# Avoid Fixed Pricing: Consume Less, Earn More, Make Clients Happy

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## ABSTRACT

Efficient management of resources in a society is a key ingredient of many multiagent systems. Self-interested agents (either human or automated) working to maximize their own benefit might make excessive use of a common resource, a situation known as the "tragedy of the commons". Therefore, game-theoretic considerations should come into play in the design of mechanisms that avoid such undesirable outcomes. In this paper, we consider two prototypical policies that are being used for the management of costly resources. In the first, consumers pay a fixed price and the provider covers the cost of the consumed resource; in the second, consumers pay according to the amount they use. It is clear that the first policy may prompt excessive and wasteful consumption. We analyze the incentives of the agents involved, assuming that all of them are self-interested and behaving strategically, and we prove that per-use pricing policy is better for the provider in the equilibrium outcome. We then show conditions under which consumers will *also* benefit from this policy on average, although some free-riders may still favor the wasteful fixed-price policy. Finally, we introduce a mechanism where consumers are allowed to choose their own policy, and show that it must converge to the efficient equilibrium where all consumers are paying according to their use.

## Categories and Subject Descriptors

J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Economics*

## General Terms

Theory, Economics

## Keywords

Mechanism design, Game theory, Public policy

## 1. INTRODUCTION

Consider a costly resource that can be managed in one of two ways, or policies. One policy is to charge consumers per use. Another policy is to charge some fixed cost, and allow consumers to use the amount they want. It is not hard to see that the latter form of resource management encourages excessive and wasteful usage. The problem is aggravated when consumption also has significant negative externalities on the society and environment, as is the case with many natural resources such as fossil fuels.

As a running example throughout this paper, we consider companies that are supplying their employees with a leased car. This benefit includes the vehicle and maintenance, and may or may not include pre-paid fuel (fuel that is paid for by the company). There are over 28 million private cars today in the UK alone, driving about 400 *billion* km each year.[1] Much of this vast amount of traffic is attributed to daily commuters, many of them driving leased cars owned by the company that employs them. The rapidly increasing number of cars on the roads exhausts global oil reserves, overloads existing infrastructure, and causes a plethora of environmental and economic problems.[2]

Given the large scale of the problem and the high price of fuel, we would expect people to use fuel economically, and decision-makers to assist them in doing so. Yet there are many companies that only offer their employees leasing deals with pre-paid fuel, thus providing them with a strong incentive to drive *more*.[3]

Situations where free access to a resource prompts excessive use are known as "the tragedy of the commons", and occur in numerous areas (see Section 1.2, below). What characterizes our case – and allows for some optimism – is that there is a way to make consumers face at least some of the externalities, by enforcing a policy in which they have to pay the full price of the resource. However, the policies are not designed and selected by some benevolent external authority. Rather, the decision of what prices to charge is in the hands of another self-interested party, which controls the resource. It is therefore necessary to study how the incentives under each policy shape the actions of all the involved agents, in order to improve resource management.

We model each of the pricing policies as a game between the provider and the consumers (formal definitions are given in Section 3.1). The policy where consumers are charged a constant sum is referred to as the *Fixed Policy* (FP). In the second policy (the *Use Policy*, or UP), agents are charged a smaller fixed sum for access to the resource, and have to cover the cost of the amount they consume. In the situations we depict, there is only one provider, which can prescribe the fixed price in each policy.

The preferences of the consumers depend of course on the cost in both cases, and on the utility they extract from using the resource, which may differ among agents. Therefore the key to the analysis

[1]Statistics are taken from the UK Department for Transport. Available from `http://tiny.cc/8logl`.

[2]These include air and ground pollution, accidents, and long-term atmospheric impact [6].

[3]Indeed, drivers of company-owned cars in the UK and elsewhere drive significantly more. In Israel, the annual mileage of a car with pre-paid fuel is $81\%$ higher on average [2].

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of such situations is the incentive of the resource provider in setting the prices. Companies provide leased cars and fuel to their employees to enable their commuting and to increase their satisfaction; governments supply water and other resources to their citizens; online service providers generate revenue from registered uses, and so on. The utility of the provider in such cases is therefore influenced both by the utility of the agents, and by the costs involved in supplying the resource.

It is important to note that negative externalities due to excessive consumption are not limited to car usage. In many computerized and online domains there are costly resources such as storage, CPU time and bandwidth. Overusing them may inflict increased congestion on computational or communication resources, but also cause concrete environmental effects [22]. Thus in some of the domains it is beneficial to the society to actively bias these systems in the direction of the Use Policy, to prevent excessive use. The purpose of this paper is to promote the Use Policy by showing that it is not only better for society, but also the rational choice.

## 1.1 Our contribution

We show that under some reasonable assumptions the Use Policy is better for the provider than the Fixed Policy, and then study the conditions under which consumers are expected to gain or lose (on average) from the Use Policy, in equilibrium.

We first demonstrate this in a game with a single consumer, starting with a simplified version where only the consumer acts strategically, and showing how a moderate bias towards the Fixed Policy (e.g. due to risk-aversion) can also be handled. In the remaining sections we show how our results extend to more realistic situations, where there are multiple consumers with different preferences.

The key assumption we make is that consumers gain some money-equivalent utility from using the resource. Moreover, this utility has *decreasing marginal value*, which is a standard assumption in economic situations. For example, driving 200 miles is better than driving 100 miles, but not twice as good, as some rides can be replaced with cheaper alternatives such as public transportation or carpooling. Alternatives for online services include local computing resources or optimizations.

While we want to bias agents towards the Use Policy in order to promote economized management of resources, we do not want to coerce their behavior. Some consumers may strongly oppose the Use Policy if they end up less satisfied, even if most agents are expected to gain. Our main result shows that in many cases merely allowing consumers to choose between the policies results in a process where *all* consumers eventually select the Use Policy, voluntarily.

We use the domain of car leasing arrangements in the text alongside our abstract model, as a concrete example. While we make some general assumptions on the behavior of involved parties, we do not assume any specific values for the parameters (such as the price of fuel or commuting distance, in the case of car leasing); our analysis can therefore be applied in various domains. Some of the proofs have been deferred to the appendix to allow continuous reading.

## 1.2 Related work

It is generally accepted that flat rate charging (corresponding to our Fixed Policy) encourages increased usage in various domains including telephony and Internet services [3, 18, 23], water consumption [20], and car usage [2].[4] However, whether this effect is considered negative for the society (as we assume) depends on the context.

The behavior of consumers in the Fixed Policy is a form of *free riding*, as they benefit at the expense of others (the environment, the provider and/or other consumers); a situation widely known in economics and game theory as the *tragedy of the commons*, popularized by Hardin [12]. In such scenarios a group of agents (the consumers), working to maximize their own utility, end up in an outcome that is worse for everyone—excess resource usage. Hardin suggests several possible solutions such as taxation and privatization, all of which are designed to make agents internalize the external effects of their behavior. Our Use Policy (in contrast to the Fixed Policy) is intended to do exactly the same. The problem we tackle is how to convince rational decision makers to take it on themselves to make the transition, preferably without coercion.

A standard approach to avoid free riding is to limit the options available to the agents, a view that is also taken by Hardin and followers. A famous example is the Braess paradox [8], which is a routing-related instantiation of the tragedy of the commons. Interestingly, we show how a policy shift can be promoted by allowing *additional options*, rather than by narrowing them. Other approaches rely on the emergence of norms without external interference [9] or on incentivizing agents to reveal their true preferences [11] (when applicable). Taxation is also often used to align the incentives of agents with those of society [13, 5].

