

**The Configuration Space Method for Kinematic Design of
Mechanisms**

Elisha Sacks and Leo Joskowicz

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Preface

This monograph presents the configuration space method for computer-aided design of mechanisms with changing contacts of parts. Configuration space is a complete and compact geometric representation of the motions and interactions of parts in a mechanism that supports the core design tasks of analysis, synthesis, and tolerancing. Our method is the first general algorithmic treatment of the kinematics of higher pairs with changing contacts. It helps designers detect and correct unexpected kinematic behaviors and design flaws, as demonstrated in automotive, micromechanical, and optical case studies.

This book is intended for students, researchers, and engineers in mechanical engineering, computer-aided design, computer science, and robotics. The presentation is self-contained and is suited for a course, a seminar, or independent study. The prerequisites are freshman mathematics and computer science.

The first part describes the configuration space framework and the algorithms for the kinematics of mechanisms. The second part describes the algorithms for kinematic analysis, tolerancing, and synthesis based on configuration spaces. The third part presents four case studies taken from industry in which the configuration space method supports the analysis and design of mechanisms. A catalog of higher-pair mechanisms is given in appendix A. Appendix B describes an open-source C++ mechanical design system, called HIPAIR, that implements some of the configuration space methods described in this book, including visualization of configuration space and kinematic simulation. HIPAIR comes with an interactive graphical user interface and many sample input files for mechanisms.

1 Introduction

Mechanisms are assemblies of moving parts that perform useful tasks. They are pervasive in modern life and include familiar mechanical systems, such as door locks, transmissions, and gearboxes, and specialized ones, such as industrial robots and microscopic mechanisms fabricated on silicon chips (MEMS). The designing of mechanisms is an important engineering task that motivates research in design methods and in computer-aided design (CAD) tools.

A key aspect of this task is kinematics. This is the branch of mechanics that studies the motion of parts independently of the forces acting on them. It assumes that parts are rigid bodies with fixed shapes. A mechanism performs a task by transforming input motions into output motions through the contact of parts. The transformation of motions is called the kinematic function of the mechanism. The design goal is to ensure that the intended and actual transformations match. Studying the kinematics is an early and crucial step in designing a mechanism. It helps answer many questions about the workings of mechanisms and is a prerequisite for further mechanical studies involving dynamics, stress, and deformation.

The main kinematic design tasks are analysis, synthesis, and tolerancing. Analysis derives the kinematic function of a mechanism from a specification of its parts' shapes and motion constraints. For example, a rotating gear wheel is specified by its profile (how many teeth and what shape) and the configuration (position and orientation) of its rotation axis. A pair of gears is analyzed to determine its gear ratio. Synthesis is the inverse task of devising a mechanism that performs a specified function. The starting point can be a prior design or a novel design concept. Analysis and synthesis disregard the imprecision of manufacturing, which causes actual parts to vary from their intended shapes and configurations. The tolerancing design task is to determine the kinematic effect of variation in manufacturing

and modify the design or the manufacturing process to ensure correct function.

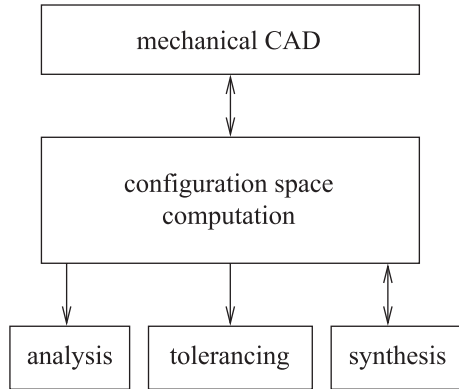
The foundation for a systematic study of mechanism kinematics was laid by the German engineer Franz Reuleaux (1829–1905). Reuleaux defined mechanisms as collections of basic building blocks, called kinematic pairs, and developed a classification system for mechanisms. A kinematic pair consists of two interacting parts. There are two types of pairs: lower and higher. Lower pairs involve a single, permanent surface contact between two parts, such as a round pin in a complementary hole. The parts have simple relative motions, such as rotation around an axis. All other pairs are higher pairs. Lower pairs are idealizations of higher pairs because real parts require a small clearance to function.

The compositional model of mechanisms uses a hierarchical analysis strategy: compute the kinematic functions of the pairs, then combine them to obtain the kinematic function of the mechanism. The kinematic function of a pair is determined by the parts' geometry in the neighborhood of the points of contact. Analysis consists of formulating and solving contact equations. These equations are highly nonlinear and can be solved in closed form only in simple cases. Combining a pair's kinematic functions entails an analysis of the interactions among the pair's contacts. This analysis is difficult, even for mechanisms with few parts, because there are many potential interactions. Synthesis and tolerancing are also performed hierarchically and pose similar challenges.

