Concurrent Auctions Across The Supply Chain

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ABSTRACT

With the recent technological feasibility of electronic commerce over the Internet, much attention has been given to the design of electronic markets for various types of electronically-tradable goods. Such markets, however, will normally need to function in some relationship with markets for other related goods, usually those downstream or upstream in the supply chain. Thus, for example, an electronic market for rubber tires for trucks, will likely need to be strongly influenced by the rubber market as well as by the truck market.

In this paper we design protocols for exchange of information between a sequence of markets along a single supply chain. These protocols allow each of these markets to function separately, while the information exchanged guarantees efficient global behavior across the supply chain. Each market form a link in the supply chain operates as a double auction, where the bids on one side of the double auction come from bidders in the corresponding segment of the industry, and the bids on the other side are synthetically generated by the protocol to express the combined information from all other links in the chain. The double auctions in each of the markets can be of several types, and we study several variants of incentive compatible double auctions, comparing them in terms of their efficiency and of the market revenue.

1. INTRODUCTION

1.1 Motivation

The recent rush towards electronic commerce over the Internet raises many challenges, both technological and conceptual. This paper deals with the conceptual challenge of coordination between electronic markets. Let us look only a few years into the technological future of electronic commerce. It seems very likely that the following two key challenges will be adequately solved by the industry:

- **Supply Chain Integration:** The enterprise information systems of businesses will be able to securely and efficiently share information and interoperate with the information systems of their suppliers, customers, and partners.
- **Electronic Markets:** Efficient, sophisticated, robust and liquid electronic markets will be available for the trade of goods in most segments of the industry. Such markets will interactively respond to changes in supply and demand, dynamically changing trade quantities and prices.

We are interested in the conceptual question of how can markets for related goods share information. Consider, for example, a fictional market for rubber tires for trucks, and the two related markets for rubber and for trucks. One can imagine the following simplified supply chain forming: rubber manufacturers placing sell bids for rubber in the rubber market; tire manufacturers placing buy orders on the rubber market and sell bids on the tire market; truck manufacturers placing buy bids in the tire market and selling trucks on the truck market; and finally customers bidding for trucks. One would expect the combination of these markets together with the information systems of the manufacturers to be able to automatically respond to markets changes in an economically efficient way. Thus, for example, a surge in demand for a certain type of trucks, will raise their price in the truck market causing manufacturers of this type of truck to automatically decide to increase production, consequently and automatically raising their electronic bids for tires. This, in turn, may increase tire prices in the tire market, etc., etc., leading, eventually and indirectly, but still completely automatically, to increased rubber production by rubber manufacturers.

Let us emphasize: the process just described occurs, slowly, in normal human trade by the combined effects of a large number of self-interested decisions by the many people involved in this supply chain. What we desire from the

*This research was supported by grants from the Israeli academy of science and the Israeli ministry of science.

†This research was supported by grants from the Israeli academy of science and the Israeli ministry of science.

EC'01, October 14-17, 2001, Tampa, Florida, USA.

combination of the participating information systems and electronic markets is to automatically, without human control, and very rapidly, within the time-frames of electronic commerce, to reach similar results, or even more economically efficient ones than what humans usually achieve. These results should be achieved despite the fact that the information systems of manufacturers will still be self-interested – optimizing the company's profit and not the global economic efficiency.

Seeing the "invisible hand" function in normal human economic activity, one would certainly expect that electronic markets reach these types of results. However, a key conceptual design challenge emerges when bidders must be *concurrently active in more than one market.* Tire manufacturers must concurrently participate as buyers in the rubber market and as sellers in the tire market. Clearly, the quantity of rubber they wish to buy at a given price is determined by the amount of tires they can sell at a given price. Thus, the price they bid for buying rubber must be intimately related to the price they bid for selling tires – any increase in one of them will lead to a corresponding increase in the other. It is theoretically impossible to define a suggested bid for one without the other. Thus, if the two markets operate independently, then the tires manufacturers are not able to reasonably participate in any of them. If they operate sequentially, say first rubber is bought and only afterwards tires are sold, then a serious exposure problem emerges: tire manufacturers must be conservative in their bids for rubber, as they do not know in advance what price they will get in the tire market.

One approach for handling this inter-dependence of markets is to run the complete supply chain as a single complex huge market. Conceptually this is in the spirit of the recently popular "vertical markets" and "vertical portals" that try to vertically integrate information and trade for a complete vertical segment of industry. The integration of all these markets into a single complex market results in a complex optimization problem, and some research has been done to address such problems as a pure optimization problems. Such a centralized solution has obvious advantages, but is also problematic due to necessity of concentrating all information, communications, and decision making at a single point. This is problematic both in the sense of distributed computing systems and in the economic sense.

In this paper we suggest an alternative approach: the supply chain is organized as a sequence of separate markets that communicate among themselves using a fixed protocol. A similar approach has been suggested in [14, 15], who formulate a general problem that is NP-complete and obtain solutions that are not provably efficient, either in the computational sense or in the economic sense. In [3] a much simpler problem was considered, but no provably efficient protocol was suggested. We consider the same problem, and obtain provably efficient protocols. In our protocols, the intermediate markets along the chain are semantically for transforming one good into another. Thus, for example, tire manufacturers place bids for the operation of "transforming a unit of rubber into a tire". The protocol between the different markets assures that the different markets reach compatible decisions, i.e. that the amount of "rubber units to tires" that was allocated is equal both to the amount of rubber manufactured and to the amount of tires needed. Furthermore, these amounts achieve global economic efficiency across the supply chain. Finally, this result does not assume from the manufactures information systems any global knowledge or behavior beyond that they know their own cost structure and are self-interested (rational).

