Time and the Prisoner's Dilemma

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Abstract

This paper examines the integration of computational complexity into game theoretic models. The example focused on is the Prisoner's Dilemma, repeated for a finite length of time. We show that a minimal bound on the players' computational ability is sufficient to enable cooperative behavior.

In addition, a variant of the repeated Prisoner's Dilemma game is suggested, in which players have the choice of opting out. This modification enriches the game and suggests dominance of cooperative strategies.

Competitive analysis is suggested as a tool for investigating sub-optimal (but computationally tractable) strategies and game theoretic models in general. Using competitive analysis, it is shown that for bounded players, a sub-optimal strategy might be the optimal choice, given resource limitations.

Keywords: Conceptual and theoretical foundations of multiagent systems; Prisoner's Dilemma

Introduction

Alice and Bob have been arrested as suspects for murder, and are interrogated in separate rooms. If they both admit to the crime, they get 15 years of imprisonment. If both do not admit to the crime, they can only be convicted for a lesser crime, and get 3 years each. However, if one of them admits and the other does not, the defector becomes a state's witness and is released, while the other serves 20 years.

This is a **"Prisoner's Dilemma" (PD)** game, a type of interaction that has been widely studied in Political Science, the Social Sciences, Philosophy, Biology, Computer Science, and of course in Game Theory. The feature of PD that makes it so interesting is that it is analogous to many situations of interaction between autonomous parties. The PD game models most situations in which both parties can benefit from playing cooperatively, but each party can get a higher gain from not cooperating when the opponent does. See Figure 1.

As an example, consider two software agents, A and B, sent by their masters onto the Internet to find as many articles about PD as they can. The agents meet, and identify that they have a common goal. Each agent can benefit from receiving information from the other, but sending information has a cost. The agents agree to send packets of information to each other simultaneously. Assume that sending an empty packet costs \$1, sending a useful packet costs \$2, and receiving a useful packet is worth \$3. This interaction is precisely a PD game with $S = -2$, $P = -1$, $R = 1$ and $T = 2$.

		B	
		$\mathcal{D}$	$\mathcal{C}$
A	$\mathcal{D}$	P P	S T
	$\mathcal{C}$	T S	R R

$$T > R > P > S$$
$$2R > T + S$$

Figure 1: **Prisoner's Dilemma game matrix**

Under the assumption of rationality, the PD game has only one equilibrium: both players defect. This result is valid even for any finite sequence of games—both players can deduce that the opponent will defect in the last round, therefore they cannot be "punished" for defecting in the one-before-last round, and by backward induction it becomes common knowledge that both players will defect in every round of the repeated game.

The "always defect" equilibrium of PD is in a sense paradoxical; it contradicts some of our basic intuitions about intelligent behavior, and stands in contrast to psychological evidence (Rapoport *et al.* 1962). The root of this paradox is the assumption of rationality, which implies unlimited computational power; it is precisely the unlimited computational power of rational agents that both allows and requires them to perform the unlimited backward induction in the repeated PD. In reality, both natural and artificial agents have limited resources. In this paper we show that once these limitations are incorporated into the interaction, cooperative behavior becomes possible and reasonable.

The idea of bounding agents' rationality is not new. The novelty of the approach presented here is in its straightforwardness. The bound on rationality is measured in the most standard scale of Computer Science: computation time. We assume that agents need time for computations, and that the game is repeated for a finite length of time, rather then a fixed number of iterations. These assumptions are sufficient to create cooperative equilibria.

These results are interesting from two points of view, the system (or environment) designer's perspective, and the agent designer's perspective. From the system designer's point of view, it gives guidelines as to how to create a cooperation-encouraging environment. From the agent designer's point of view, it enables him to design a strategy that will impose cooperation on his agent's opponent.

Related Work

A thorough and comprehensive survey of the basic literature on bounded rationality and repeated PD appears in (Kalai 1990). Axelrod (Axelrod & Hamilton 1981; Axelrod 1984) reports on his famous computer tournament and analyzes social systems accordingly.