The mainstream approach to managing the complexity of kinematic design is to identify special cases that restrict the interactions of parts. Kinematic pairs are assumed to be lower pairs or to have fixed, closed-form contact relations. Mechanisms are limited to assemblies of such pairs. Consequently, a mechanism's kinematic function is specified by a fixed set of equations that can be solved with an efficient numerical algorithm. The most common categories are linkages, cams, and gear mechanisms. Linkages consist of lower pairs. Cam mechanisms consist of cams in permanent contact with followers. Gear mechanisms consist of meshed gears. These categories exclude many important mechanisms.

Pairs with multiple contacts are more versatile than fixed-contact pairs because they can perform multiple functions through changes in their contacts. Higher-pair mechanisms are typically cheaper, lighter, and more compact than lower-pair mechanisms. Examples include sewing machines, copiers, cameras, and compact disc players.

In this book we present a general computational theory of kinematic design that covers all types of mechanisms. In this approach, the contacts of

**Figure 1.1**

Kinematic design of a mechanism using configuration spaces.

parts are studied in a geometric representation called configuration space. We have developed algorithms for analysis, synthesis, and tolerancing within the representation of configuration space. The algorithms provide novel computer-aided design tools that can help designers detect unexpected behaviors, correct design flaws, and study the kinematic effects of variations in manufacturing.

Figure 1.1 illustrates our kinematic design paradigm. Mechanical CAD packages are used to create, modify, and visualize mechanism models. The computation of configuration space generates a kinematic model that is used in analysis and tolerancing, and is modified in synthesis.

The rest of this chapter is organized as follows. In section 1.1, we introduce kinematic function. In section 1.2, we discuss and illustrate kinematic design. In section 1.3, we describe the content and organization of this book.

1.1 Mechanisms and Kinematic Function

The kinematic function of a mechanism is determined by the shapes, configurations, and motions of its parts. For example, consider the indexer mechanism in figure 1.2. It is composed of six parts: a driver, an indexer, a pawl, a pin, a lever, and a frame. The driver is an offcenter cylinder that acts as a cam. The indexer is a gear with 24 teeth shapes as trapezoids. The pawl has a triangular tip shaped to follow the indexer's teeth. The pin is a rectangular block. Each of these parts rotates on a shaft mounted on a cylindrical hole in the frame. The lever has a rounded triangular tip shaped to engage

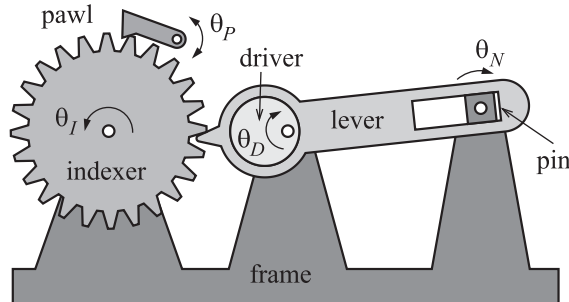


Figure 1.2
Indexer mechanism.

the indexer's teeth, a cylindrical hole that fits over the cam, and a rectangular slot that fits over the pin. Its motion is constrained by the driver and the pin. The frame holds the parts together. The indexer mechanism has six lower pairs (indexer, pawl, driver and pin mounted on the frame, lever-pin, and driver-lever), and two higher pairs (indexer-pawl, indexer-lever). The rotations of the parts are as indicated by the arrows. (The following conventions for arrows are used throughout the book. The intended part rotation and translation directions are indicated with thick-headed arrows. The rotation angle is always measured in the standard counterclockwise direction even when the arrow points clockwise. The translation is always measured along the standard right-handed coordinate axis even when the arrow points left. A double thick-headed arrow indicates back-and-forth motion.)

The kinematic function of the indexer mechanism is to advance the indexer wheel by one tooth (15°) for every turn of the driver. As the driver turns clockwise, the lever tip traces a closed trajectory whose form is determined by the relative position of the driver and pin rotation axes, the driver's offset, and the length of the lever. This causes the indexer to rotate counterclockwise by 15° . The pawl prevents clockwise rotation of the indexer. Figure 1.3 shows four snapshots of one cycle: (a) the start configuration, (b) the lever tooth engaging the wheel tooth, (c) the lever driving the wheel, and (d) the lever tooth disengaging the wheel tooth.

In designing a mechanism, the kinematic function is usually derived first because it sets the stage for the study of other physical phenomena. In our example, dynamical analysis determines the driver and indexer torques based on their mass and shape. Stress analysis determines the indexer tip deformation based on its load and material properties. In both analyses, the kinematic function constrains the motions and contacts of the parts.