1.2 Technical Scenario of this Paper

This paper focuses on the case of a simple linear supply chain for discrete units of goods, where each manufacturer is able to transform a single unit of one good into a single unit of another good, incurring some cost for the transformation. We shortly mention how our results extend to the case of multi-unit bids and to more complicate forms of "chains" such as trees.

Our markets all take the form of double-auctions ([6]), and we consider several such variants. These variants address three issues: *Incentive Compatibility:* The double auction rules motivate self-interested players (the manufacturers) to follow truthfully the protocol rules. (We use the standard notions of dominant strategies from Mechanism Design [10, 8].) *Economic Efficiency:* The desired outcome should optimize the total utility of all participants; specifically, this means that all buyers with valuations above the market clearing price should trade with all sellers whose valuations are below the clearing price. *Budget Balance:* The buyer payments are not necessarily equal to the seller payments, but we wish to ensure that the market mechanism itself does not subsidize the trade, and thus the total buyer payments should be at least the total amount given to sellers. It is well known that these three conditions cannot apply simultaneously [8], and thus we consider variants that trade-off the last two conditions. Specifically, we suggest several double auction rules, some of these rules are randomized, that obtain budget balance or surplus but with a slight loss of efficiency. Such a deterministic rule was previously suggested in [9] but, surprisingly, this rule turned out to not be compatible with our supply-chain protocols. We provide some simulation results comparing the efficiency and budget surplus of the different variants of double auctions.

The main contribution of this paper is the description of two alternative protocols that allow a supply chain formed from such double-auctions to operate efficiently. These protocols are computationally efficient in terms of communication and computation time. We prove that when these protocols are applied with our double auction variants, the resulting system exhibits the following three key properties:

1. Global economic efficiency (not optimal, but as good as the underlying double auctions.)
2. Incentive compatibility. (in the sense of dominant strategies)
3. Budget balance. (in expectation)

Paper Organization

In section 2 we give a complete self contained example of organizing a simple supply chain and demonstrate the types of calculations and information transfer needed. In section 3 we summarize the properties of several variants of incentive-compatible double auctions. Details regarding these double auction rules appear in appendix A. In section 4 we present two alternative protocols for supply chain coordination between markets, and prove the properties achieved by these protocols. Finally, in section 5, we mention extensions.

2. THE LEMONADE-STAND INDUSTRY

Charlie Brown has decided to draw on his vast experience in the lemonade stand industry and transform the whole industry by bringing it online to the Internet. Charlie Brown has already struck partnerships with strategic players from the three core segments of the industry:

- **Lemon Pickers:** Alice, Ann, and Abe can pick a lemon from the neighborhood lemon tree (one lemon maximum per day).
- **Lemonade squeezers:** Bob, Barb, and Boris know how to squeeze a single lemon and make a glass of lemonade from it (one glass maximum per day).
- **Lemonade customers:** Chris, Carol, and Cindy want to buy one glass of lemonade each.

Charlie Brown has obtained a preliminary version of this paper and has built his Internet systems accordingly, using the *symmetric protocol* suggested here. Charlie Brown has created three communicating electronic markets: A *lemon market* through which Alice, Anna, and Abe will sell lemons. A *squeezing market* in which Bob, Barb, and Boris will offer their squeezing services, and a *juice market* in which Chris , Carol, and Cindy can buy lemonade.

In the first day of operations each of the participants logged into his or her market and has entered a bid: Alice is asking for 3$ in order to pick a lemon, while Ann wants 6$, and Abe (who lives farthest from the tree) wants 7$. Bob, Barb, and Boris are asking for, respectively, 1$, 3$, and 6$ in order to squeeze a lemon, while Chris, Carol, and Cindy, are willing to pay, respectively, 12$, 11$, and 7$ for their glass of lemonade. Let us follow the operation of the system and see how it manages to reach socially efficient allocations: how it decides how many lemons should be picked to be squeezed into lemonade.

Lemon Market		Squeezing Market		Juice Market	
Supply (S^L)	Demand (D^L)	Supply ($S^{L\to J}$)	Demand ($D^{L\to J}$)	Supply (S^J)	Demand (D^J)
3		**1**			**12**
6		**3**			**11**
7		**6**			**7**

Figure 1: The Supply Chain bids

In the first stage, the markets send information to each other in two phases. In the first phase, the lemon market aggregates the supply curve for lemons, S^L, and sends this information to the squeezing market. The squeezing market aggregates the supply curve for squeezing services, $S^{L\to J}$, adds this vector, point-wise, to S^L, and sends the sum to the juice market. For the juice market, this sum represents the supply curve for juice, S^J, aggregated over the complete supply chain.

Lemon Market		Squeezing Market		Juice Market	
Supply (S^L)	Demand (D^L)	Supply ($S^{L\to J}$)	Demand ($D^{L\to J}$)	Supply (S^J)	Demand (D^J)
3		**1**		4	**12**
6		**3**		9	**11**
7		**6**		13	**7**

$\xrightarrow{S^L}$ $\xrightarrow{S^L+S^{L\to J}}$

Figure 2: The Supply Chain after supply graphs propogation

In the second phase, the juice market send the demand curve for juice, D^J to the squeezing market, who subtracts from it, point wise, the supply curve for squeezing services, sending the difference vector to the lemon market, where this is interpreted as the demand curve for lemons, D^L, aggregated over the complete supply chain.