Resource allocation in general has been widely studied in the literature, where externalities in the form of congestion are often explicitly modeled. Some research has shown how self-interested consumers may end up in suboptimal outcomes, albeit in models that are quite different from ours [21, 14]. When considered as a mechanism design problem, resource allocation can be optimally handled using the Vickrey-Clark-Groves (VCG) mechanism [10]. However, such a mechanism requires an external central authority, and also does not model the externalities on the environment (which is not a player in the game). Somewhat closer to the spirit of our paper, some resource allocation problems converge to the optimal solution when both providers and consumers are acting strategically [4].

A preliminary version of this paper that focused on leasing agreements was presented in the 6th Workshop on Agents in Traffic and Transportation [16].

# 2. PRELIMINARIES

A *game* consists of a set of agents $N$, a set of strategies for each agent $\{A_i\}_{i\in N}$, and a utility function for each agent $U_i : \times_{j\in N} A_j \rightarrow \mathbb{R}$. The set of strategies $A_i$ does not have to be discrete. For example, the strategy may be to decide on an amount of (continuous) money to spend. A joint selection of strategies for each agent $\mathbf{a} = \{a_j \in A_j\}_{j\in N}$ is called a *strategy profile*. The profile of all agents *except* $i$ is denoted by $a_{-i} = \{a_j \in A_j\}_{j\neq i}$.

**Equilibrium.** We say that the strategy profile $\mathbf{a}$ is an *equilibrium*, if no agent can gain by choosing a different strategy, assuming that all other agents keep theirs. Formally, $\mathbf{a}$ is an equilibrium if for any agent $i$ and any strategy $a'_i \neq a_i$, we have that $U_i(\mathbf{a}) \geq U_i(a_{-i}, a'_i)$. Our definition coincides with the standard definition of a *pure Nash equilibrium*, as we do not allow agents to randomize among strategies.

**Dominant strategies.** $a^*_i \in A_i$ is a *dominant strategy* of $i$ if agent $i$ always prefers $a^*_i$, regardless of the choices of other agents. Formally, for all $\mathbf{a}$, $U_i(a^*_i, a_{-i}) > U_i(\mathbf{a})$. Note that if some player has a unique dominant strategy, then all other players can assume that this strategy will be played. If we can continue to remove strategies

[4] There is a large debate over the extent to which this effect exists in health care services. See e.g. [15, 19].

from the game until there is only one strategy profile left, we say that the game is *iterated dominance solvable*. The outcome $\mathbf{a}^*$ is called the *iterated dominant strategy equilibrium*, and it is also the unique Nash equilibrium of the game.

For more background in game theory, see, for example, [17].

**Concavity.** A (continuously differentiable) function $h(y)$ is *concave* if its derivative $h'(y)$ is non-increasing (or, equivalently, if $h''(y) \leq 0$). Another definition of a concave (increasing) function is in terms of decreasing marginal value. That is, for all $y < z$ and $\epsilon > 0$, $h(y+\epsilon) - h(y) \geq h(z+\epsilon) - h(z)$. We say that $h$ is *strictly concave* if the inequality is strict. Similarly, a function $h(y)$ is *convex* if $-h(y)$ is concave.

# 3. THE SINGLE CONSUMER CASE

## 3.1 Initial Model

In the simplest case, we model the interaction between a single provider and a single consumer, where only the consumer acts strategically.

### *The Fixed Policy.*

The utility of the consumer, denoted by $u$, is composed of two factors. One factor is the amount of consumed resource, denoted by $x$; the other factor is the fixed cost of access to the resource, denoted by $s$. While the cost $s$ is prescribed by the producer, the consumer is free to choose how much of the resource to use; thus her strategy space is $\mathbb{R}_+$. The utility of the consumer in the Fixed Policy (FP) thus can be decomposed as $u(s,x) = f(x) - s$.

That is, there is some function $f$ that makes the two factors comparable. As explained in the introduction, we make the following assumption:

ASSUMPTION 1. *The consumer has decreasing marginal utility from increasing usage, and there is a maximal amount that the consumer has no reason to exceed. Formally:*

*(a) $f$ is non-decreasing and continuous.*

*(b) There is some $x^*$ s.t. $f$ has a maximum in $f(x^*)$.*

*(c) $f$ is strictly concave in the range $[0, x^*]$.*

To make the above concrete, we can think of an employee (the consumer) driving a leased car owned by the company (the provider) as part of her employment conditions. The resource $x$ represents the monthly mileage, or consumed fuel, whereas $s$ represents the sum deducted from the salary of the employee for the benefit of using a company-owned car. Other factors affecting the utility of the employee, such as her income or satisfaction from work, can be embedded as constant factors inside the function $f$.

Clearly, in FP the dominant strategy of the employee is to drive $x^*$, thus maximizing utility. This holds for any fixed cost $s$, and therefore the company has no influence on the strategy of the employee regarding her mileage.

We next consider the expense of the company (i.e., the provider), although for now we do not treat it as a player in the game. That is, we will not consider the rationality of the provider's actions. The provider gets a fixed amount $s$ from the consumer, but pays a variable amount $\alpha \cdot x$ to cover the cost of the used resource. In our example, $\alpha$ can be thought of as the fuel cost per kilometer. The total profit of the company from the deal is therefore $s - \alpha x$.

We can assume that the consumer follows her dominant strategy. Thus we get that in FP, the utility of the consumer is $u(s, x^*)$, while the provider earns $s - \alpha x^*$.

### *The Use Policy.*

We define a new game for the Use Policy (UP), with the same strategy spaces but different utility functions.

The utility of the consumer in UP can be similarly decomposed, to a fixed cost $r$, and a component that depends on the actual consumption:

$$v(r,x) = f(x) - r - \alpha \cdot x.$$

The best strategy for the consumer in UP depends on both $\alpha$ and $f$, and we make the following observations:

- $v(r,x)$ is concave in $x$, and has a peak at some $x' < x^*$.
- Regardless of $r$, the dominant strategy in UP is to consume $x'$.
- The utility of a consumer using the dominant strategy is $v(r,x') = f(x') - r - \alpha \cdot x'$.

In our leasing setting, this means that the employee will forgo trips whose marginal value per kilometer is lower than $\alpha$. The profit of the provider in UP is simply $r$.

If we assume that the employee follows her dominant strategy, we have that in UP, the company may reduce the fixed price of the leasing deal and still gain (compared to FP). This supplies us with a simple formalization of the intuition given earlier.[5]

PROPOSITION 1. *The Use Policy can be better for both provider and consumer. Formally, for all $r < s$ s.t. $f(x^*) - (f(x') - \alpha x') < s - r < \alpha x^*$, we have that $r > s - \alpha x^*$ (i.e., the provider earns more), and $v(r,x') > u(s,x^*)$ (i.e., the consumer earns more).*

PROOF. The provider side follows directly from the condition $s - t < \alpha x^*$. Also, as long as $s - r > f(x^*) - f(x') + \alpha x'$, the consumer benefits, as

$$\begin{aligned} v(r,x') - u(s,x^*) =& f(x') - r - \alpha \cdot x' - (f(x^*) - s) \\ =& (s-r) - (f(x^*) - f(x') + \alpha x') > 0. \end{aligned}$$

It is thus left to show that both constraints can be satisfied at the same time, i.e., that $\alpha x^* > f(x^*) - (f(x') - \alpha x')$. Indeed, recall that $x'$ is such that $f(x') - \alpha x' = \max_{x \geq 0}(f(x) - \alpha x)$. In particular, $f(x') - \alpha x' > f(x^*) - \alpha x^*$. $\square$

For example, the first employee in Example 3 prefers UP as long as $r < s + 40 - (38 - 5) = s + 7$.