Most of the work on this subject has centered on various automata as models of bounded rationality; (Papadimitriou 1992; Gilboa & Samet 1989; Fortnow & Whang 1994) and others (see (Kalai 1990) for an extensive bibliography) deal with finite state automata with a limited number of states, and (Megiddo & Wigderson 1986) examines Turing machines with a bounded number of states. The main drawback of the automata approach is that cooperative behavior is usually achieved by "exhausting" the machine—designing a game pattern that is so complex that the machine has to use all its computational power to follow it. Such a pattern is highly non-robust and will collapse in the presence of noise.

Papadimitriou, in (Papadimitriou 1992), analyzes a 3-player variant of the PD game. This game is played in two stages: first, every player chooses a partner, then if two players choose each other, they play PD. Two sections of this paper deal with a similar variation on the repeated PD game, in which players have the possibility of opting out.

Several researchers (see (Nowak & Sigmund 1993; Binmore & Samuelson 1994) for examples and further bibliography) took Axelrod's lead and investigated evolutionary models of games. Most, if not all, of these works studied populations of deterministic or stochastic automata with a small number of states. The reason for limiting the class of computational models investigated was mainly pragmatic: the researchers had limited resources of computer space and time at their disposal. We hope that this paper might give a sounder motivation for the focus on "simple" or "fast" strategies. We claim that the same limitations that hold for researchers hold for any decision maker, and should be treated as an inherent aspect of the domain.

The task of finding papers on the Internet, as presented in the example above, can be seen as a distributed search problem. The power of cooperation in such problems has been studied in (Hogg & Hubermann 1993).

Other examples of domains for which work of the type presented here is relevant can be found in (Fikes *et al.* 1995) and in (Foner 1995). Foner's system explicitly relies on autonomous agents' cooperation. He describes a scenario in which agents post ads on a network, advertising their wish to buy or sell some item. Ads (or, in fact, the agents behind them) find partners by communicating with other agents, and sharing with them information on ad-location acquired in previous encounters. As Foner himself states:

> Such behavior is altruistic, in that any given agent has little incentive to remember prior ads, though if there is uniformity in the programming of each agent, the community as a whole will benefit, and so will each individual agent.

Game theory predicts that such behavior will not prevail. The results presented in this paper suggest that the computational incompetence of the agents can be utilized to make sure that it does.

Outline of the Paper

The section "Finite Time Repeated Prisoner's Dilemma" presents and examines the finite time repeated PD game. The main result of this section is in Theorem 4, which shows weak conditions for the existence of a cooperative equilibrium.

The following section introduces the possibility of *opting out*, parting from an unsuitable partner. In that section we mainly develop tools for dealing with opting out, and show conditions under which opting out is a rational choice.

In the section "Sub-Optimal Strategies" we use *competitive analysis* to show that, in a sense, opting out strengthens the cooperative players. Without it, a noncooperative player can force his opponent into noncooperative behavior. The possibility of opting out changes the balance of forces, allowing a cooperative player to force an opponent into cooperative behavior.

Finite Time Repeated Prisoner's Dilemma

In this section we deal with a game of PD that is repeated for a finite time (which we call the *FTPD* game). Previous work (Kalai 1990) focused on finite or infinite iterated PD (*IPD*). The basic idea is that two players play PD for N rounds. In each round, once both have made their move (effectively simultaneously), they get the payoffs defined by the PD game matrix. In our version of the game, the players play PD for a fixed amount of (discrete) time. At each tick of the clock, if both made a move, they get the PD payoff. However, if either player did not make his move yet, nothing happens (both players get 0). See Figure 2. Readers who are familiar with the game theoretic literature on PD, might notice a problem that this perturbation entails. If $H = 0$, it creates a new equilibrium point, thus technically eliminating the paradox (this is, in fact, why the payoff when both players wait is labeled H and not defined as 0). However, this is a technical problem, and can be overcome by technical means, for instance by setting $P = 0$ or $H = -\epsilon$. See (Mor 1995) for further discussion.