Lemon Market		Squeezing Market		Juice Market	
Supply (S^L)	Demand (D^L)	Supply ($S^{L\to J}$)	Demand ($D^{L\to J}$)	Supply (S^J)	Demand (D^J)
3	11	**1**		4	**12**
6	8	**3**		9	**11**
7	1	**6**		13	**7**

$\xleftarrow{D^J-S^{L\to J}}$ $\xleftarrow{D^J}$

Figure 3: The Supply Chain after demand graphs propogation

The net demand curve for squeezing services, $D^{L\to J}$ can now be calculated by the squeezing market to be $D^J - S^L$. At this point all three markets have both a supply curve and a demand curve, and they conduct a double auction in each market.

Lemon Market		Squeezing Market		Juice Market	
Supply (S^L)	Demand (D^L)	Supply ($S^{L\to J}$)	Demand ($D^{L\to J}$)	Supply (S^J)	Demand (D^J)
3	11	**1**	9	4	**12**
6	8	**3**	5	9	**11**
7	1	**6**	0	13	**7**

Figure 4: The Supply Chain after constructing the supply and demand graphs

Being an Internet startup, Charlie Brown has decided to subsidize the tarde in his markets, ignoring the sections of this paper that aim to eliminate any budget deficit of the markets. He is thus using the Generalized Vickrey Auction rules (a.k.a. VCG rules [13, 5, 7]) in the markets. Each market now operates separately according to the double auction VCG rule. In the lemon market two lemons are sold (by Alice and Ann) for 7$ each (computed as the minimum of 7, the first non-winning supply bid, and, 8, the last winning demand bid. In the Squeezing market, two squeezing contracts are awarded (to Bob and Barb) for 5$ each ($min(5, 6)$), and in the juice market the VCG rules award two glasses of lemonade (to Chris and Carol) for the price of 9$ each ($max(7, 9)$).

Charlie Brown is thrilled: the different markets have all reached the same allocation amount, 2, which, he has verified is indeed the social optimum: Society's net gains from trade in his system are (12+11)-(1+3)-(3+6) = 10$, which can't be beaten. Charlie Browns' investors are somewhat worried by the fact that the system subsidized every glass of lemonade by 3$ (7+5-9), but Charlie Brown assures them that changing the double-auction rules to one of the other double auction rules suggested in this paper can lead to a budget balance or even surplus. The nine trading partners have evaluated carefully the operation of this chain of markets and have assured themselves that they are best served by always bidding their true cost structure.

3. INCENTIVE COMPATIBLE DOUBLE AUCTIONS

Each of the markets along our supply chain performs a double auction (a study about double auctions can be found in [6]). Our protocols for supply chains can function with a wide selection of double auction rules so in this section we consider several double auction rules. All these rules start by constructing the supply and demand curves for the market by sorting the supply bids $S_1 \leq S_2 \leq ...$ and the demand bids $B_1 \geq B_2 \geq ....$ At this point the optimal trade quantity l is defined to be maximal such that $B_l \geq S_l$. Most real markets proceed by choosing a market clearing price anywhere in the range $S_l ... B_l$ (e.g. $(B_l + S_l)/2$ – "the 1/2-DA" which is a special case of **The k-Double Auction** [16, 4, 11]). This pricing scheme is *not* incentive compatible but it has been shown to perform reasonably well in many cases under strategic behavior of the participants [1, 12].

One may alternatively use **The VCG Double Auction** rule, setting separate prices for the winning demand bids $p_B = max(S_l, B_{l+1})$ and for the winning supply bids $p_S = min(S_{l+1}, B_l)$. This ensures incentive compatibility but leads to a budget deficit by the market since $p_B < p_S$. It is well known that as long as "participation constraints" are met (e.g. if non-traders pay 0) every incentive compatible mechanism that always achieves the optimal trade quantity must lead to a budget deficit. We demonstrate several incentive compatible double auction rules that achieve budget balance (or even surplus) at the price of achieving slightly sub-optimal trade quantity. Here we only describe the rules shortly, more details can be found in appendix A. The simplest auction with this property is **The Trade Reduction DA**. In this auction $l - 1$ units of the good are traded, each trading buyer pays B_l, and each trading seller receives S_l.

We suggest two new randomized double auction, which capture the tradeoff between the auction efficiency and the budget balance with one parameter α. In **The α Reduction DA**, for a fixed $0 \leq \alpha \leq 1$, the bids are submitted and then with probability α the Trade Reduction DA rule is used, and with probability $1 - \alpha$ the VCG DA rule is used. This randomized double auction is *universally incentive compatible* which means that the agents bid truthfully even if they know the randomization result.

The α Payment DA is another randomized double auction which has the same distribution of the allocation as the former auction, and the payment of each agent is his expected payment of The α Reduction DA. This auction is incentive compatible; but not universally.

In both of these auctions, as α grows from zero to one, the expected revenue increases from negative to positive and the expected efficiency decreases. If the distribution of the agents types is known prior to the beginning of the auction, the parameter α can be chosen such that the expected revenue is zero. We denote this value of α as α^*.