### *Risk aversion.*

Consider a setting where the consumer is risk averse. That is, she prefers to have a fixed level of expenses over uncertain expenses, even if the expected variable cost is somewhat lower. Such bias would give an advantage to the Fixed Policy, where expenses do not depend on (variable) usage. Suppose we quantify this risk aversion in the term $\delta$, then the actual experienced utility in the Use policy is $v(r,x) = f(x) - \delta - r - \alpha \cdot x$. It is not hard to see that Proposition 1 still holds if $f(x^*) - f(x') < \alpha(x^* - x') - \delta$, i.e. if $f$ is sufficiently concave so as to overcome the bias caused by risk aversion.

How much concavity is required? Let $\mu = -\sup_{x' \leq x \leq x^*} f''(x) > 0$, and denote the risk aversion factor by $\delta$. We show (see Lemma 9 in the appendix) that if $\delta < \frac{1}{2}\mu(x^* - x')^2$, then $f(x^*) - f(x') < \alpha(x^* - x') - \delta$, and thus Prop. 1 still holds. In the remainder of this paper we will assume risk neutrality to simplify the presentation. However results still holds under risk aversion provided that all consumers' utility functions are sufficiently concave.

[5]In some cases providers may set a limit on the maximal consumption in FP to some $x^{**}$. This will not change our analysis as long as $x' < x^{**}$.

### 3.2 Weaknesses of the initial model

Our basic model provides some formal flavor for the intuition that win-win situations can be achieved simply by transferring the cost from the provider to the consumer, with very few additional assumptions. Unfortunately, such a simple analysis still suffers from several weaknesses.

First, although the consumer had a dominant strategy in both policies, we did not analyze the actions available to the provider from a strategic point of view. Therefore it is possible in principle that the new state is not an equilibrium.

Second, we only modeled a single consumer. Although the results hold if we add *identical* consumers, we have a problem when consumers have different utility functions. For example, if one employee lives closer to the train station, or likes to travel on weekends, then the utility of 100km for her might be lower than the utility for her colleague. In the following sections, we address these issues by refining our model.

### 3.3 The resource game

We first note that our original definition of the consumer's utility, which only considered the cost, ignored an important factor. Both sides must gain something from the interaction, where the utility of a company depends on its employees'/clients' satisfaction. Formally, we denote the gain function by $g(u)$, where $u$ is the utility of the employee.

ASSUMPTION 2. *$g(u)$ has a well-defined third derivative. Moreover, the provider has a decreasing marginal profit from the utility of the consumer. Formally:*

*(a) $g(u)$ is non-decreasing and continuous, i.e., $g'(u) \geq 0$.*

*(b) $g(u)$ is strictly concave, i.e., $g''(u) < 0$.*

While the first requirement is merely technical and aimed at simplifying the analysis, the decreasing marginal profit is a standard economic assumption. In the context of company-employee interaction, it can be interpreted as "happy employees work harder" (or "better conditions attract better employees"), but the benefit is diminishing. In the context of a service supplied by the government, this assumption means that the provider aims to make the consumer happier, but prefers to prevent misery over encouraging luxury. We do not search for the "correct" interpretation, as the implication is the same.

We can now formalize the utility of the provider, considering the utility of the consumer in FP and UP:

$$U(s,x) = g(u(s,x)) + s - \alpha x = g(f(x) - s) + s - \alpha x, \quad \text{and}$$
$$V(r,x) = g(v(r,x)) + r = g(f(x) - r - \alpha x) + r.$$

Recall that in either policy, the consumer has a dominant strategy (either $x^*$ or $x'$). Assuming that the consumer is indeed using her dominant strategy, we expect the provider to optimize the cost, so as to maximize its profit. Thus there is an optimal fixed cost $s^*$ that maximizes $U(s,x^*)$, and the strategy profile $(x^*,s^*)$ is the (unique) iterated dominant strategy equilibrium of FP.[6]

Similarly, there is an optimal fixed cost $r^*$ which maximizes the gain of the provider in the Use Policy. We denote the difference $s^* - r^*$ by $\Delta$.

[6]While this is really a 2-step game in extensive form, the consumer has a dominant strategy and will thus play the same in every path. We therefore use a simpler presentation as a normal-form game, which is equivalent in this case.

PROPOSITION 2. *The equilibrium profile $(r^*, x')$ in the Use Policy is preferred by the provider to the equilibrium $(s^*, x^*)$ in the Fixed Policy. The consumer is indifferent between the two. Formally, $v(r^*, x') = u(s^*, x^*)$, and $V(r^*, x') > U(s^*, x^*)$.*

PROOF SKETCH. For the first part, we show that $\Delta = f(x^*) - f(x') - \alpha x'$ by deriving $U(s, x^*)$ and $V(r, x')$ to get $s^*, r^*$. As for the provider,

$$\begin{aligned} V(r^*, x') &= g(v(r^*, x')) + r^* = g(u(s^*, x^*)) + (s^* - \Delta) \\ &= g(u(s^*, x^*)) + s^* + \alpha x' + f(x') - f(x^*) \\ &= U(s^*, x^*) + (f(x') - f(x^*)) - \alpha(x^* - x') > U(s^*, x^*) \end{aligned}$$

where the inequality follows since $f'(x') = \alpha$, and $f'(x) < \alpha$ for all $x > x'$. □

In other words, a single employee has no reason to prefer the pre-paid deal. Although she will end up driving less, she is only avoiding trips whose (marginal) benefit is below the real cost of the fuel. With the money saved (due to the reduced cost), she can now use cheap alternatives or enjoy activities she values more than driving.

It is important to emphasize that we did not include congestion and other externalities in the utility of the agents. Thus consumers should at least have a slight preference toward the Use Policy when they are otherwise indifferent.

## 4. MULTIPLE EMPLOYEES

Adding *identical* consumers to the game makes no difference, as the equilibrium described previously will satisfy all of them independently. Unfortunately (at least from an analytic point of view) different people do have different preferences, which are reflected in our model as a different function $f_i$ for each consumer $i \in N$.

In the general formulation of the multi-consumer problem, there are $n$ consumers, i.e. a total of $n + 1$ players.

### 4.1 Strategies and utilities

The strategy space of each consumer is her consumption $x_i$, as in Section 3.3. The strategy vector of all consumers is denoted by $\mathbf{x} = (x_1, \dots, x_n)$. The provider in the Fixed Policy controls a single cost parameter $s$, which affects all consumers. That is, the utility of each consumer is $u_i(s, \mathbf{x}) = f_i(x_i) - s$. The gain of the provider from the satisfaction of each consumer is the (concave) gain function $g$.

