ROBUST MARKET DESIGN:
INFORMATION & COMPUTATION

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The Field of Market Design

- Study of resource allocation with dispersed information by markets and auctions
- Remarkably successful applications, 2012 Nobel Prize
- Computer science involved in all aspects

This talk focuses on:

- Contributions of theoretical computer science to the theory of market design:
  - Relaxing assumptions
  - Tackling informational and computational challenges
- In the context of indivisible resources, prices, welfare/revenue maximization
# Major Challenges & Simplifying Assumptions

<table>
<thead>
<tr>
<th>Context</th>
<th>Fundamental challenge for market design</th>
<th>Mitigating assumption in classic market design theory</th>
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<tbody>
<tr>
<td>Revenue-maximization</td>
<td>Extracting revenue when values are private information</td>
<td>Seller has information on the distributions of values</td>
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<tr>
<td>Welfare-maximization</td>
<td>Achieving a welfare-maximizing allocation of indivisible resources</td>
<td>Values have structure, e.g., substitutes</td>
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# Classic Results Rely on Simplifying Assumptions

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<th>Context</th>
<th>Assumption</th>
<th>Classic result depending on assumption</th>
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<td>Revenue-maximization</td>
<td>Seller knows distributions</td>
<td>Characterization of optimal revenue [Mye’81]</td>
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<td>Welfare-maximization</td>
<td>Resources viewed as substitutes</td>
<td>Existence of welfare-maximizing equilibrium in markets with indivisible resources [KC’82]</td>
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Motivation for More Robust Market Design

• The **common knowledge** assumption is too stringent:

  *Contrary to much of current theory, the statistics of the data we observe shift very rapidly*

  [Google Research white paper]

• The **substitutes** assumption is too stringent:

  *Complements across license valuations exist and complicate the design process*

  [Byk’00 on spectrum licenses]

  *Substitutes valuations form a zero-measure subset of (even submodular) valuations*

  [Lehmann-Lehmann-Nisan’06]
Research Goal

Use computer science theory to understand when central results in market design theory hold (at least approximately) in a robust way, without stringent assumptions.

What is really driving classic positive results?
My Research Domains

Revenue-maximization and information:
- Matching markets
  - Interdependent and correlated buyers
  - Ad auctions

Welfare-maximization with complements:
- Feasibility constraints in double auctions
  - Walrasian equilibrium
- Bundling equilibrium
- Purely computational and informational aspects
Plan for Rest of Talk

1. Model

2. Robust revenue-maximization in matching markets [RTY’12]
   - Main result: “Vickrey with increased competition” approximately extracts the (unknown) optimal revenue (a multi-parameter “Bulow-Klemperer” result)
   - Computational tools: approximation, probabilistic analysis

3. Non-existence of welfare-maximizing market equilibrium [RT’15]
   - Main result: Computational complexity explanation for non-existence of equilibrium
   - Computational tools: reduction, LP, complexity hierarchy
MODEL
A Resource Allocation Problem

$m$ indivisible items, priced

$n$ buyers, where buyer $i$ has:
- Private valuation $v_i: 2^m \rightarrow \mathbb{R}_{\geq 0}$
- Demand $S \subseteq 2^m$
  - $S$ maximizes $i$’s quasi-linear utility: $v_i(S) - i$’s payment

Bayesian assumption for revenue:
- Values for item $j$ are i.i.d. draws from a regular distribution $F_j$

To solve, find allocation $(S_1, \ldots, S_n)$ and set prices
- Maximize welfare (sum of values $\sum v_i(S_i)$) or revenue (sum of payments)
Example: Unit-Demand Valuations (& Item Prices)

**Unit-demand valuations**

\[ v_i(S) = \max_{j \in S} v_i(j) \]

**Allocation—a matching**

Buyers

- \( v_3 = $10 \)
- \( v_3 = $2 \)

**Items**

- \( p = $3 \)
- \( p = $5 \)
- \( p = $6 \)

**Item prices**
Example: More Complex Valuations (& Prices)

General valuations

 Allocation (not a matching)

Personalized bundle prices

\[ v_3 = 15, \]
\[ v_3 = 4, \]
\[ ... \]

\[ p_3 = 9 \]
\[ p_3 = 6 \]
Example: Bayesian Assumption

Buying Items

\[ \nu_3 \sim F_2 \]
\[ \nu_4 \sim F_2 \]

"Regular" distributions
INFORMATIONAL CHALLENGES IN REVENUE MAXIMIZATION
Matching Settings and Revenue