Another such rule which is an extension of the Trade Reduction rule, was previously suggested by McAfee in [9] but it turned out not to be compatible with our protocols – we extend on this in appendix B.

In figure 5 we present a summary of the properties of the DA rules. Further details appear in appendix A and in [2].

DA rule	Incentive compatible	Revenue	effiecincy loss
k-DA	no	0	0
VCG	yes	deficit	0
Trade Reduction	yes	surplus	LFT
α^* *Reduction*	yes, universally	0 (expected)	α^* LFT (expected)
α^* *Payment*	yes, not universally	0 (expected)	α^* LFT (expected)
McAfee	yes	surplus	not more then LFT

Figure 5: Double Auction Rules comparison table
Notes : a) in the k-DA we assume that the agents bid truthfully b) LFT means Least Favorable Trade which is $B_l - S_l$.

We have also ran simulations comparing the different DA rules with respect to the market revenue and to the total social efficiency. In figures 6 and 7 we show the results for the average of 100 random auctions with a given number of buyers and sellers, with all bids drawn uniformly at random in the range [0..1].

4. THE PROTOCOLS

We suggest two protocols that can be used to conduct an auction in a chain of markets in a distributed manner, **The Symmetric Protocol** and **The Pivot Protocol**. Both protocols run on servers connected by a network with chain topology. Each server represents one market and receives bids from only one type space. In the Symmetric Protocol, each of the markets conducts a double auction, after constructing its demand and supply graphs. In the Pivot Protocol, only one market constructs its demand and supply graphs, this market applies the double auction allocation and payments rule, and sends the results of the auction to the rest of the markets. Each market uses this information to calculate its allocation and payments.

We denote the supply market as M^1 and each conversion market from a good r to the following good $r + 1$ as $M^{r \to r+1}$. The demand market is marked by M^t. Each of the

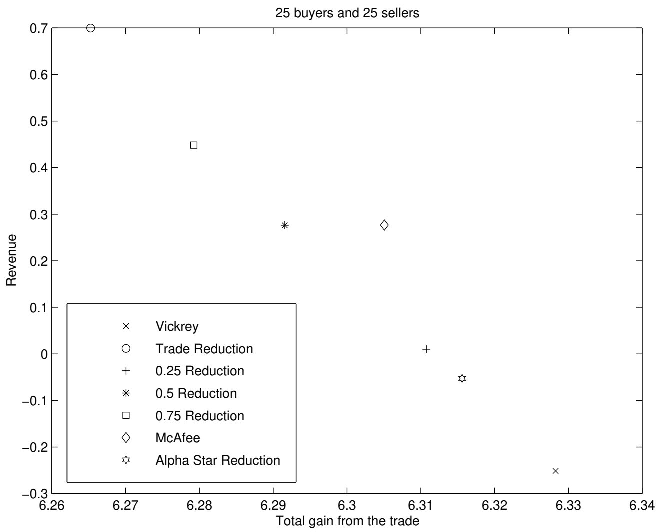

Figure 6: 25 buyers and 25 sellers
Revenue/Efficiency tradeoff simulations results

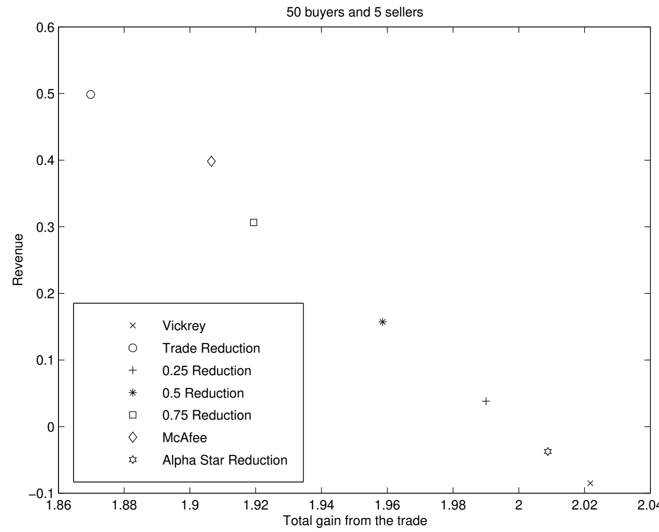

Figure 7: 50 buyers and 5 sellers
Revenue/Efficiency tradeoff simulations results

conversion markets is connected with bi-directional communication channels to the market that supplies its input good and to the market which demands its output good. We denote the supply and demand graphs for a good r as S^r and D^r respectively, and the supply and demand graphs for the conversion of a good r to a good $r+1$ as $S^{r\rightarrow r+1}$ and $D^{r\rightarrow r+1}$ respectively.

Our protocols are generic and can operate with various DA rules. The protocols are presented as using an abstract non-discriminating double auction rule that takes supply and demand graphs as inputs, and returns the trade quantity q and the prices for the sellers and the buyers P_S and P_B respectively.

4.1 The Symmetric Protocol

As we have seen in the example in section 2, the Symmetric Protocol starts with supply graphs propagation along the supply chain from the first supply market to the consumers (demand) market. The protocol continues with demand graphs propagation along the supply chain in the other way (demand graphs propagation may be done concurrently with the supply graphs propagation). During this process each of the markets builds its supply and demand graphs from the information it receives. At this point each of the markets has its supply and demand graphs, and a double auction is conducted. If the rule is randomized, the random choice is shared between the markets. The formal protocol for the supply market M^1 is described in figure 8. The formal protocol for the conversion markets is described in figure 9 and the formal protocol for the demand market M^t is described in figure 10.