We define the *social welfare* in FP as the average of consumers' utilities (excluding the provider), that is,

$$SW_{FP}(s, \mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}(f_i(x_i) - s) = \left(\frac{1}{n}\sum_{i=1}^{n} f_i(x_i)\right) - s.$$

The dominant strategy of each consumer does not depend on $s$, nor on the actions of the other consumers. For example, we can continue to assume that employee $i$ drives $x_i^*$ kilometers in the Fixed Policy. The total utility of the provider from all interactions is given by

$$U(s, \mathbf{x}) = \sum_{i=1}^{n} \left(g(f_i(x_i) - s) + s - \alpha x_i\right).$$

We can similarly write the social welfare in UP:

$$SW_{UP}(r, \mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} v_i(r, \mathbf{x}) = \left(\frac{1}{n}\sum_{i=1}^{n} f_i(x_i) - \alpha x_i\right) - r,$$

and the utility of the provider in UP

$$V(r,\mathbf{x}) = \sum_{i=1}^{n} \left(g(f_i(x_i) - r - \alpha x_i) + r\right).$$

## 4.2 Who gains from the Use Policy?

In contrast to the single-consumer case, the UP might not be better for everyone when there are several consumers.

EXAMPLE 3. *Consider a company with two employees, and suppose that the fuel cost is $\alpha = 1$. For the first employee, $x_1^* = 10$; $f_1(x_1^*) = 40$; $x_1' = 5$; $f_1(x_1') = 38$. For the second employee $x_2^* = 20$; $f_2(x_2^*) = 50$; $x_2' = 11$; $f_2(x_2') = 45$. The gain function is $g(u) = 10\sqrt{u}$.*

*We get that in the Fixed Policy $s^* \approx 19.2$, the utilities of the employees are $u_1(s^*,\mathbf{x}^*) \approx 20.8$ and $u_2(s^*,\mathbf{x}^*) \approx 30.8$, and the profit of the company is $U(s^*,\mathbf{x}^*) \approx (45.6 + s^* - \alpha x_1^*) + (55.5 + s^* - \alpha x_2^*) \approx 109.5$.*

*In the Use Policy, the utilities of the employees from driving drop by $f_1(x_1^*) - f_1(x_1') + \alpha x_1' = 7$ and $f_2(x_2^*) - f_2(x_2') + \alpha x_2' = 16$. Therefore the fixed cost must drop by at least 16 (to $r < 3.2$) in order to keep both employees satisfied. Their new utilities will be 29.8 (see hollow point in Fig. 1(a)) and 30.8. Recomputing the utility of the company in the Use Policy, we get that $V(r,\mathbf{x}') < (54.6 + 3.2) + (55.5 + 3.2) = 106.4 < U(s^*,\mathbf{x}^*)$. That is, either at least one employee is less satisfied in the Use Policy, or the company loses money. The equilibrium outcome is shown in Fig. 1(b))*

As some consumers may gain while others are damaged in UP, we are interested in the effect on the average consumer, i.e., on the social welfare.

Before computing the equilibrium of the Use Policy, we offer a strategy for the provider. Recall that in Section 3.3 we showed that setting $r = s^* - \Delta$ is good for everyone. Let $\Delta_i = f_i(x_i^*) - f(x_i') + \alpha x_i'$, and $\bar{\Delta} = \frac{1}{n}\sum_{i\leq n} \Delta_i$, and assume that the provider charges $r = s^* - \bar{\Delta}$ from each consumer in the Use Policy. It can be shown that as in the first part of Prop. 2, the social welfare is the same in both policies:

$$SW_{UP}(s^* - \bar{\Delta},\mathbf{x}') = \sum_{i\in N} \frac{v_i(s^*,\mathbf{x}')}{n} + \Delta_i = \frac{1}{n}\sum_{i\leq n} v_i(s^* - \Delta_i,\mathbf{x}')$$
$$= \sum_{i\in N} \frac{u_i(s^*,\mathbf{x}^*)}{n} = SW_{FP}(s^*,\mathbf{x}^*). \qquad (1)$$

Unfortunately, even though the expenses of the provider are lower *and* the social welfare is higher (suggesting that consumers are happier on average) in UP, it is not guaranteed that the overall utility of the consumer increases. This is due to the non-linearity of the gain function $g$. Suppose that there is an embittered employee or customer whose satisfaction now deteriorates significantly and hurts the company. More generally, a small decrease in the utility of a single consumer may drag down the average gain. A closer look at this scenario reveals that not every consumer can have such a negative effect. The happier that consumers are (in FP), the smaller their effect on the change in the average gain (due to the concavity of $g$). If indeed the Use Policy is more profitable to those who are initially worse off, then the increase in social welfare will induce an increase in the average gain—and hence in the utility of the company. Moreover, it is quite reasonable to assume that in reality, the consumers who benefit the most from the Fixed Policy are indeed those who exploit it the most by excessive use. Thus these consumers will indeed benefit less than others from the Use Policy, as is the case in Example 3. We now formalize and prove this intuition.

ASSUMPTION 3. *Happier consumers in the Fixed Policy are still happier in UP, but by a smaller margin. Note that the assumption does not depend on the cost in either policy, as it affects all consumers in the same way. Formally, if for some $s$ it holds that $u_i(s,\mathbf{x}^*) \geq u_j(s,\mathbf{x}^*)$, then for any $r$ $v_i(r,\mathbf{x}') \geq v_j(r,\mathbf{x}')$, and $v_i(r,\mathbf{x}') - u_i(s,\mathbf{x}^*) \leq v_j(r,\mathbf{x}') - u_j(s,\mathbf{x}^*)$.*

Assumption 3 holds, for example, if the utility of each employee from using the car (ignoring the fixed cost) drops by some constant fraction when paying for her own fuel.

PROPOSITION 4. *When $r = s^* - \bar{\Delta}$, the provider strictly prefers the Use Policy, while the social welfare of the consumers is the same in both policies. Then*

$$SW_{UP}(s^* - \bar{\Delta},\mathbf{x}') = SW_{FP}(s^*,\mathbf{x}^*). \qquad (2)$$

$$V(s^* - \bar{\Delta},\mathbf{x}') > U(s^*,\mathbf{x}^*). \qquad (3)$$

The proof relies mainly on the following lemma, which is proved in the appendix.

LEMMA 5. $\sum_{i\leq n} g(v_i(s^* - \bar{\Delta},\mathbf{x}')) > \sum_{i\leq n} g(u_i(s^*,\mathbf{x}^*))$.

PROOF OF PROPOSITION 4. The consumer side, i.e. Eq. (2) follows directly from Eq. (1). As for Eq. (3),

$$V(s^* - \bar{\Delta},\mathbf{x}') = \sum_{i\in N} g(v_i(s^* - \bar{\Delta},\mathbf{x}')) + (s^* - \bar{\Delta})$$
$$> \left(\sum_{i\in N} g(u_i(s^*,\mathbf{x}^*)) + s^*\right) - n\bar{\Delta} \qquad \text{(from Lemma 5)}$$
$$= \sum_{i\in N} g(u_i(s^*,\mathbf{x}^*)) + s^* - \Delta_i \qquad (\Delta_i < \alpha x_i^*)$$
$$> \sum_{i\in N} g(u_i(s^*,\mathbf{x}^*)) + s^* - \alpha x_i^* = U(s^*,\mathbf{x}^*). \qquad \square$$

## 4.3 Equilibrium with multiple clients

Proposition 4 shows that the provider *can* set the fixed cost $r$ s.t. it strictly gains in UP without hurting social welfare. However, the provider might benefit more by setting a higher cost, thereby hurting social welfare.