B

A		$\mathcal{D}$	$\mathcal{C}$	$\mathcal{W}$
	$\mathcal{D}$	P, P	T, S	0
	$\mathcal{C}$	S, T	R, R	0
	$\mathcal{W}$	0	0	H

$H \leq 0$

Figure 2: **FTPD Game Payoff Matrix**

Rules of the FTPD Game:

- 2 players play PD repeatedly for N clock ticks, N given as input to players.
- At each round, players can choose C (cooperate), or D (defect). If they choose neither, W (wait) is chosen for them by default.
- The payoff for each player is his total payoff over N rounds (clock ticks).

Theorem 1 *If all players are (unboundedly) rational, and $P > 0 > H$, the FTPD game is reduced to the standard IPD game.*

Proof. The W row (column) is dominated by the D row (column), and thus can be eliminated. □

We now proceed to define our notion of bounded rationality, and examine its influence on the game's outcome. Theorem 2 is presented to enable the reader to compare our approach to the more standard one of automata models.

Definition 1 *A* **Complexity Bounded (CB)** *player is one with the following bound on rationality: each "compare" action takes the player (at least) one clock tick.*[1]

Unless mentioned otherwise, we will assume this is the only bound on players' rationality, and that each "compare" requires exactly one clock tick.

Theorem 2 *The complexity bound on rationality is weaker than restricting players to Turing Machines. That is to say, any strategy realizable by a Turing Machine can be played by CB players.*

Proof. Assuming that every read/write a Turing Machine performs takes one clock tick, a Turing Machine is by definition complexity bounded. □

From now on we will deal only with complexity bounded players. Furthermore, we must limit either design time or memory to be finite (or enumerable).[2] Otherwise, our bound on rationality becomes void: a player can "write down" strategies for any N beforehand, and "jump" to the suitable one as soon as it receives N.

The main objective of this section is to show how the complexity bound on rationality leads to the possibility of cooperative behavior. To do this, we must first define the concepts of equilibrium and cooperativeness.

Definition 2 *A* **Nash equilibrium** *in an n-player game is a set of strategies, $\Sigma = \{\sigma^1, ..\sigma^n\}$, such that, given that for all i player i plays σ^i, no player j can get a higher payoff by playing a strategy other then σ^j.*

Definition 3 *A* **Cooperative Equilibrium** *is a pair of strategies in Nash equilibrium, such that, when played one against the other, they will result in a payoff of $R * N$ to both players.*

Definition 4 *A* **Cooperative Strategy** *is one that participates in a cooperative equilibrium.*

Theorem 3 *A cooperative player will Wait or Defect only if his opponent Defected or Waited at an earlier stage of the game.*

Proof. If the player is playing against its counterpart in the cooperative equilibrium and it Waits, its payoff is no more then $(N-1) * R$, in contradiction to the definition of cooperative equilibrium.

For any other player, as long as the opponent plays C, the cooperative player cannot distinguish it from its equilibrium counterpart. □

Theorem 4 is the main result of this section.

Theorem 4 *If $R > 0$, then there exists a cooperative equilibrium of the FTPD game.*

Proof. Consider the strategy GRIM:

[1] Formally, we have to say explicitly what we mean by a "compare". The key idea is that any computational process takes time for the player. Technically, we will say that a CB player can perform at most k binary XORs in one clock tick, and $k < \log_2 N$.

[2] Papadimitriou (Papadimitriou 1992) makes a distinction between design complexity and decision complexity. In our model, decision complexity forces a player to play W, while design complexity is not yet handled. The choice of strategy is made at design time; switching from S_a to S_b at decision time can be phrased as a strategy S_c: "play S_a, if... switch to S_b".

1. Begin by playing C, continue doing so as long as the opponent does.
2. If the opponent plays anything other then C, switch to playing D for the remainder of the game.[3]

Note that GRIM requires one "compare" per round, so having it played by both players results in N rounds played, and a payoff of $N * R$ for each player.