Symmetric Protocol for the Supply Market M^1.

1. $S^1 \leftarrow$ sort the list of supply bids in non-decreasing order.
2. Send S^1 to demand market $M^{1\rightarrow 2}$.
3. Receive D^1 from demand market $M^{1\rightarrow 2}$.
4. apply DA Rule on (S^1, D^1) to obtain (q, P_S, P_B).
5. Output: The q lowest bidders sell for price P_S.

Figure 8: Symmetric Protocol for the Supply Market

Since we require *material balance*, which means that every conversion market produces the same number of items as it consumes, we require that all markets decide on the same trade size. A double auction rule with this property is called *consistent*.

DEFINITION 1. *A double auction rule that is used in all the markets in the Symmetric Protocol is called* **consistent**, *if for any bids of the agents, it decides on the same trade size in all the markets.*

Note that the optimal trade size l is the same in all the markets (since the difference between the i-th supply and

Symmetric Protocol for a Conversion Market $M^{r\rightarrow r+1}$

1. $S^{r\rightarrow r+1} \leftarrow$ sort the list of supply bids in non-decreasing order.
2. When receiving S^r from $M^{r-1\rightarrow r}$,
   send $S^{r+1} = S^r + S^{r\rightarrow r+1}$ to $M^{r+1\rightarrow r+2}$.
3. When receiving D^{r+1} from $M^{r+1\rightarrow r+2}$,
   send $D^r = D^{r+1} - S^{r\rightarrow r+1}$ to $M^{r-1\rightarrow r}$.
4. Construct market's demand graph $D^{r\rightarrow r+1} = D^{r+1} - S^r$.
5. apply DA Rule on $(S^{r\rightarrow r+1}, D^{r\rightarrow r+1})$ to obtain (q, P_S, P_B).
6. Output: The q lowest bidders sell for price P_S.

Figure 9: Symmetric Protocol for a Conversion Market

Symmetric Protocol for the Demand Market M^t

1. $D^t \leftarrow$ sort the list of demand bids in non-increasing order.
2. Send D^t to supply market $M^{t-1\to t}$.
3. Receive S^t from supply market $M^{t-1\to t}$.
4. apply DA Rule on (S^t, D^t) to obtain (q, P_S, P_B).
5. Output: The q highest bidders buy for price P_B.

Figure 10: Symmetric Protocol for the Demand Market

demand bids in each market is always the same from the way the graphs are build by the protocol), therefore any double auction rule which decides on a trade size which is a function of l only, is consistent. For example the VCG rule and the Trade Reduction rules are consistent since the trade size is l and $l-1$ respectively.

Not all double auction rules are consistent, for example McAfee's DA rule is not consistent as we can seen in the example in appendix B.

In case that the double auction rule is consistent and incentive compatible, the symmetric protocol creates an incentive compatible mechanism:

THEOREM 4.1. *If the double auction rule used by the symmetric protocol is incentive compatible and consistent, then the mechanism created by the symmetric protocol is materially balanced and incentive compatible.*

PROOF. See appendix C.1 □

4.2 The Pivot Protocol

In the Pivot Protocol, one of the markets is chosen as a pivot and the DA is only held there. Any market may be chosen as a pivot; we describe the case where the pivot is the consumers market.

The Pivot Protocol starts with supply graphs propagation along the supply chain as in the Symmetric Protocol. At this point the consumers market has its supply and demand graphs, and a double auction is conducted in this market only.

After that, starting with the consumers market which is now viewed as the demand market of its predecessor, each of the demand markets sends to it's supply market the size of trade and the critical value which is the highest price that the demand market is willing to pay for one unit of good without reducing the trade quantity.

The protocol for the pivot market is described in figure 11. The protocol for the conversion markets is described in figure 12 and the protocol for the supply market is described in figure 13.

The following theorem shows how the properties of the mechanism created by the pivot protocol are derived from the double auction rule used in the pivot market.

Pivot Protocol for Pivot Market M^t

1. $D^t \leftarrow$ sort the list of demand bids in non-increasing order.
2. Receive S^t from supply market $M^{t-1\to t}$.
3. apply DA Rule on (S^t, D^t) to obtain (q, P_S, P_B).

// Send results to the other markets:

4. Send (P_S, q) to market $M^{t-1\to t}$.
5. Output: The q highest bidders buy for price P_B.

Figure 11: The Pivot Protocol for Pivot Market

Pivot Protocol for a Conversion Market $M^{r\to r+1}$

1. $S^{r\to r+1} \leftarrow$ sort the list of supply bids in non-decreasing order.
2. When receiving S^r from $M^{r-1\to r}$,
   send $S^{r+1} = S^r + S^{r\to r+1}$ to $M^{r+1\to r+2}$.
3. When receiving the pair (V, q) from $M^{r+1\to r+2}$,
   send $(V - S_q^{r\to r+1}, q)$ to $M^{r-1\to r}$.
4. Output: The q lowest bidders sell for price $min(V - S_q^r, S_{q+1}^{r\to r+1})$.