Our first primary result (see appendix for proof) is to characterize a condition that specifies which agents benefit in the Use Policy.

THEOREM 6. *Let $(r^*,\mathbf{x}')$ be the Nash equilibrium of the Use Policy, under all the assumptions stated so far. Then*

*(a) The provider gains more in UP than in FP.*

*(b) If $g$ has a positive third derivative (i.e., $g'(u)$ is convex), then the social welfare is (weakly) lower in UP.*

*(c) If $g$ has a negative third derivative (i.e., $g'(u)$ is concave), then the social welfare is (weakly) higher in UP.*

The following example (illustrated in Figure 1) can provide some intuition for Theorem 6. We take Example 3, where $g(u) = 10\sqrt{u}$ (i.e., with convex derivative). Recall that in the Fixed Policy $s^* \approx 19.2$, $SW_{FP}(s^*,\mathbf{x}^*) \approx (20.8+30.8)/2 = 25.8$, and $U(s^*,\mathbf{x}^*) \approx 109.5$. In the Use Policy, the optimal fixed cost drops to $r^* \approx 8.5$. Then the social welfare slightly drops to $SW_{UP}(r^*,\mathbf{x}') \approx (24.5 + 25.5)/2 = 25$, and the utility of the company increases to $V(r^*,\mathbf{x}') \approx (49.5 + r^*) + (50.5 + r^*) \approx 116$.

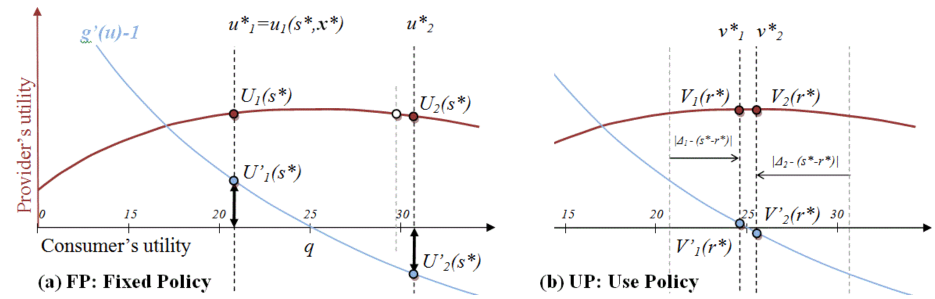


**Figure 1: The dark (red) curve represents the utility of the company from each interaction with an employee in FP (Fig. (a)), and the light curve is the derivative of the utility.[7] The optimal cost $s^*$ maximizes the sum of heights of the dark dots—thus the distance of the two light dots from $0$ on the Y axis is the same (vertical heavy arrows). On the X axis, employee 2 is further away from $q$, since $U'$ is convex. When changing the policy to UP (Fig. (b)), employees' utilities become much closer to one another (this is due to Assumption 3). We can now see the intuitive explanation for Theorem 6: the company gains since when dots are closer they are also higher on the dark utility curve. We can also see that the benefit of employee 1 from UP (left horizontal arrow) is smaller than the loss of employee 2. Therefore the social welfare is decreasing.**

On the other hand, since $g$ is an increasing function, its derivative is lower bounded by 0 (and decreasing). Thus $g'$ cannot be concave on its entire range. However, clearly the results still hold if consumers' utilities happen to be restricted to a range in which $g'$ is concave. In such a case, the social welfare is still guaranteed to increase.

## 5. ALLOWING FREE CHOICE

We have established that the Use Policy is better for the provider and for the society, and may sometimes benefit the average consumer as well. Still, some consumers may strongly object to UP, and a provider (e.g., a company) might be reluctant to enforce a policy change on its clients or employees, or cannot do so because of legal restrictions. However, the provider may still offer the two policies as options, allowing each client to choose. We will show that this simple mechanism initiates a gradual transition of *all* consumers to the Use Policy.

### *Game definition.*

We modify our game as follows. Each consumer, in addition to deciding on her consumption level, also chooses which of the two policies she prefers. Thus the strategy of each consumer $i$ is now $(x_i, t_i)$, where $t_i \in \{FP, UP\}$. The provider may set the fixed cost for each group separately, thus its strategy is a pair $(s, r)$. Each agent is aiming to maximize its own utility as defined in the previous sections.

Our second primary result shows that this game converges to the best possible outcome (from our perspective).

THEOREM 7. *Under the assumptions stated so far, the described game has a unique Nash equilibrium,[8] where all consumers select the Use Policy. Moreover, from any state of the game there is a sequence of "best replies" that leads to this equilibrium.*

In the remainder of this section we sketch the proof of the theorem (see appendix for the full proof). However we should first define exactly what we mean by "best reply".

[7] Note that there are no numbers on the Y axis. This is because in fact each consumer has an additional constant as part of $U_i(s^*)$. We present all consumers on the same dark curve only for the purpose of illustration. The derivative is the same for all consumers, and their height on the light curve is proportional to $U'_i(s^*)$.

[8] Up to changes that do not affect utilities.

### *Game dynamics.*

Given the current fixed costs $s$ and $t$, each consumer selects the better policy for her, assuming optimal consumption (either $x^*_i$ or $x'_i$). If utilities are equal, then UP is preferred (e.g., due to ecological awareness). For the provider, we assume the following dynamics, which reflect a simple decision-making process. At each step the provider considers each policy, and optimizes the fixed cost myopically, i.e., setting the cost that maximizes the average utility from an interaction with consumers in this group (ignoring how consumers will react to this change). If the policy currently has no consumers, then the average utility is meaningless. In that case we assume that the provider is looking just one step ahead, setting the cost based on the the reaction of consumers to the new cost.

We next provide a proof sketch that highlights the main elements of the analysis. The full proof appears in the appendix.

PROOF SKETCH OF THEOREM 7. We denote by $S \subseteq N$ the set of consumers that prefer FP over UP. The remaining consumers who prefer UP are denoted by $R = N \setminus S$. As in previous sections, $s$ is the cost to set $S$, and $r$ is the cost to set $R$.

Given some initial state, sort consumers according to decreasing utility $u_i(s, \mathbf{x}^*)$ (order is independent of $s$). It holds by Assumption 2 that $(v_i(r, \mathbf{x}'))_{i \in N}$ is also decreasing, whereas $(\Delta_i)_{i \in N}$ is increasing. Note that $i$ prefers group $R$ (i.e., UP) if and only if $\Delta_i \leq s - r$. Thus if consumers best-reply to the current costs $(s, r)$, there is $k \leq n + 1$ s.t. $R = \{1, \ldots, k-1\}$ and $S = \{k, \ldots, n\}$.

The provider now optimizes the costs of both sets (assume that both are nonempty). With optimal costs, consumers in either set are clustered around the point $q$ where $g'(q) - 1 = 0$ (the peak of the provider utility curve, see also Figure 1(a).). The utility of the most unsatisfied consumer in $S$ is $u_k(s^*, \mathbf{x}^*) \leq q$. In contrast, by moving to $R$, $k$ will be the most-satisfied consumer, and in particular will gain *at least* $q$. Informally, $k$ feels that she is subsidizing her colleagues in $S$, and prefers to pay for her own resource. We can iteratively continue until all consumers move to $R$.

It remains to handle the cases where either $S$ or $R$ are empty. Suppose first that $R$ is empty. Then the provider has a clear incentive to set the cost $r_\emptyset$ low enough, so that some consumers will choose to switch. To see this, note that if *all* consumers switch, the provider is guaranteed to strictly gain (by Prop. 4 and Theorem 6).