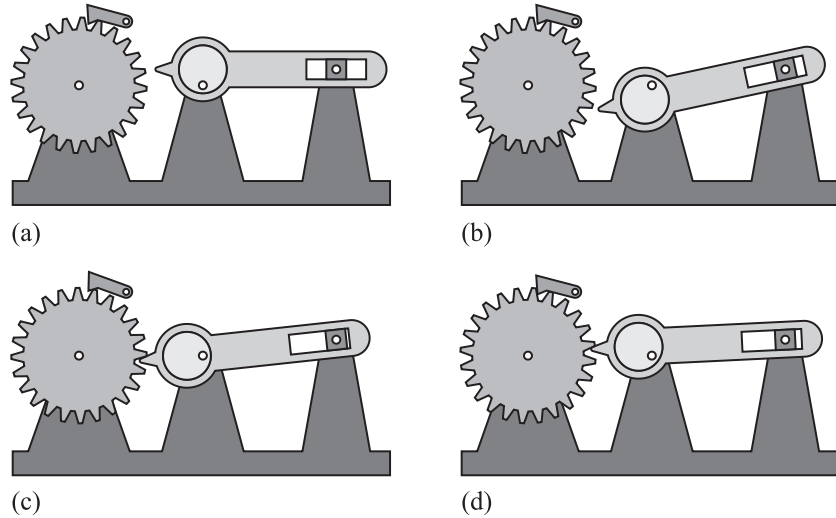


Figure 1.3
Snapshots of the kinematic function of an indexer mechanism.

Kinematic design is an iterative process consisting of five steps (figure 1.4). Conceptual design consists of selecting a design concept that captures the desired kinematic function. The concept determines the structure of the mechanism: the parts, the kinematic pairs, and the intended kinematic function. Parametric design consists of building a parametric model of the mechanism that encodes the shapes and configurations of its parts. Parameter values are chosen to achieve the intended kinematic function in the absence of manufacturing variation. The resulting design is called a nominal mechanism. Analysis, tolerancing, and synthesis are then performed as described earlier. They are iterated until a satisfactory design is obtained. When this is impossible, the parametric model is revised or an alternative design concept is elaborated.

1.2 Kinematic Design

We illustrate the design cycle using the indexer mechanism. Figure 1.5 shows a parametric model for the conceptual design described here. The parameters include the relative positions of the fixed axes, the offset of the driver, and the radii of the indexer teeth. The parts are modeled in the xy plane because they have a constant cross-section along the z axis. Likewise, their motions are modeled in the xy plane. The indexer, pawl, driver, and pin are mounted on cylindrical shafts, so each is modeled with a pin joint.

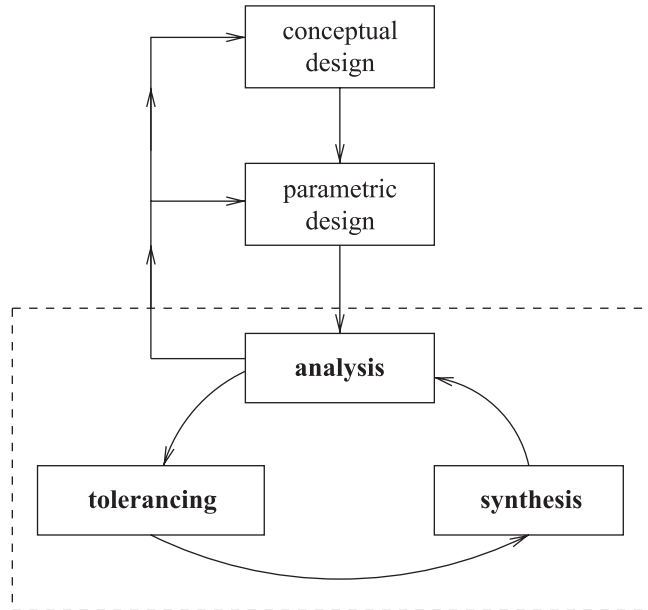


Figure 1.4
Kinematic design cycle. Boldface indicates tasks covered in this book.