- Unit-demand valuations, Bayesian assumption
- **1 item**: Myerson’s mechanism maximizes revenue (dominant-strategy) truthfully
  - Runs Vickrey (2\textsuperscript{nd} price) auction after using $F$ to fine-tune a reserve price
- **>1 items**: No such mechanism known
  - (Revenue-maximizing, dominant-strategy truthful [cf. Cai-Daskalakis-Weinberg’12])
Simple Robust Approach: Increasing Competition

• Design the market to determine pricing through competition

... then maximize welfare by running simple special case of VCG

Technical challenge:

• VCG is inherently robust; it’s left to quantify how much to increase competition such that VCG’s revenue is comparable to the ("unknown") optimal revenue
Main Results: Revenue

• Define:
  • $\text{OPT} = \text{Optimal revenue with known distributions subject to (dominant-strategy) truthfulness}$

**Theorem:** For $m$ items and $n$ buyers, assuming symmetry and regularity,
\[ \mathbb{E}[\text{revenue of VCG with } n + m \text{ buyers}] \geq \text{OPT} \]

• “Multi-parameter Bulow-Klemperer theorem” [cf. BK’96]
  • $m$ more buyers is tight in worst-case

**Theorem:** For $m$ items and $n$ buyers, assuming symmetry and regularity,
\[ \mathbb{E}[\text{revenue of VCG with supply limit } n/2] \geq \alpha \cdot \text{OPT} \]

• $\alpha$ is at least $\frac{1}{4}$ when $m \leq n$
Approximation Guarantee

Approximation ratio = \( \min_{F_1, \ldots, F_m} \left\{ \frac{\mathbb{E}_{F_1, \ldots, F_m} \left[ \text{revenue of VCG}^+ \right]}{\mathbb{E}_{F_1, \ldots, F_m} \left[ \text{revenue of OPT}_{F_1, \ldots, F_m} \right]} \right\} \)

Where

- \( F_1, \ldots, F_m \) = Arbitrary regular distributions
- \( \text{VCG}^+ \) = Vickrey with increased supply, no knowledge of \( F_1, \ldots, F_m \)
- \( \text{OPT}_{F_1, \ldots, F_m} \) = “Unknown” optimal mechanism for \( F_1, \ldots, F_m \)

\( \mathbb{E}[\text{revenue of } \text{VCG} \text{ with } n + m \text{ buyers}] \geq \text{OPT} \)
\( \mathbb{E}[\text{revenue of } \text{VCG} \text{ with supply limit } n/2] \geq \alpha \cdot \text{OPT} \)
Related Work

- **Information-robustness as a goal**
  - Scarf’58, Wilson’87, Bertsimas-Thiele’14, …

- **Prior-independent mechanism design**
  - Bulow-Klemperer’96, Segal’03, Bergemann-Morris’05, Dhangwatnotai-et-al.’11, Devanur’11, Chawla-et-al.’13, Bandi-Bertsimas’14, …

- **Simple versus optimal mechanisms**
  - Hartline-Roughgarden’09, Chawla-et-al.’10, Hart-Nisan’12, Babioff-et-al.’15, …
How to Limit Supply

• Limiting supply of homogeneous items is a standard marketing method
• How to limit supply of heterogeneous items?

• Instead, let the market decide

• Definition: \textit{VCG with supply limit }$\ell$
  • \textbf{Allocation}: Welfare-maximizing matching limited to $\ell$ items
  • \textbf{Pricing}: As usual ("externalities")
Proof Idea

1. VCG with supply limit approximates the revenue of VCG with added buyers
   - Idea: Limiting the supply creates the same supply-demand ratio as adding buyers

2. To show that VCG with added buyers is comparable to OPT:

<table>
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<th>VCG(^+) revenue from item (j)</th>
<th>OPT revenue from item (j)</th>
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<td>At least what the (n) unmatched buyers would pay for (j)</td>
<td>At most what (n) buyers would pay for (j) [Chawla-et-al.'10]</td>
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- The challenge: Showing that the unmatched buyers lead to high prices for sold items
- Use “principle of deferred decision” and stability of matching
Summary: Revenue

Generalizations: VCG with increased competition also works well for

- Items with multiple copies
- Asymmetric buyers
- Interdependent buyers and single item

Take-aways:

- Competition replaces knowledge-intensive pricing in complex settings
- (a) Participation and (b) setting quantities are 1st order design decisions when designing for revenue; there is a formal connection between them
- Relaxing knowledge assumption leads to simple mechanisms
COMPUTATIONAL CHALLENGES IN WELFARE MAXIMIZATION
A Fundamental Question in Economics

• Which classes of markets are guaranteed to have a market equilibrium?
  • [Milgrom’00, Gul-Stacchetti’99]: Substitutes are an almost necessary & sufficient condition for existence
  • [Sun-Yang’06] show existence for markets with complements
  • [Teytelboym’13, Ben-Zwi’13, Sun-Yang’14, Candogan’14, Candogan-Pekec’14, Candogan’15], ...

