1 Introduction

Ten to fifteen years ago the Internet emerged into our lives, and effected almost all aspects of them, from social to economical and much more. In many fields the changes are just starting to be observed and it will take ten to thirty years, before the full influence of the Internet in those areas would be seen and could be analyzed.

In this course we would try to understand how future economy would look like, by understanding computer based commerce. Our starting point will be today’s economy and markets, and our focus will be on what can be done in the future, taking advantage of the computerized systems’ expected capabilities. We will concentrate on how to set up markets and bids in the modern world.

**Definition 1 (Market)** A Market is a social arraignment that allows buyers and sellers to discover information and carry out voluntary exchange of goods or services.

1.1 Examples of current computerized markets and bids

**Example 1** Financial markets.
*Relatively simple and well structured, such as*

- Stock market.
- Foreign currency.
- Goods market.

*The connectivity between such markets creates the global market and adds a new level of complexity.*

**Example 2** Ebay.

*An on-line auction and shopping website where people and businesses buy and sell goods and services worldwide.*
Example 3 Google Adwords.

On-line, real-time bidding system. Currently the most common bid (occurs about a million times per day). In each Google search result page, there is a limited number of sponsored links, related to the search topic. Each time a search is performed using Google, a bid takes place between advertisers for a place in the sponsored links section. The actual payment is on a cost-per-click basis.

Example 4 The selling of governmental permits for RF communication (in the USA).

The Federal Communications Commission (FCC) is an independent United States government agency that is charged with regulating interstate and international communications by radio, television, wire, satellite and cable. Since 1994, the Federal Communications Commission has conducted auctions of licenses for electromagnetic spectrum. These auctions are open to any eligible company or individual. FCC auctions are conducted electronically and are accessible over the Internet.\(^1\)

Example 5 Car parts acquisition management.

Each automotive company has many suppliers and second (and third) level suppliers for the various parts needed to assemble a car. The need arises for optimizing inventory levels vs production rate. Using computerized systems, this goal can be achieved while purchasing the needed parts at the minimum available price.

Our goal will be to create a mathematical and economical foundation for the analysis of such systems, based on computer science and economic principles.

2 Modeling a Simple Electronic Market

We will use a Bazooka gum market as an example of a simple market. We will assume the following:

1. All Bazookas are identical.
2. There are different Bazooka manufacturers with various production capabilities.
3. There are different buyers with various demands and budgets.
4. All buy/sell instructions are gathered from the different parties during a specified period of time, after which all transactions are determined for the following year.

We would like to determine the Bazooka production and acquisitions for the following year in an optimal way using a web site. Before attending to this task we notice that a number of critical issues regarding the optimum criteria arise:

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\(^1\)FCC website: www.fcc.gov
• What do we want to achieve?
• Do we want a maximum number of Bazookas to be exchanged?
• Do we want to maximize the volume of trade?
• How can we measure how much a party “needs” to participate in a transaction?
• How can we determine the distribution of production and acquisitions?

We will not view these problems from the buyer’s or seller’s perspective, but from the point of view of a “global government” interested in social welfare. We will measure the utility of each party for a chosen set of transactions and try to maximize the overall utility. The utility and how it can be distributed needs to be measurably defined. We will try to do so while continuing with our Bazooka example.

Back to our website, we will have two input formats:

<table>
<thead>
<tr>
<th>Seller Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount</td>
</tr>
<tr>
<td>Price per unit</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Buyer Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount</td>
</tr>
<tr>
<td>Price per unit</td>
</tr>
</tbody>
</table>

Selling of a partial amount is possible. Price per unit is identical for each amount.
Buying of a partial amount is possible. Price per unit is considered as maximal price.

Price per unit is considered as minimum price.

After acquiring all data we are able to construct a buyer list and a seller list.

Example 6 Buyer and seller lists.

<table>
<thead>
<tr>
<th>Sellers</th>
<th>Buyers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seller ID</td>
<td>Amount</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can now plot a supply graph and a demand graph on the same plot (the amount offered/required as a function of price per unit). Figure 1 shows such a graph, based on the
data from Example 6. Notice the meeting point of the two functions in the figure. This is the point where supply meets demand. Using this point we can resolve our problem, that is, we can determine the distribution of production and acquisitions. 8 pieces of gum will be sold for a price of 50$ per unit. What remains is to decide which buyers and sellers will participate in transactions. The buyers will be those who bid the most, and the sellers those who requested the smallest price, in our example it would be buyers 1 (3 units) and 3 (5 units) and sellers 2 (4 units) and 3 (4 units of 17 suggested).

Our decision process can be formalized into a greedy algorithm as illustrated by Algorithm 1.