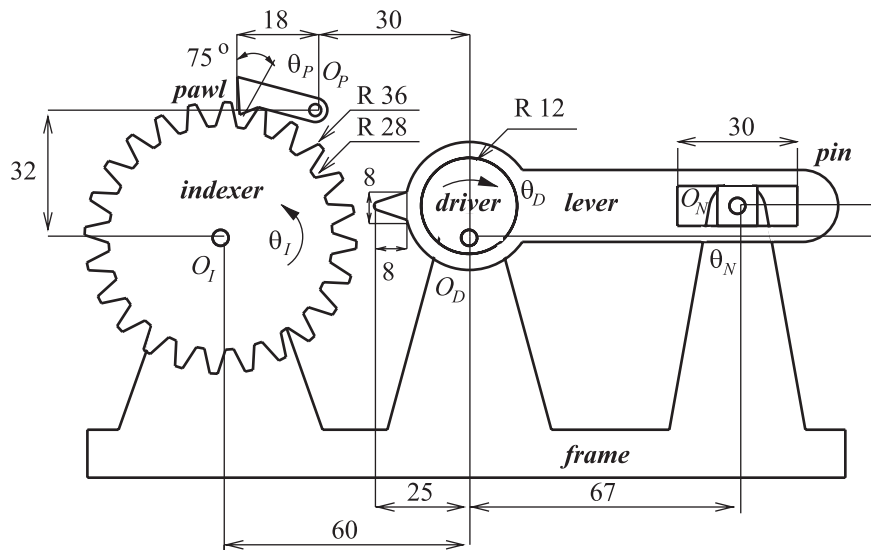


Figure 1.5
Indexer mechanism: functional parameters (in millimeters).

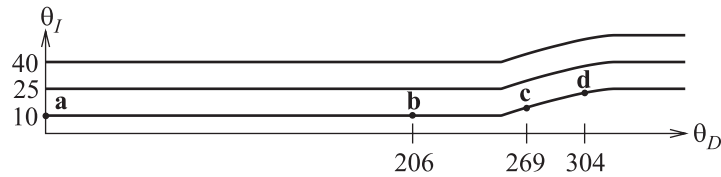


Figure 1.6
Kinematic function of the indexer mechanism.

Their configurations are specified with orientation parameters with respect to fixed axes: θ_I and O_I for the indexer, θ_P and O_P for the pawl, θ_D and O_D for the driver, and θ_N and O_N for the pin. The intended rotation directions are indicated with arrows. The lever is not mounted on a shaft, so its configuration is specified with two position parameters and an orientation parameter. The pin fits into the lever's rectangular slot, so the pin-lever contact is modeled as a slider joint. The driver and lever have a permanent contact between a circle and a matching circular hole. The pawl-indexer and lever-indexer interactions are more complex because they involve multiple changes of contact.

Having created the parametric model, the designer faces several key questions. Will the nominal mechanism function as intended or can it fail, owing to unexpected motions of parts? What modifications are necessary to make the mechanism function properly despite manufacturing variation? What tolerances are required? Can they be loosened, which reduces manufacturing cost, by modifying the nominal design? To answer these questions, the designer analyzes the current design and synthesizes alternative designs.

The analysis derives the kinematic function of the nominal mechanism from its parametric model. The kinematic function is quantified by plotting the relation between the motion parameters. Figure 1.6 shows the plot for our example. The functional relation is between the driver's orientation angle, θ_D , and the indexer's orientation angle, θ_I . The initial values are $\mathbf{a} = (0^\circ, 10^\circ)$. Angle θ_D increases clockwise and θ_I increases counterclockwise. The bottom curve represents the work cycle shown in figure 1.3 with points $\mathbf{a-d}$ corresponding to the snapshots. The indexer is at rest on the horizontal part of the curve and is driven on the diagonal part. At the end of the cycle, θ_D equals 360° and θ_I has increased to 25° . The middle curve represents the second cycle, in which θ_I increases from 25° to 40° , and the top curve represents the third cycle. Since the indexer has 24 teeth, there are 24 such segments at 15-degree intervals.

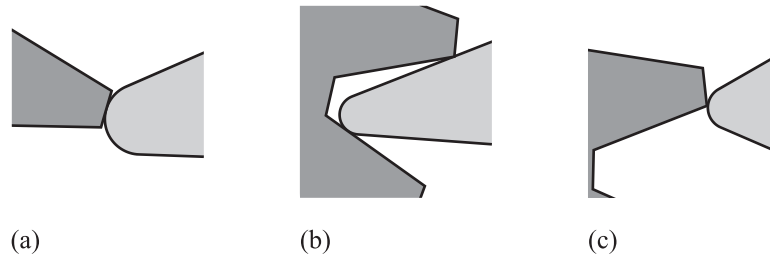


Figure 1.7

Lever pin failures: (a) tip too wide, (b) tip too long, (c) tip too short.