Figure 12: Pivot Protocol for a Conversion Market

THEOREM 4.2. *If the double auction rule used in the pivot market is incentive compatible and non-discriminating, then the mechanism created by the Pivot Protocol is incentive compatible and non-discriminating.*

PROOF. See appendix C.2. □

In appendix B we present an example of a double auction rule (McAfee's rule) which has revenue surplus, but the pivot protocol using that rule creates a mechanism with revenue deficit. In section 4.4 we examine several chain auctions created by different double auction rules, and list their properties.

In case that the DA rule is consistent, incentive compatible and non-discriminating, the Symmetric Protocol and the Pivot Protocol create the same mechanism:

THEOREM 4.3. *A double auction rule which is incentive compatible, non-discriminating and consistent, if used by the pivot protocol and the symmetric protocol creates the same mechanisms.*

PROOF. See appendix C.3. □

4.3 Communication Complexity

Note that in all of the double auction rules that we have seen, the size of trade q is easily calculated from the Optimal

Pivot Protocol for the Supply Market M^1

1. $S^1 \leftarrow$ sort the list of supply bids in non-decreasing order.
2. Send S^1 to demand market $M^{1\to 2}$.
3. Receive the pair (V, q) from the demand market $M^{1\to 2}$.
4. Output: The q lowest bidders sell for price $min(V, S_{q+1}^1)$.

Figure 13: Pivot Protocol for the Supply Market

Trade Quantity l (it is either the same or one unit smaller). The payments rules are dependent only on the l and $l+1$ items in the supply and demand graphs. We can exploit this to reduce the communication complexity of the protocol:

THEOREM 4.4. *If the double auction rule which is used in the pivot protocol is incentive compatible, non-discriminating and the trade size decided by this rule is a function of the Optimal Trade Size l only, then the pivot protocol can be implemented with only $O(log(l))$ messages for each market, where each message contains $O(1)$ prices.*

PROOF. See appendix C.4. □

The idea is to use binary search to find l and thus send only the few values needed from the supply graphs, instead of passing the entire supply graphs along the chain.

4.4 Global properties of the different DA types

We examined the properties of the mechanisms created by the protocols using different double auction rules, and we summarize the results in the following theorem.

THEOREM 4.5. *The properties of chains of auction are as appear in figure 14.*

PROOF. See [2]. □

DA rule	Incentive Comp.	Revenue	Effiecincy loss
k-DA	no	deficit	0
VCG	yes	deficit	0
Trade Red.	yes	surplus	LFT
α^* *Reduction*	yes, universally	0 (expected)	α^* *LFT* (expected)
α^* *payment*	yes, not universally	0 (expected)	α^* *LFT* (expected)
McAfee	yes	surplus or deficit	not more then LFT

Figure 14: Supply Chain Auctions

Notes: a) In the k-DA we assume that the agents bid truthfully. b) McAfee's rule can not be used by the symmetric protocol, the table presents its properties under the pivot protocol. c) LFT (least favorable trade) in the context of supply chain, means the net total utility of the least favorable item.

When comparing the properties of the chain mechanisms in figure 14 with the properties of the original double auction rules in figure 5, one can see the following. The incentive compatible and the efficiency loss properties are preserved by the protocols, while the revenue is not preserved as we have seen in McAfee's rule example. The consistency property which enables chaining of the markets by the symmetric protocol applies to the first three deterministic rules, and the two randomized rules under the assumption of common coin toss. It does not exist in McAfee's rule since this rule sets its trade quantity as a function of the bids submitted, in such a way that the trade quantity can be different in two different markets.

Note that the supply chain auction created by the VCG rule, is a VCG auction in the broad sense - each of the agents pays the damage his bid caused to the other agents.

5. EXTENSIONS

5.1 Multi-Unit Bids

A simple extension to the model we have seen above is that agents can bid for multiple units instead of only one unit. For example, a selling bid in this case can be of the form (q, p) which means that this agent is willing to sell up to q units for at least p per unit. Most double auction rules presented in this paper can be extended to support this model as well.

Specifically, in order for the Trade Reduction rule to remain with revenue surplus, the reduction must be of size Q, where Q is the a-prior maximal amount requested by any agent.

The symmetric protocol can use those extended double auction rules to create a supply chain auction. To extend the pivot protocol, Q values must be sent backwards in the supply chain, instead of only one value. For more details see [2].

5.2 Trees of Auctions

Clearly a linear topology supply chain is only the simplest form of a supply chain, which in general may take the form of an arbitrary directed acyclic graph. Our Pivot Protocol is easily extended to the case of trees. In the linear topology supply chain that we have seen so far, a good is created by conversion from one specific good. In the tree extension, each good can be created from one of several goods. For example, we might have a lemon market in Florida and another lemon market in California, and also separate squeezing markets in both states, but only one lemonade market. The tree of auctions decides how many lemonade glasses will be produced, and how many of the glasses will be produced from Florida lemons and from California lemons. For more details about the extension of the Pivot protocol to this case see [2]. We do not know how to extend our results to the general case of DAGs.

Acknowledgments: We thank Hila Wallerstein for her comments on this paper.

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APPENDIX

A. DA RULES

In this appendix we survey some double auction (DA) rules. In all the rules, supply and demand graphs are created by sorting the bids in non-descending and non-ascending order respectively. Then the trade quantity and the payments to the agents are determined.