When $S = \emptyset$, we need to show that the provider cannot gain by setting a cost $s_\emptyset$ that will tempt some consumers to choose the

Fixed Policy. The proof shows (as expected) that setting such a low cost must mean that the utility of the provider will drop. □

# 6. DISCUSSION

We showed that transferring the cost of a resource from the central provider to the consumers has more benefits than "just" reducing congestion, helping the environment, etc. It can actually leave both sides richer and happier. This result holds under some realistic assumptions on the preferences of the involved parties.

The underlying idea that is responsible for this situation is *marginal benefit vs. marginal cost*. When a consumer does not pay for a resource, her marginal cost of additional use of that resource is 0, which gives her an incentive to do so even when the marginal benefit is negligible. On the other hand, using the resource does not really come for free—it does have a cost, which is externalized and incurred on the provider (and on the environment). The provider, in turn, imposes some of this cost on the consumers, "hidden" inside the fixed cost of the deal.

In cases where there are multiple consumers with different usage patterns, those who are currently free riding the system may object to change. Adopting a standard where consumers may choose their own policy prompts a gradual transition towards the Use policy. This is somewhat similar to the effect known as "health insurance death spiral", where light consumers decline public health insurance programs, due to their unwillingness to subsidize heavy service consumers. While these indirect subsidies arguably benefit the society in the domain of health care (and hence the term "death spiral"), in our case such subsidies clearly have a negative effect, and the converging behavior described in Section 5 is welcome.

### *Barriers that hinder the Use Policy.*

Our results suggest that theoretically providers always have an incentive to offer, and maybe even enforce, the Use Policy to consumers. However, at least in the domain of leasing arrangements (which we used as an example), this is not always the case in practice. Understanding the barriers that prevent decision makers from adopting the Use Policy is important if we want to promote better resource management. We provide several possible explanations for the adoption of the inefficient Fixed Policy. These explanations should be taken only as preliminary suggestions, as this question is not the focus of this paper.

The first reason is that taxation and regulatory policy (set by the government) can make the Fixed Policy more profitable for companies, thus effectively subsidizing resources that are paid for by providers. Examples include taxation of company-owned cars in the UK [25],[9] and the deregulation of the telecom market in some countries, which triggered a transition from flat-rate pricing to various usage based tariffs [23].

A second reason is that even if the social welfare increases, some consumers that are free-riding the system can complain and hinder the adoption of the Use Policy.

A third related issue is natural, but irrational, decision patterns (see [24]). Effects such as default-bias and loss-aversion could possibly account for the reluctance of consumers to adopt the Use Policy, whereas companies refrain from a policy change that is perceived as hurting consumers. When these effects are not too strong, they can be handled in a similar way to risk-aversion, as explained in Section 3.1. Moreover, people almost never object to having more options (see [7], p. 25). We therefore expect that our mechanism in Section 5 would be less prone to such biases, at least from the company side.

Lastly, the cost of many online resources such as bandwidth and computation power is not always linear in consumption. Certain cost functions (e.g., economies of scale) make it more beneficial for the producer to encourage increased consumption.

[9]It is interesting to note that the treasury in Israel has recently decided to revise its taxation policy for this very reason [1].

## 6.1 Conclusions and Future Work

This paper deals with the optimal behavior of *rational* players with no external intervention, in settings where pre-paid resources and services encourage excessive usage. Our paper demonstrates how economic theory supports a policy that eliminates externalities affecting providers, consumers, and the environment. It emphasizes the importance of increasing the availability of alternatives to fuel (and other resources), which reflect in the concavity of the consumer's utility function. This importance is further accentuated under conditions of risk aversion.

In this paper we only considered two extreme policies, where either the consumer or the provider pays the full cost of the resource. Of course, other pricing methods also exist in the market, and should be considered in future extensions of this work.

A key factor not treated in this work is *competition*. When there are multiple providers (e.g., several competing cloud services) the incentive structure changes. Future work should therefore combine standard models of competition (like Bertrand competition), with a focused analysis of fixed vs. per-use pricing. Finally, our work should be complemented by experimental studies on the effects of the suggested policy transition on real providers in order to validate our assumptions, and in particular to test the effectiveness of our proposed mechanism.

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## Appendix: Proofs

LEMMA 8. $\Delta = f(x^*) - f(x') - \alpha x'$.

PROOF. The optimal fixed cost $s^*$ is the cost maximizing $U(s, x^*) = g(f(x^*) - s) + s - \alpha x^*$. As $s$ appears in the argument of $g$ with a negative sign, $\frac{\partial U(s,x^*)}{\partial s} = \frac{\partial g(f(x^*)-s)}{\partial s} + 1 = -g'(f(x^*) - s) + 1$. We get that in the optimal cost, $g'(f(x^*) - s^*) = 1$. Note that $g'$ is monotone and therefore injective. By derivating $V(r, x')$ we similarly get that $g'(f(x') - r^* - \alpha x') = 1 = g'(f(x^*) - s^*)$, and thus $f(x') - r^* - \alpha x' = f(x^*) - s^*$. □

LEMMA 9. *Let $\mu = -\sup_{x' \leq x \leq x^*} f''(x)$, and denote the risk aversion factor by $\delta$. If $\delta < \frac{1}{2}\mu(x^* - x')^2$, then $f(x^*) - f(x') < \alpha(x^* - x') - \delta$.*

PROOF. For every $x \in [x', x^*]$, we can write

$$f'(x) = f'(x') + \int_{y=x'}^{x} f''(y)dy = \alpha + \int_{y=x'}^{x} f''(y)dy \leq \alpha + \int_{y=x'}^{x} -\mu dy = \alpha - \mu(x - x').$$

Similarly,

$$f(x^*) = f(x') + \int_{x=x'}^{x^*} f'(x)dx \leq f(x') + \int_{x=x'}^{x^*} (\alpha - \mu(x - x'))dx = f(x') + \alpha(x^* - x') - \mu \int_{x=x'}^{x^*} (x - x')dx.$$

Expanding the last term,

$$\begin{aligned}
\mu \int_{x=x'}^{x^*} (x - x')dx =& \mu \int_{x=x'}^{x^*} x\, dx - \mu x'(x^* - x') \\
&= \mu[x^2/2]_{x=x'}^{x^*} - \mu x'(x^* - x') = \mu \frac{1}{2}((x^*)^2 - (x')^2) - \mu x' x^* + (x')^2 \\
&= \mu \frac{1}{2}((x^*)^2 - (x')^2) - \mu x' x^* + (x')^2 = \frac{1}{2}\mu((x^*)^2 + (x')^2 - 2x'x^*) \\
&= \frac{1}{2}\mu(x^* - x')^2 > \delta. \qquad \text{(by our assumption)}
\end{aligned}$$

Thus

$$f(x^*) - f(x') \leq \alpha(x^* - x') - \mu \int_{x=x'}^{x^*} (x - x')dx < \alpha(x^* - x') - \delta,$$

as required. □

PROPOSITION 2. *The equilibrium profile $(r^*, x')$ in the Use Policy is preferred by the provider to the equilibrium $(s^*, x^*)$ in the Fixed Policy. The consumer is indifferent between the two. Formally,*

$$v(r^*, x') = u(s^*, x^*), \text{ and } V(r^*, x') > U(s^*, x^*).$$

PROOF. For the first part, by Lemma 8

$$v(r^*, x') = f(x') - r^* - \alpha x' = f(x^*) - s^* = u(s^*, x^*).$$

As for the provider,

$$\begin{aligned}
V(r^*, x') &= g(v(r^*, x')) + r^* = g(u(s^*, x^*)) + (s^* - \Delta) \\
&= g(u(s^*, x^*)) + s^* + \alpha x' + f(x') - f(x^*) \\
&= U(s^*, x^*) + (f(x') - f(x^*)) - \alpha(x^* - x') > U(s^*, x^*)
\end{aligned}$$

where the inequality follows since $f'(x') = \alpha$, and $f'(x) < \alpha$ for all $x > x'$. □

LEMMA 5. $\sum_{i \leq n} g(v_i(s^* - \bar{\Delta}, \mathbf{x}')) > \sum_{i \leq n} g(u_i(s^*, \mathbf{x}^*))$.