Assume that both players decide to play GRIM. We have to show that no player can gain by changing his strategy.

If player A plays D in round k, $k < N$, then player B plays D from that round on. A can gain from playing D if and only if he plays D in the Nth round only. In order to do so, he has to be able to count to N. Since N is given as input at the beginning of the game, A can't design a strategy that plays GRIM $N-1$ rounds and then plays D. He must compare some counter to $N-2$ before he defects, to make sure he gains by defection. Doing so, he causes B to switch to playing D. Therefore, his maximal payoff is:

$$(N-1) * R + (P - R)$$

A will change his strategy if and only if $P - 2R > 0$. We assumed that $R > P$. If $R > 0$, then $2R > R$, and A will not switch. Else, by assumption, $2R > T + S$, so A will switch only if $P > T + S$, and will not switch if $T - P > -S$. □

Finite Time Repeated Prisoner's Dilemma with Opting Out

In this section we study a variant of the FTPD game, which we call OPD, in which players have the option of Opting Out—initiating a change of opponent (see Figure 3). A similar idea appears in (Papadimitriou 1992). While in the previous section we only altered the nature of the players and the concept of iteration, here we change the rules of the game. This requires some justification.

The motivation for OPD is that it allows players greater flexibility in choosing strategies. Consider player A whose opponent plays GRIM. In IPD or FTPD, once A defects, the opponent will defect forever after. From this point on, the only rational strategy for A is also to defect until the end of the game. The possibility of opting out enables A to return to a cooperative equilibrium. Generally, the existence of "vengeful strategies" (like GRIM) is problematic in the standard game context. Other researchers (Gilboa & Samet 1989; Fortnow & Whang 1994) have dealt with these strategies by explicitly removing them from the set of strategies under consideration. Once opting out is introduced, this is no longer necessary.

[3] The strategy GRIM assumes a player can react to a Wait or Defect in the next round, i.e., without waiting himself. This can be done, for example, if the strategy is stated as an "if" statement: "If opponent played C, play C, else, play D." This strategy requires one "compare" per round.

		B			
		$\mathcal{D}$	$\mathcal{C}$	$\mathcal{O}$	$\mathcal{W}$
A	$\mathcal{D}$	P, P	T, S	Q	0
	$\mathcal{C}$	S, T	R, R	Q	0
	$\mathcal{O}$	Q	Q	$\hat{Q}$	0
	$\mathcal{W}$	0	0	0	H

Figure 3: **OPD Game Matrix**

The possibility of opting out also makes the game less vulnerable to noise, and provides fertile ground for studying learning in the PD game context. These issues are beyond the scope of the current paper; see (Mor 1995) for further discussion.

One last motivation for this line of work is sociological: "breaking up a relationship" is a common way of punishing defectors in repeated human interactions. Vanberg and Congleton, in (Vanberg & Congleton 1992), claim that opting out ("Exit" in their terminology) is the *moral* choice, and show that Opt-for-Tat is more successful than Tit-for-Tat in Axelrod-type tournaments (Axelrod 1984).

We start by comparing the OPD game to traditional approaches, namely to games played by rational players and to the IPD game.

Theorem 5 *If all players are (unboundedly) rational, and $P \geq Q \geq 0$ (and $\hat{Q} < 0$, $H < 0$), the OPD game is reduced to the standard IPD game.*

Proof. The W row (column) is dominated by the D row (column), and thus can be eliminated.

Playing O will result in a payoff of Q, $Q < P$, and a switch in partner. Since all players are rational, this is equivalent to remaining with the same partner. A player cannot get a higher payoff by playing O, hence the O row and column are eliminated. □

Theorem 6 *In the IPD game with opting out, if $P > Q > \hat{Q}$ and $H < 0$, then the only Nash equilibrium is $< D^N, D^N >$.*

Proof. The standard reasoning of backward induction works in this game; see (Mor 1995) for the full proof. □

From now on we will deal only with the OPD game. For simplicity's sake, we will assume $Q = 0$, and $\hat{Q} = H = -\epsilon$.