We denote by S_i the i-th supply bid (in the sorted order), and by B_j the j-th demand bid (in the sorted order). We denote by l *the optimal trade quantity*, l is maximal such that $B_l \geq S_l$. Trading l units maximizes the efficiency if the agents bids are the agents true types. In all the following double auctions, non-trading agents pay zero, and once the trade quantity q is set, the trading agents are the first q sellers and q buyers. We call a double auction rule *non-discriminating* if the price paid by all trading buyers is the same, and the price paid to all trading sellers is the same (but we do not require that the buyers price will be the same as the sellers price as in a uniform priced auction).

It is known that a double auction mechanism can not be efficient, incentive compatible and budget balanced, under the condition that non trading agents pay zero. The VCG double auction is efficient and incentive compatible, but is not budget balanced. The k-DA (presented below) is an efficient double auction with zero revenue but is not budget balanced. McAfee [9] suggested an incentive compatible and budget balanced DA, with some reduction in efficiency in some cases. We suggest to extend this concept of trading efficiency with revenue, and to use randomization in the allocation and payments rules in order to achieved high efficiency with expected revenue of zero.

We present the different double auction types and their properties:

- The VCG DA - l units of the good are traded, each trading buyer pays $max(S_l, B_{l+1})$, and each trading seller receives $min(S_{l+1}, B_l)$.
- The k-DA [16, 4]- Before the auction begins, a parameter k is chosen such that $k \in [0, 1]$. k is used to calculate a clearing price $P = k \cdot S_l + (1 - k) \cdot B_l$. l units of the good are traded at the price of P.
- The Trade Reduction DA - $l - 1$ units of the good are traded, each trading buyer pays B_l, and each trading seller receives S_l. (this is a simple version of McAfee's DA)
- McAfee's DA [9]- if a suggested clearing price $p = \frac{S_{l+1}+B_{l+1}}{2}$ is accepted by the l buyer and seller ($p \in [S_l, B_l]$) then l units of the good are traded at the price of p, otherwise the Trade Reduction rule is used.

We suggest two new randomized double auction rules, which capture the tradeoff between the auction efficiency and the budget balance with one parameter α:

- The α Reduction DA - Before the auction begins, a parameter α is chosen such that $\alpha \in [0, 1]$. Then the bids are submitted. With probability α the Trade Reduction DA rule is used, and with probability $1 - \alpha$ the VCG DA rule is used.
- The α Payment DA - A parameter α is chosen as in the α Reduction DA. Then the bids are submitted and the allocation and payments are decided. $l - 1$ units are traded between buyers which pay $\alpha \cdot B_l + (1 - \alpha) \cdot max(B_{l+1}, S_l)$ and sellers which receive $\alpha \cdot S_l + (1 - \alpha) \cdot min(S_{l+1}, B_l)$. With probability α another unit of the good is traded between a buyer which pays $max(B_{l+1}, S_l)$ and a seller which receives $min(S_{l+1}, B_l)$. (Note that the trade size and the allocation have the same distribution as in the α Reduction DA and each trading agent pays his expected payment in the α Reduction DA.)

In both of these auction, as α grows from zero to one, the probability of trade reduction increases, and as a result the expected revenue increases and the expected efficiency decreases. The VCG auction has revenue deficit and the Trade Reduction auction has revenue surplus, so if the distribution of the agents types is known prior to the beginning of the auction, the parameter α can be chosen such that the expected revenue is zero. We denote this value of α as α^*.

B. PROBLEMS WITH CHAINING MCAFEE'S AUCTION RULE

The following example of chain auction using McAfee's rule shows that not all double auction rules are consistent. Figure 15 shows the supply and demand graphs in the three markets after the supply and demand graphs propagation.

Lemon Market	
Supply (S^L)	Demand (D^L)
10	20
20	10

Squeezing Market	
Supply ($S^{L\to J}$)	Demand ($D^{L\to J}$)
5	15
7	-3

Juice Market	
Supply (S^J)	Demand (D^J)
15	**25**
27	**17**

Figure 15: Symmetric Protocol using McAfee's Rule

The optimal trade quantity in this example is one as can be seen in Figure 15. McAfee's rule in the lemonade market says that one item should be traded, since $\frac{27+17}{2} = 22 \in [15, 25]$. On the other hand, if we use McAfee's rule on the conversion market, trade reduction should be made since $\frac{7+(-3)}{2} = 2 \notin [5, 15]$ and the trade quantity is zero, in contradiction to the previous decision. We conclude that the symmetric protocol can not use all double auction rules.

The following example in figure 16 shows that the fact that the DA rule has revenue surplus, does not ensure that the pivot protocol using this rule has revenue surplus as well.

Lemon Market	
Supply (S^L)	Demand (D^L)
10	17
20	

$\xleftarrow{1,17}$

Squeezing Market	
Supply ($S^{L\to J}$)	Demand ($D^{L\to J}$)
5	12
7	

$\xleftarrow{1,22}$

Juice Market	
Supply (S^J)	Demand (D^J)
15	**25**
27	**17**

Figure 16: Pivot Protocol using McAfee's rule

In the example presented in figure 16 McAfee's rule on the lemonade market says that one item should be traded, since $\frac{27+17}{2} = 22 \in [15, 25]$. The buyer in this market should pay 22. By following the pivot protocol, the converter which bid 5 should be paid $7 = min(7, 22 - 10)$ and the supplier in the supply market which bid 10 should be paid $17 = min(22 - 5, 20)$. The total revenue is $22 - 7 - 17 = -2$ which means that there is revenue deficit.