PROOF. Denote by $u_1^* \leq \cdots \leq u_n^*$ the utility of all employees in the FP, i.e. $u_i^* = u_i(s^*, \mathbf{x}^*)$. Note that $SW_{FP}(s^*, \mathbf{x}^*) = \sum_{i \in N} u_i^*$.

We similarly denote by $v_1', \ldots, v_n'$ the utility of all employees in the UP (where $v_i' = v_i(s^* - \bar{\Delta}, \mathbf{x}')$), and by $d_i = v_i' - u_i^*$ the gain of client $i$ from switching between the policies. W.l.o.g. $d_i \neq 0$, as we can ignore clients whose utility did not change (they do not affect the average). By Assumption 3, the order $v_1' \leq \cdots \leq v_n'$ is kept, and also $d_1 \geq \cdots \geq d_n$.

By Eq. (1), $\sum_{i \in N} d_i = SW_{UP} - SW_{FP} = 0$. As the sequence $(d_i)_{i \in N}$ is non-increasing, let $k$ be the highest index $i$ s.t. $d_i \geq 0$. Note that

$$d \equiv \sum_{i \leq k} d_i = -\sum_{i > k} d_i > 0. \tag{4}$$

Next, we define $z_i$ to be the slope of $g$ in the segment $[u_i^*, v_i']$, i.e.,

$$z_i = \frac{g(v_i') - g(u_i^*)}{d_i} = \frac{g(u_i^* + d_i) - g(u_i^*)}{d_i}.$$

Note that $g$ is increasing and thus $z_i > 0$ for all $i$. Moreover, since the segments are ordered and $g$ is concave, then $z_1 > \cdots > z_n$ (i.e., the slope is becoming moderate as $i$ increases).

Thus,

$$\begin{aligned}
&\sum_{i\in N} g(v_i') - \sum_{i\in N} g(u_i^*) = \sum_{i\in N} g(v_i') - \sum_{i\in N} g(u_i^*) \\
&= \sum_{i\in N} (g(v_i') - g(u_i^*)) = \sum_{i\in N} z_i \cdot d_i \\
&= \overbrace{\sum_{i\leq k} z_i \cdot d_i}^{\text{positive}} + \overbrace{\sum_{i>k} z_i \cdot d_i}^{\text{negative}} \\
&\geq \sum_{i\leq k} \min_{j\leq k}\{z_j\} \cdot d_i + \sum_{i>k} \max_{j>k}\{z_j\} \cdot d_i \\
&= z_k \sum_{i\leq k} d_i + z_{k+1} \sum_{i>k} d_i && ((z_j)_{j\in N} \text{ is decreasing}) \\
&= d \cdot (z_k - z_{k+1}) > 0 && (\text{by Eq. (4)})
\end{aligned}$$

□

THEOREM 6. *Let $(r^*, \mathbf{x}')$ be the Nash equilibrium of the Use Policy, under all the assumptions stated so far. Then*

*(a) The provider gains more in UP than in FP.*

*(b) If $g$ has a positive third derivative (i.e., $g'(u)$ is convex), then the social welfare is (weakly) lower in UP.*

*(c) If $g$ has a negative third derivative (i.e., $g'(u)$ is concave), then the social welfare is (weakly) higher in UP.*

PROOF. Part (a) follows immediately from Prop. 4, as the dominant strategy of the provider ($r^*$) is at least as good as $s^* - \bar{\Delta}$, and thus its profit can only grow.

We denote by $u_1^* \leq \cdots \leq u_n^*$ and $v_1^* \leq \cdots \leq v_n^*$ the utility of all consumers in the respective equilibria of FP and UP. We define the function $U_i(s)$ to be the utility of the provider from the interaction with an optimal consumer $i$ in FP, i.e., $U_i(s) = g(u_i(s, \mathbf{x}^*)) + s - \alpha x_i^*$. The function $V_i(s)$ is similarly defined w.r.t. the policy UP.

As the utility $u_i$ is composed of the cost $s$ (with a negative sign) and some constant $c_i$ (assuming consumer $i$ plays her optimal strategy), we have that $U_i'(s) = \frac{\partial g(c_i - s) + s}{\partial s} = -g'(u_i) + 1$. By a similar derivation, $V_i'(r) = -g(v_i) + 1$ (where $v_i$ is the optimal utility of an employee with fixed cost $r$ in UP). Since the cost $s^*$ is set to optimal, we have that $U(s^*, \mathbf{x}) = \sum_{i\in N} U_i(s^*)$ is at its maximum, i.e., $\sum_{i\in N} g'(u_i^*) = n$. Note that the cost $r^*$ in UP is also optimal (unlike the case in Lemma 5). Therefore $\sum_{i\in N} g'(v_i^*) = n$ as well.

Let $d_i = v_i^* - u_i^*$ be the gain of client $i$ from switching to UP. By Assumption 3, $d_1 \geq \cdots \geq d_n$. W.l.o.g., $d_i \neq 0$, as we can ignore clients whose utility did not change (they do not affect the average).

Next, we define $y_i$ to be the slope of $g'$ in the segment $[u_i^*, v_i^*]$, i.e.,

$$y_i = \frac{g'(v_i^*) - g'(u_i^*)}{d_i} = \frac{g'(u_i^* + d_i) - g'(u_i^*)}{d_i}.$$

Note that since $g$ is concave, $g'$ is decreasing (and convex) and thus $y_1 < \cdots < y_n < 0$ (i.e., the slope is becoming more moderate as $i$ increases). It holds that

$$\sum_{i\in N} y_i \cdot d_i = \sum_{i\in N} g'(v_i^*) - \sum_{i\in N} g'(u_i^*) = 0. \tag{5}$$

Assume, towards a contradiction, that the social welfare is higher in UP. Thus $0 < SW_{UP} - SW_{FP} = \sum_{i\in N} d_i$. As the sequence $(d_i)_{i\in N}$ is non-increasing, let $k$ by the highest index $i$ s.t. $d_i \geq 0$.

$$\begin{aligned}
&\sum_{i\in N} y_i d_i = \overbrace{\sum_{i\leq k} y_i d_i}^{\text{negative}} + \overbrace{\sum_{i>k} y_i d_i}^{\text{positive}} \leq \sum_{i\leq k} \min_{j\leq k}\{y_j\} d_i + \sum_{i>k} \max_{j>k}\{y_j\} d_i \\
&< y_1 \sum_{i\leq k} d_i + y_n \left(-\sum_{i\leq k} d_i\right) = \underbrace{\left(\sum_{i\leq k} d_i\right)}_{\text{positive}} \underbrace{(y_1 - y_n)}_{\text{negative}} < 0,
\end{aligned}$$

where the first inequality exists since $(y_j)_{j\in N}$ is increasing and the second since $-\sum_{i\le k} d_i < \sum_{i>k} d_i < 0$. This is a contradiction to Eq. (5), which proves part (b).