Let us define the full context of the game.

Rules of the OPD Game:

1. A population of $2K$ players is divided into pairs.
2. For every pair of players $< i, j >$, every clock tick, each player outputs an action α, $\alpha \in \{C, D, O\}$. If he does not output any of these, W is assigned by default.

3. If either outputs O, the pair is split and both get Q, regardless of the other player's action. Otherwise, if both play C or D, they get the PD payoff, and continue playing with one another. In any other case, both get 0 and remain paired.
4. Both players can observe their payoff for the previous round. We assume both do, and therefore ignore this monitoring action in our computations, i.e., assume this is done in 0 time.
5. Every t clock ticks, all unpaired players are randomly matched.
6. The payoff to a player is the sum of payoffs he gets over N clock ticks.

We can now make a qualitative statement: opting out can be a rational response to a defection. This is the intuition behind Theorem 7. In the next section we will attempt to quantify this claim.

Theorem 7 *The expected payoff when playing against a cooperative opponent is higher than when playing against an unknown one.*

Proof. (Sketch - See (Mor 1995) for a more detailed proof.) Whatever A's move is for the first round, his payoff against a cooperative opponent is at least as high as against an unknown one. If he plays anything other then C, his opponent becomes unknown, and the proof is complete. If he plays C, we continue by induction. □

Theorem 8 *If there is a positive probability of at least one of the other players being cooperative, and rematching is instantaneous, then a player in the OPD game has a positive expected gain from opting out whenever the opponent waits.*[4]

Proof. This theorem follows directly from Theorem 3 and Theorem 7. The full proof can be found in (Mor 1995). □

Sub-Optimal Strategies

In considering whether to opt out or not, player A has to assess his expected payoff against his current opponent B, the probability B will opt out given A's actions, and his expected payoff after opting out. Such calculations require extensive computational resources, and thus carry a high cost for a CB player. Furthermore, they require vast amounts of prior knowledge about the different players in the population. Although complete information is a standard assumption in many game theoretic paradigms, it is infeasible in most realistic applications to form a complete probabilistic description of the domain.

As an alternative to the optimizing approach, we examine satisfying strategies.[5] Instead of maximizing their expected payoff, satisfying players maximize their worst case payoff. The intuition behind this is, that if maximizing expected payoff is too expensive computationally, the next best thing to do is to ensure the highest possible "security level," protecting oneself best against the worst (max-min).

In order to evaluate satisfying strategies, we use the method of competitive analysis. Kaniel (Kaniel 1994) names Sleator and Tarjan (Sleator & Tarjan 1984) as the initiators of this approach. The idea is to use the ratio between the satisfying strategy's payoff and that of a maximizing strategy as a quantifier of the satisfying strategy's performance.

We begin by defining the concepts introduced above.

Definition 5 *The* **Security Level** *of a strategy S is the lowest payoff a player playing S might get. Formally, if Γ is the set of all possible populations of players, and $\mu(S)$ is the expected payoff of S then the security level $SL(S)$ is:*

$$\min_{\gamma\in\Gamma} \{\mu| \text{ the population is } \gamma\}$$

Definition 6

- *A* **Maximizing** *player is one that plays in a way that maximizes his expected payoff.*
- *A* **Satisfying** *player is one that plays in such a way that maximizes his security level.*

Definition 7 *Let A be a satisfying player, and let S be A's strategy. Let $h(S)$ be the expected payoff of A, had he been a maximizing player. The* **Competitive ratio** *of S is:*

$$CR(S) = \frac{SL(S)}{h(S)}.$$

The following two theorems attempt to justify examination of satisfying rather than maximizing strategies.

Theorem 9 *If there is a probability of at least $q > 0$ of being matched with a cooperative player at any stage of the game, then a player in the OPD game can ensure himself a security level of $N * R - const$.*

[4] The problematic condition is instantaneous rematching. There are 2 necessary and sufficient conditions for this to happen:

1. Opting out can be done in the same round the opponent waits.
2. If a player opted out in this round, he will be playing against a (possibly new) opponent in the next round, i.e., there is no "transition time" from one opponent to another.