• Is there a systematic way to study such questions?
Computational Approach

**Computational complexity** is useful for studying **economic equilibrium existence in a systematic way**

- Computation is clearly relevant for **finding** an equilibrium
- This work: also relevant for equilibrium **existence**

**Non-existence** of equilibria follows from **complexity** of related computational problems

- Assuming $P \neq NP$, i.e., a hierarchy of computational problems
Theorem Statement

A necessary condition for guaranteed existence of a Walrasian equilibrium in markets with valuations in class $\mathcal{V}$:

Utility-maximization for $\mathcal{V}$ given item prices is not computationally easier than welfare-maximization for $\mathcal{V}$

Contrapositive: If, assuming $P \neq NP$, utility-maximization is easier than welfare-maximization for $\mathcal{V}$, then there is a market with valuations in $\mathcal{V}$ and no Walrasian equilibrium
Walrasian Equilibrium (WE)

An allocation $S_1, \ldots, S_n$ and item pricing $p$ such that, given $p$,
1. every consumer $i$’s bundle $S_i$ maximizes $i$’s utility;
2. the market clears

• Remarkable properties:
  • Pricing is succinct, anonymous
  • Pricing coordinates stable allocation among selfish players
  • First welfare theorem: Allocation maximizes welfare
  • Guaranteed to exist for markets with valuations in $\mathcal{V}$ where $\mathcal{V}$ is the class of GS valuations
Classes of Valuations

Unit-demand: \( v_i(S) = \max_{j \in S} v_i(j) \)

Additive: \( v_i(S) = \sum_{j \in S} v_i(j) \)

Gross substitutes [algorithmic definition]:
- Assume item prices
- Consider buyer’s problem of utility-maximization (demand)
- \( v_i \) is GS \( \iff \) greedily selecting items by marginal utility is optimal

Substitutes are a computational issue!
Applying the Theorem: Example

- Let $\mathcal{V}$ be the class of capped-additive valuations
  - $\nu(S) = \min\{c, \text{sum of values of items in } S\}$
  - Well-studied (e.g. in context of online ad markets)
- Demand (utility maximization) problem:
Applying the Theorem: Example

• Let \( \mathcal{V} \) be the class of capped-additive valuations
  
  \( \nu(S) = \min\{c, \text{sum of values of items in } S\} \)

• Welfare maximization problem (symmetric version):
Applying the Theorem: Example

- If $P \neq NP$, welfare maximization is generally harder
  - For polynomially-bounded item values and caps:

- Corollary:
  - There exists a market with capped-additive valuations and no WE

- Other applications in paper
Generalizations

Generalizations:

• Robust non-existence of a WE
  • If \((1 + \epsilon)\)-approximation of welfare is harder than demand

• Pricing equilibria with anonymous pricing
• Pricing equilibria with succinct pricing
Beyond Walrasian Equilibrium

General markets

Markets with WE

GS valuations

Sun-Yang

Unit-demand

(Not to scale…)

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Summary: Welfare

• Complements are a computational issue
• Computational complexity can be used to study equilibrium existence with complements, in a systematic way
• 2 computational problems are naturally associated with economic markets
• Methodology: Equilibrium existence means that utility-maximization is as hard as welfare-maximization
• This method is used in 2 ways:
  1. To easily prove generic non-existence of equilibria, using results from the extensive complexity literature
  2. To shed light on the elusiveness of interesting market equilibria beyond Walrasian
CONCLUSION
Summary

• Major results of market design assume away informational and combinatorial/computational challenges

• Concepts from computer science are useful in addressing these challenges, to get a robust theory that is more applicable in complex settings

• Robust simple approaches give qualitative understanding of revenue and welfare in complex settings

• In some areas (interdependence, complements), we’ve only scratched the surface…
Future Directions

• What other assumptions that are a standard part of our current models should we try to free ourselves from?

• As economic transactions become increasingly computer-driven, how else can we harness computer science to design better markets? And to better understand their limitations?
THANK YOU!