We omit the proof of part (c), which is similar. $\square$

THEOREM 7. *Under the assumptions stated so far, the described game has a unique Nash equilibrium,*[10] *where all employees select the Use Policy. Moreover, from any state of the game there is a sequence of "best replies" that leads to this equilibrium.*

PROOF. We denote by $S \subseteq N$ the set of employees that prefer UP over FP. The remaining agents who prefer FP are denoted by $R = N \setminus S$. It will be convenient to treat the producer as two separate agents (agent $S$ and agent $R$), each controlling the cost of one type of contract, aiming to maximize the producer's utility *per consumer* in its own scope. As in the previous sections, $s$ is the cost to set $S$, and $r$ is the cost to set $R$.

The utility of agent $S$ (when $S$ is not empty) is therefore $\bar{U}(s,S) = \frac{1}{|S|}\sum_{i\in S} U_i(s)$ when $|S| \ge 1$. Similarly, the utility of agent $R$ is $\bar{V}(r,R) = \frac{1}{|R|}\sum_{i\in R} V_i(r)$ when $|R| \ge 1$. In the cases where $R$ or $S$ are empty, the cost cannot be optimized locally. In these cases, the company is looking one step forward, considering the total utility of the producer.

Suppose that the current costs are $s$ and $r$. Sort consumers according to decreasing utility $u_i(s,\mathbf{x}^*)$ (this is independent of $s$). It holds by Assumption 2 that $(v_i(r,\mathbf{x}'))_{i\in N}$ is also decreasing, whereas $(\Delta_i)_{i\in N}$ in increasing. Note that $i$ prefers group $R$ (i.e., UP) iff $\Delta_i \le s - r$. Thus if consumers best-reply to the current costs $(s,r)$, there is $k \le n+1$ s.t. $R = \{1,\dots,k-1\}$ and $S = \{k,\dots,n\}$. For convenience, we rename these sets as $R_k$ and $S_k$.

The producer (represented by the two agents $S$ and $R$) now optimizes the costs of both sets. Let $s^*_k$ and $r^*_k$ be the optimal costs for $S_k$ and $R_k$, respectively (suppose that neither one is empty). Denote by $q$ the point (i.e., consumer's utility, see also Figure 1) for which $g'(q) = 1$. As $s^*_k$ is maximizing $\bar{U}(s,S_k)$, we have that $\sum_{i\in S_k} U'_i(s^*_k) = 0$. Thus for some consumers (including $k$), $-g'(u_i(s^*_k,\mathbf{x}^*)) + 1 = U'_i(s^*_k) \le 0$, i.e., $g'(u_k(s^*_k,\mathbf{x}^*)) \ge 1$. Since $g'(u)$ is decreasing, $u_k(s^*_k,\mathbf{x}^*) \le q$. On the other hand, the cost $r$ is also optimized to $r^*_k$, so that $\sum_{i\in R_k} V'_i(r^*_k) = 0$, and thus

$$v_k(r^*_k,\mathbf{x}') \ge v_{k-1}(r^*_k,\mathbf{x}') \ge q \ge u_k(s^*_k,\mathbf{x}^*).$$

In the step of the consumers, at least one consumer ($k$) is switching to the Use Policy, i.e., from $S$ to $R$. In particular, this means that $s^*_k - r^*_k \ge \Delta_k > s - r$, i.e., the gap between the costs strictly increased. Thus no consumers want to move from $R$ to $S$. The producer is replying by optimizing the costs according to the new groups, and so on. Since in every iteration the size of $R$ is strictly increasing, the game converges after at most $n$ iterations (where in each iteration all agents reply).

It remains to handle the cases where either $S$ or $R$ are empty. Recall that in these cases we assume that the producer is looking one step ahead, trying to maximize its total utility given the expected reaction of the consumers. Suppose first that $R$ is empty, i.e., all consumers are choosing FP. Then the producer has a clear incentive to set the cost $r_\emptyset$ low enough, so that some consumers will choose to switch. To see this, note that if *all* consumers switch, the producer is guaranteed to strictly gain (by Proposition 4 and Theorem 6). Once $r_\emptyset$ is set, the convergence continues as described above.

Next, suppose that $S$ is empty, and $R = N$. The cost of the set $R$ is already optimized to $r^*$, and agent $S$ (representing the producer) is required to set a cost for the Fixed Policy. Suppose that $S$ is setting the cost to $s_0$, where $s_0$ is such that a subset $S_0 \ne \emptyset$ is tempted to switch to FP. By the argument above, $s_0 < r^* + \min_{i\in S_0}\Delta_i$. We argue that $\bar{U}(s_0,S_0) < \bar{V}(r^*,S_0)$, i.e., that after the reaction of the consumers, the producer will strictly lose.

We denote by $v^*_i = v_i(r^*,\mathbf{x}') = f_i(x'_i) - \alpha x'_i - r^*$ the current utility of each consumer in $R = N$. Let $\delta = s_0 - r^* < \min_{i\in S_0}\Delta_i$. Recall that $\sum_{i\in R} g'(v^*_i) = 0$, as shown in the proof of Theorem 6. Since $g'(u)$ is decreasing and $v^*_1 \le \dots v^*_n$, then $(g'(v^*_i))_{i\in R}$ is a decreasing sequence. Thus if $k = \min\{i \in S_0\}$, then

$$\sum_{i\in S_0} g'(v^*_i) = \sum_{i\ge k} g'(v^*_i) \le 0. \tag{6}$$

[10] Up to changes that do not affect utilities.

Once the consumers of $S_0$ reply by switching to the Fixed Policy, the utility of the producer changes by

$$\begin{aligned}
&\bar{U}(r^* + \delta, S_0) - \bar{V}(r^*, S_0) = \sum_{i \in S_0} U_i(r^* + \delta) - V_i(r^*) \\
&= \sum_{i \in S_0} g(f_i(x_i^*) - r^* - \delta) + r^* + \delta - \alpha x_i^* \\
&\quad - (g(f_i(x_i') - \alpha x_i' - r^*) + r^*) \\
&= \sum_{i \in S_0} g(v_i^* + \Delta_i - \delta) - g(v_i^*) + \delta - \alpha x_i^* \\
&< \sum_{i \in S_0} g(v_i^* + \Delta_i - \delta) - g(v_i^*) && \text{(since } \delta < \Delta_i < \alpha x_i^*\text{)} \\
&\leq \sum_{i=k}^{n} g'(v_i^*)(\Delta_i - \delta) && (g \text{ is concave}) \\
&\leq \underbrace{\left(\sum_{i=k}^{n} g'(v_i^*)\right)}_{\text{non-positive by (6)}} \underbrace{\left(\frac{1}{n-k} \sum_{j=k}^{n} (\Delta_j - \delta)\right)}_{\text{positive}} < 0.
\end{aligned}$$

The penultimate inequality follows since $g'(v_i^*)$ is a decreasing sequence, and $(\Delta_i - \delta)$ is a positive increasing sequence. $\square$