h = 0.707$ and $h = \sqrt{2}/2$.



Summary and future research

Summary

- We have proved a new coding theorem for the 2-user Gaussian MAC where both users are constrained to use the same linear code.
- This result was utilized in order to find an achievable rate region for the K-user integer-interference channel at finite SNR.
- The derived rate agrees with previous asymptotic results. For moderate values of SNR it is robust to slight variations of the channel gains.

Future research

- We would like to apply our results for general (non-integer) interference channels.
- Transforming an arbitrary interference-channel to an integer-interference channel is sometimes possible using time extensions, with some loss.