**Algorithm 1 Assigning transactions**

1. Sort buyers in descending order by the suggested price per unit.
2. Sort sellers in ascending order by the requested price per unit.
3. Traverse the two lists from top to bottom and perform transactions while maintaining the total amount sold equal to the total amount bought.
4. Stop when the offered price in the buyer’s list is lower than the requested price in the seller’s list.

Let’s observe Figure 1 and find out what is the total profit that was gained using this algorithm.

If we color all the areas limited by the two graphs and the Price per unit axis, coloring in blue for the “sell” areas and in pink for the “buy” areas, we get Figure 2.
The colored area is the space in which profitable transactions have taken place. This area represents the gain from trade, i.e. social welfare.

Our method didn’t cause any damage: no seller had to sell bellow his minimal price, and no buyer had to buy above his maximal price. Further more, a number of parties had actually gained profit from their transactions. Therefore, our method increased social utility/welfare.

Let’s explain a few points that our discussion revealed.

**Definition 2 (Utility)** *In economics, utility is a measure of the relative happiness or satisfaction (gratification) gained by consuming different bundles of goods and services. Given this measure, one may speak meaningfully of increasing or decreasing utility, and thereby explain economic behavior in terms of attempts to increase one’s utility. The theoretical unit of measurement for utility is the util.*

**Definition 3 (Shortage problem)** *The shortage problem is a basic term in economy which describes the difference between the unlimited desires and needs of individuals and the limited resources of the economy.*

Our Bazooka example shows that we are able to increase the social utility by decreasing the shortage in Bazooka. But is the suggested solution optimal? Why should the highest bidder be allowed to buy and the cheapest seller to sell? We also need to pay attention to the fact that our problem definition hasn’t yet included money, but money was part of the solution. The need arises to express the utility of buyers and sellers in a formal mathematical manner.

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\(^2\text{Wikipedia.org}\)
**Definition 4 (Utility function)** In economics utility function is \( u : X \rightarrow \mathbb{R} \) when \( X \) is a consumption set - a set of items that could be consumed. The utility function defines the consumer preferences, that is if \( u(x) > u(y) \) for \( x, y \in X \) then the consumer prefers \( x \) over \( y \).\(^3\)

Let’s apply the above definition to our simple market case.

**Definition 5 (Value function)** The Value Function is the consumer utility function. \( V_i : \mathbb{N} \rightarrow \mathbb{R} \) is the \( i \)th buyer’s value/utility, measured in terms of money. \( V_i(n) \) is the value, for \( i \), of purchasing \( n \) units.

**Example 7** If a buyer wants 20 units at 300$ per unit, then his utility function (see also Figure 3) would be

\[
V_i(n) = \begin{cases} 
0 & n < 0 \\
300n & 0 \leq n \leq 20 \\
6000 & 20 < n
\end{cases}
\]

Notice that an assumption was made here: We assume that there is no added cost to the buyer for receiving more than the wanted amount of Bazookas.

We can specify a similar function for our sellers.

**Definition 6 (Cost function)** The Manufacturer’s cost function \( C_i : \mathbb{N} \rightarrow \mathbb{R} \cup \{\infty\} \), is the \( i \)th seller’s cost measured in terms of money. \( C_i(n) \) is the cost, for \( i \), to manufacture \( n \) units.

\(^3\)University of texas ,equilibrium theory course, lect1.
Example 8 If a seller wishes to sell 30 units at 100$ per unit, then his cost function (see also Figure 4) would be

\[ C_i(n) = \begin{cases} 
0 & n < 0 \\
100n & 0 \leq n \leq 30 \\
\infty & 30 < n 
\end{cases} \]

We could have expressed more complex needs of the parties by more complex functions or graphs. Note that in that case, we would have to change Algorithm 1 to handle such input. For now, we will confine our discussion to the simple value and cost functions as presented in Example 7 and Example 8.

3 The Relation Between Utility and Money

Can we really measure utility by money? Is this measure homogeneous to all people?

Assumption 7 (Money) Money exists and it can be used to measure utility.

The assumption that money can be used as a uniform measure of utility is not trivial. When is such an assumption valid? Usually when the stakes are low, that is, when the items involved don’t have a significant meaning to individuals or parties (unlike medical life saving treatment for example) and their price is relatively low.

Assumption 8 (Transference of utility) Utility can be transferred using money.
As we noted, these are philosophically profound assumptions that we will side step in this course. The place to look for innovations in this respect is technology.

4 Summery

We defined and agreed on the following

- Each buyer has a value function measured by money.
- Each seller has a cost function measured by money.
- We confined these functions to a certain type.
- We accepted the assumptions that relate money with utility.
- We accepted the assumption that the utility function measures welfare/satisfaction correctly.

We have not yet formalized the optimal solution, the condition for it to be possible and the optimum criteria for the simple market problem.