The second condition is unjustifiable in any realistic setting. The first condition returns to a player's ability to "watch the opponent" without waiting, mentioned in Footnote 3. In (Mor 1995) we show different assumptions that make this condition possible.

[5] Herbert Simon (Simon 1969; 1983) coined the term "Satisficing" as an alternative to "Maximizing." Although our approach is close in spirit to his, it differs in its formalism. Therefore we prefer to use a slightly different term.

Proof. Consider the strategy Opt-for-Tat (OFT). This is the strategy of opting out whenever the opponent does not cooperate and cooperating otherwise. The expected number of times a player A playing OFT will opt out is $\frac{1}{q}$, after which he will be matched with a cooperative player, and receive R for each remaining round of the game.

Actually, it is easy to show that $const = \frac{1}{q} * [(r+1)R - S]$ where r is the expected number of rounds a player has to wait for a rematch (Mor 1995). □

Theorem 10 *If there is a probability of at least $q > 0$ of being matched with a cooperative player, and all players in the population are satisfying or optimizing players, a player in the OPD game cannot receive a payoff higher then $N * R + const$.*

Proof. Assume there exists a strategy θ that offers a player A playing it a payoff greater then $N * (R + \beta)$, $\beta > 0$. This means that in some rounds A defects and receives T. However, as soon as A defects, he identifies himself as a θ player. His opponent can infer that playing against A he will receive a payoff lower then the security level $N * R - const$, and will opt out. Let r be the expected number of rounds a player waits for a rematch. If $(r+1) * R < T$ then A loses every time he defects, and will get a payoff $< N * R$. If $(r+1) * R > T$ then "defect always" is the only equilibrium strategy, and the probability of being rematched with a cooperative player becomes 0 (in contradiction to our assumption). □

From Theorems 9 and 10 we get that as N and q grow, the competitive ratio of a satisfying strategy in this game approaches 1. If the time needed to compute the optimizing strategy is proportional to N, a satisfying strategy is *de facto* optimal for a CB player.

Related work revisited

The possibility of opting out makes cooperative, non-vengeful strategies even stronger. This tool can be further developed into a strategy-designing tool (Mor 1995). We wish to demonstrate this claim using the examples presented in the introduction of this paper.

In domains like the paper-searching agents or Foner's ad-agents, cooperative information sharing is a desirable, if not required, behavior. We propose the following guidelines for designers of such an environment:

- Ensure a large enough initial proportion of cooperative agents in the domain (e.g., by placing them there as part of the system). Make the existence of these agents known to the users of the system.
- Advise the users to use the following protocol in informational transactions:
  - Split the information being transferred to small packets.
  - Use an OFT strategy, i.e., keep on sending packets as long as the opponent does, break up and search for a new opponent as soon as a packet does not arrive in time.
- Inform the users of the satisfying properties of this strategy.

As shown in the section "Sub-Optimal Strategies," it is reasonable to assume that a large proportion of the users will implement cooperative strategies in their agents.

Conclusions

We introduced the *finite time repeated* PD game, and the notion of *complexity bounded* players. In doing so, we encapsulated both the player's utility and his inductive power into one parameter: his payoff in the game. Cooperative equilibria arise as an almost inherent characteristic of this model, as can be seen in Theorem 4. Furthermore, the common knowledge of limited computational power enables an agent to control his opponent's strategy. If the opponent spends too much time on computations, he is probably planning to defect.

In the sections that followed, we introduced and studied *opting out* in the PD game. We discussed both theoretical and intuitive motivations for this variation on the standard game description. In the section "Sub-Optimal Strategies," we used the tool of *competitive analysis* to show that the possibility of opting out makes cooperative, non-vengeful strategies even stronger. We then demonstrated the usefulness of these results with relation to the examples presented in the introduction.

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