C. PROOFS OF PROTOCOLS PROPERTIES

C.1 The Symmetric Protocol is Incentive Compatible

THEOREM C.1. *If the double auction rule used by the symmetric protocol creates an incentive compatible double auction, and the rule is consistent, then the mechanism created by the symmetric protocol is incentive compatible. If the rule is non-discriminating, then the mechanism is also non-discriminating.*

PROOF. The double auction rule is incentive compatible in each market, therefore any agent bids truthfully since from his point of view he submits his bid to an incentive compatible double auction, and therefore the mechanism created by the symmetric protocol is incentive compatible. Since the double auction rule is consistent, the outcome of the symmetric protocol is a consistent trade, which means that the outcome is materially balanced. If the double auction rule is non-discriminating, then the payment for all the winning agents of the same type in each of the markets is the same, and the mechanism is non-discriminating. □

C.2 The Pivot Protocol is Incentive Compatible

THEOREM C.2. *If the double auction rule used in the pivot market is incentive compatible and non-discriminating, then the mechanism created by the Pivot Protocol is incentive compatible and non-discriminating.*

PROOF. Since the double auction rule in the pivot market is incentive compatible, the buyers in this market bid truthfully. We will prove that any seller in the supply market bids truthfully, the proof for the converters is similar (so it will be omitted), and we will conclude that the mechanism is incentive compatible.

Assume a seller bids X to the supply market M^1 and assume we then run the pivot protocol using an incentive compatible and non-discriminating DA rule that results in trade size q and in P_S being the payment to the "sellers" in the pivot market. By the pivot protocol the payment to a winning seller in the supply market is

$$min(P_S - \sum_{r=1}^{t-1} S_q^{r\to r+1}, S_{q+1}^1)$$

We now show that this is the critical value of the seller that bid X. We show that if $X < S_{q+1}^1$ and $X < P_S - \sum_{r=1}^{t-1} S_q^{r\to r+1}$ then the seller trades, and if $X > S_{q+1}^1$ or $X > P_S - \sum_{r=1}^{t-1} S_q^{r\to r+1}$ then the seller doesn't trade.

- If $X > S_{q+1}^1$, then he losses the auction, since the trade size is q and he is not one of the first q bidders in his market.
- If $X > P_S - \sum_{r=1}^{t-1} S_q^{r\to r+1}$, then if $X > S_{q+1}^1$ we are back in the previous case, and if $X < S_{q+1}^1$ then it is impossible that the trade size is q since $S_q^1 \geq X$ and by the pivot protocol $S_q^t = S_q^1 + \sum_{r=1}^{t-1} S_q^{r\to r+1}$ and therefore $S_q^t \geq X + \sum_{r=1}^{t-1} S_q^{r\to r+1} > P_S$ which is a contradiction to the fact that the rule is incentive compatible and $S_q^t \leq P_S$.
- If $X < S_{q+1}^1$ and $X < P_S - \sum_{r=1}^{t-1} S_q^{r\to r+1}$, then the seller is the i in his market for $i \leq q$ and he wins the auction.

We conclude that any seller in the supply market which wins the auction receives his critical value, and therefore the auction is incentive compatible. □

C.3 Protocols equivalence

THEOREM C.3. *A double auction rule which is incentive compatible, non-discriminating and consistent, if used by the pivot protocol and the symmetric protocol creates the same mechanisms.*

PROOF. Since the supply and demand graphs of the demand market M^t are built in the same way and the rule used in the demand market is the same in both protocols, the trade size decided by the rule in this market is the same in both mechanisms. The symmetric protocol is consistent, therefore the trade size and the allocation are the same in all the markets in both protocols.

We now show that the payments are the same in both mechanisms. Since both mechanism are incentive compatible and have the same allocation, and the losing agents always pay zero, each agent pays his critical value in both mechanisms. The critical values of each agent must be the same in both mechanisms, since the allocation is the same. Assume in contradiction that an agent has two different critical values c_1 and c_2 in the two mechanisms, such that $c_1 < c_2$. If he bids T which satisfies $c_1 < T < c_2$, then the agent wins in one mechanism and losses on the other, in contradiction to equivalence in the allocations of the two mechanisms.

We conclude that the allocation and payments of the two mechanisms created by both protocols are the same, and therefore the mechanisms are the same. □

C.4 Communication Complexity

THEOREM C.4. *If the double auction rule which is used in the pivot protocol is incentive compatible, non-discriminating and the trade size decided by this rule is a function of the Optimal Trade Size l only, then the pivot protocol can be implemented with only $O(log(l))$ messages for each market, where each message contains $O(1)$ prices.*

PROOF. The Pivot Protocol can be improved by using binary search to find l and thus sending only the few values needed from the supply graphs, instead of passing the entire supply graphs along the chain. The search for l can be preformed by sending only $O(log(l))$ messages between any two consecutive markets. This is done by binary search, in the improved protocol first $S^t_{\frac{n'}{2}}$ is passed to the pivot market, for n' which is the number of bids in the pivot market. Then the pivot checks if this value is smaller or greater than $D^t_{\frac{n'}{2}}$ and asks for the $\frac{3 \cdot n'}{4}$ or the $\frac{n'}{4}$ element of the supply graphs respectively. The pivot market receives the requested value and continues in a similar way with the search, until l is found. Then the elements needed from the supply graph (which are only a function of l by our assumption) are requested by the pivot market and sent to it. □