The kinematic function rarely has a closed form, especially for mechanisms with changes in contact. Numerical kinematic simulation is often used instead. Kinematic simulation takes a driving motion of a part as input and computes motions for the other parts according to the principle that rigid parts cannot overlap. The snapshots in figure 1.3 were generated from a simulation of the indexer mechanism. Rotation of the driver is the driving motion. The lever is assigned a motion that preserves its contacts with the driver and the pin. When the lever tip engages the indexer, the indexer is assigned an angular velocity that prevents tip and tooth overlap.

The drawback of simulation is that it only provides information for one input motion. Unexpected behaviors can be missed. In our example, the simulation assumes that the only driving motion is the driver's rotation. An unexpected force can make the pawl rotate clockwise, disengage from the indexer, and so fail to perform its function. Unexpected behaviors usually involve changes in the contact of parts.

Once the nominal function of the mechanism has been achieved, the next step is to assign tolerances. The goal is to ensure that the kinematic function is preserved for small variations in the shape and configuration of parts. Figure 1.7 shows three examples in which failures occur that are due to small variations of the lever tip. The nominal tip width is 1 mm and its nominal length is 7.5 mm. Too wide a tip (1.8 mm) causes the tip to hit the tooth top (figure 1.7a). Too long a tip (11 mm) causes blocking (b). Too short a tip (5 mm) fails to engage the indexer tooth (c). Tolerance analysis derives the kinematic variation for given tolerances. Synthesis adjusts the parameter values and tolerances to prevent these failures.

Kinematic design can be difficult and time-consuming. The designer has to optimize the design to comply with multiple, conflicting requirements. Many parameter values and tolerances must be adjusted. The adjustments require extensive analysis of many design instances. The analysis involves

many contacts of parts, with complicated contact equations. Unintended contacts can arise from variation in parts and can lead to failure modes that coexist with or supersede the nominal function.

A key property of mechanisms is that the interactions of parts are tightly coupled and nonlinear. This property implies that mechanisms cannot be decomposed into linear functional modules with narrow interfaces, whereas other engineering disciplines, notably circuit design, rely heavily on decomposition. The difficulty of mechanism design motivates research in design algorithms.

1.3 Content and Organization of the Book

This book describes our configuration space method for mechanism design, which we have developed over the past 20 years. The presentation is self-contained and tutorial, with references to related work by ourselves and others. It is a research monograph rather than a textbook, so the reader should be prepared to work out some technical details.

The book is organized as follows. In chapters 2–4 we describe our configuration space representation of kinematics. In chapter 2, we review basic geometry concepts, describe the configuration space representation, and discuss kinematic pairs and classification of mechanisms. The shape of a part is defined in terms of geometric features that form its boundary. In chapter 3, we study contacts between two features. We formulate contact equations for basic features, provide closed-form solutions for some cases, and provide general numerical solutions. In chapter 4, we study the configuration spaces of kinematic pairs and describe efficient algorithms for computing them.

In chapters 5–7 we present algorithms for kinematic analysis, tolerancing, and synthesis based on configuration spaces. In chapter 5 we present algorithms for kinematic analysis of a mechanism based on examination of configuration space and kinematic simulation. In chapter 6 we describe tolerance analysis based on a parametric worst-case tolerance model. In chapter 7 we present two methods for kinematic optimization of planar mechanisms.

In chapter 8 we illustrate the configuration space method for designing mechanisms using four case studies taken from industry. The case studies are an optical filter mechanism, a manual transmission gearshift, a MEMS torsional ratcheting actuator, and an automotive asynchronous spatial reverse gear pair. In chapter 9 we conclude with a summary and research directions.

The book contains two appendices. The first is a catalog of 30 representative higher-pair mechanisms. Each mechanism is depicted by a sketch and described with a brief narrative. Its kinematic function is described using configuration spaces. The second appendix contains the users' guide to an open-source C++ software package, HIPAIR, which implements some of the mechanism design methods discussed in this book using configuration spaces.

1.4 Notes

There are numerous introductory and advanced textbooks on kinematic design of mechanisms [31, 22, 78, 59]. Other books focus on linkage [27], cam [4, 26], and gear [53] mechanisms and their design. For an introduction and description of modern mechanical CAD, see Dimarogonas [19].

Commercial packages for mechanical computer-aided design include Catia-CADAM, Pro/Engineer, AutoCAD, I-DEAS, and ADAMS. These packages support drafting, manipulation, and visualization, and include limited support for kinematic analysis and tolerancing of mechanisms whose parts interact via a fixed set of contacts.

Books on robot kinematics include those by Paul [62] and McCarthy [55]. For recent work on robotic manipulation and computational kinematics, see Mason [54] and Angeles et al. [3]. The configuration space method for planning robot motion is described in Latombe [47] and Choset et al. [16].