Contract Theory: A New Frontier for AGT

Part II: Modern Approaches

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Overview

- Part I (Inbal): Classic Theory
  - Model
  - Optimal Contracts
  - Key Results
- Break (5-10 minutes)
- Part II (Paul): Modern Approaches
  - Robustness
  - Approximation
  - Computational Complexity

1. Robustness

Motivation

The classic principal-agent model [Holmström 1979, Grossmann and Hart 1983] suggests optimal contracts that

- Are rather complex and intransparent
- Exhibit undesirable properties (e.g., non-monotonicity)
- Do not resemble contracts used in practice (which tend to be simple, often linear)

Linear contract: $t(r) = \alpha \cdot r, \alpha \in [0,1]$

Milgrom-Holmström [1987]

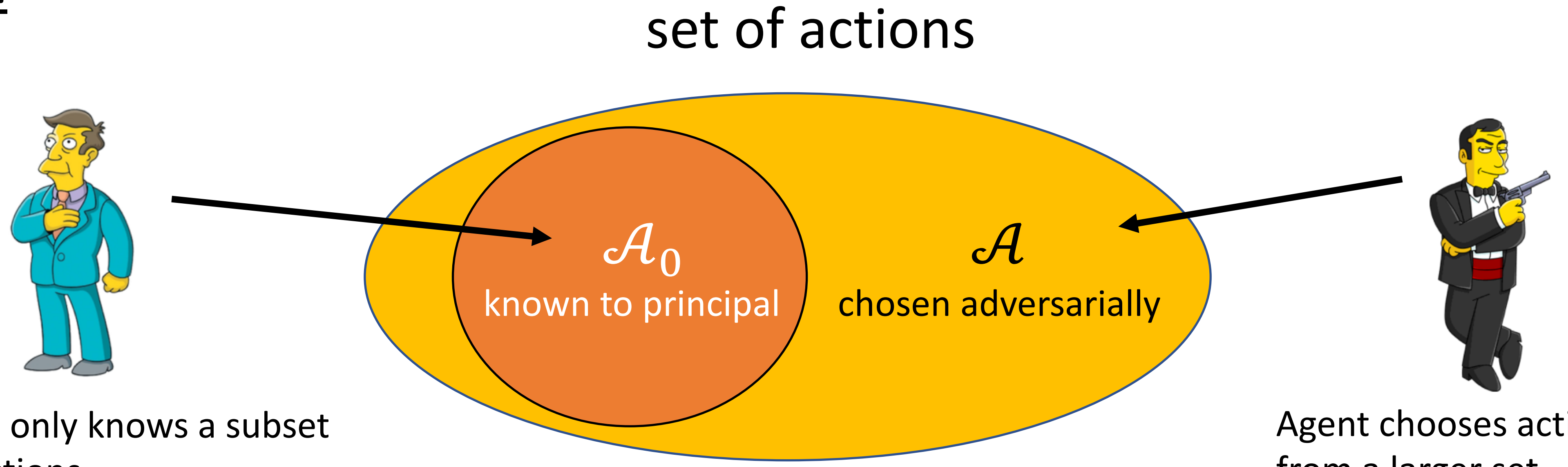
"It is probably the great robustness of linear rules based on aggregates that accounts for their popularity.

That point is not made as effectively as we would like by our model; we suspect that it cannot be made effectively in any traditional Bayesian model."

Carroll's Model [2015]

Recall: Action a_i is specified by distribution $F_{i,j}$ over rewards r_j, and a cost c_i

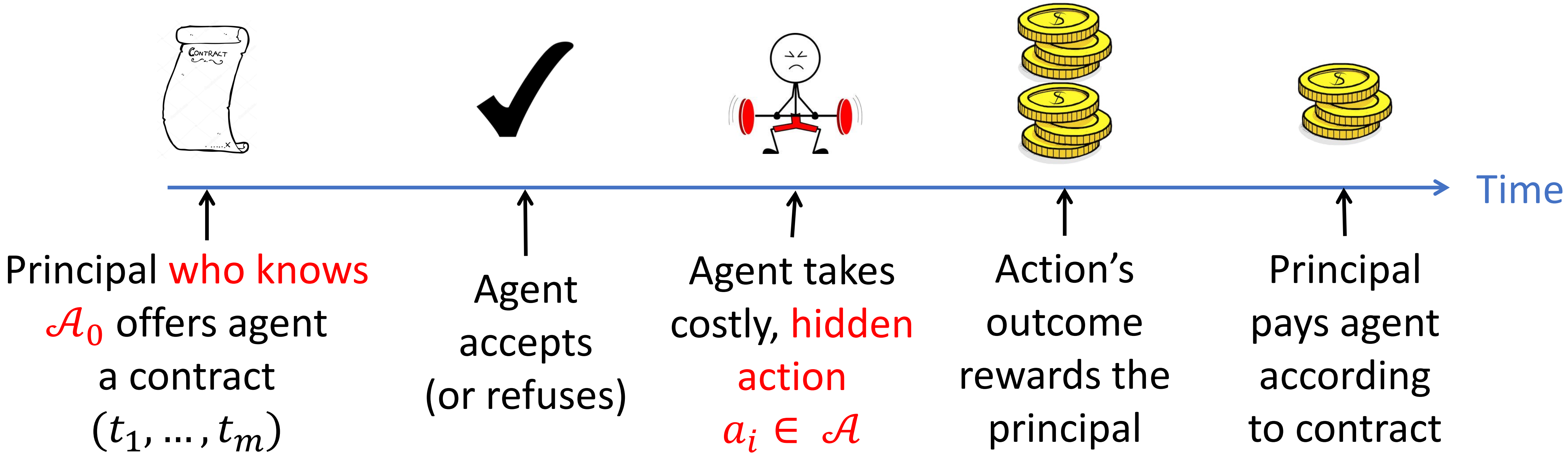
Twist:



Principal only knows a subset of the actions

Agent chooses action from a larger set

Timing


CONTRACT
Time
Principal who knows $\mathcal{A}_0$ offers agent a contract $(t_1, \ldots, t_m)$
Agent accepts (or refuses)
Agent takes costly, hidden action $a_i \in \mathcal{A}$
Action's outcome rewards the principal
Principal pays agent according to contract

The Agent's Perspective

- The agent chooses action a^* from $\mathcal{A}$ that maximizes expected payment minus cost

$$a^* \in argmax_{a=(F,c)\in\mathcal{A}}(\mathbb{E}_{r\sim F}[t(r)] - c)$$

$\Rightarrow$ agent utility $V_A(t|\mathcal{A})$

- <u>Note</u>: The agent can guarantee himself a certain expected utility by only maximizing over $\mathcal{A}_0$

"reserve agent utility" $V_A(t|\mathcal{A}_0)$

The Principal's Perspective

- Denote the set of actions that maximize the agent's utility for a given contract t and set of actions $\mathcal{A}$ by

$$A^*(t|\mathcal{A}) = argmax_{a=(F,c)\in\mathcal{A}}(\mathbb{E}_{x\sim F}\,[t(r)] - c)$$

- Then the principal solves the following max-min problem

$$sup_t\ inf_{\mathcal{A}\supseteq\mathcal{A}_0}\ max_{a=(F,c)\in A^*(t|\mathcal{A})}\mathbb{E}_{r\sim F}\,[r - t(r)]$$

V_P

principal payoff $V_P(t|\mathcal{A})$

Reserve Principal Payoff?

- With a linear contract $t(r) = \alpha \cdot r$, for any action $a = (F, c)$:

welfare pie

$$\mathbb{E}_{r\sim F}[t(r)] = \alpha \cdot \mathbb{E}_{r\sim F}[r]$$

$$\mathbb{E}_{r\sim F}[r - t(r)] = (1 - a) \cdot \mathbb{E}_{r\sim F}[r]$$

- So for every linear contract $t(r) = \alpha \cdot r$ and incentivized action a $= (F, c)$:

$$V_P \geq \frac{1-\alpha}{\alpha} \cdot \mathbb{E}_{r\sim F}[t(r)] \geq \frac{1-\alpha}{\alpha} \cdot (\mathbb{E}_{r\sim F}[t(r)] - c)$$

$$\Rightarrow V_P \geq \frac{1-\alpha}{\alpha} \cdot V_A(t|\mathcal{A}_0)$$

Maximizing the RHS gives max-min optimal contract

Max-Min Robustness

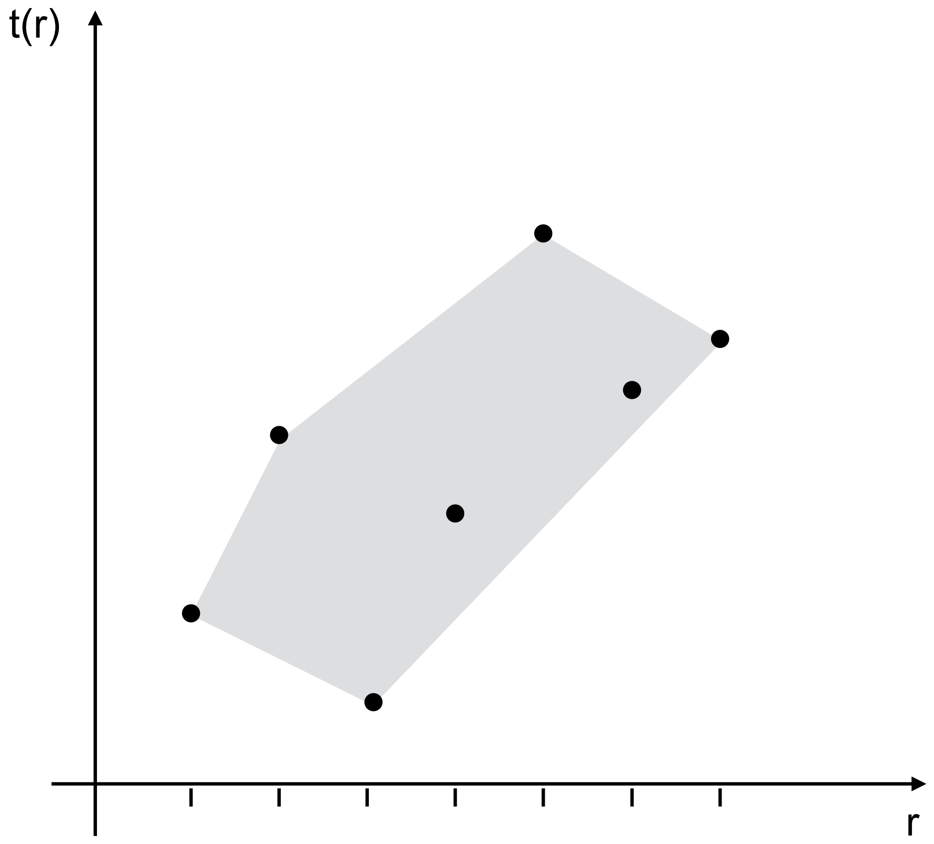
Theorem [Carroll'15]

For all partially specified principal agent-settings with rewards $r_1, \ldots, r_m$ and known action set $\mathcal{A}_0$ there exists a linear contract that maximizes V_P.

Key Steps in Proof

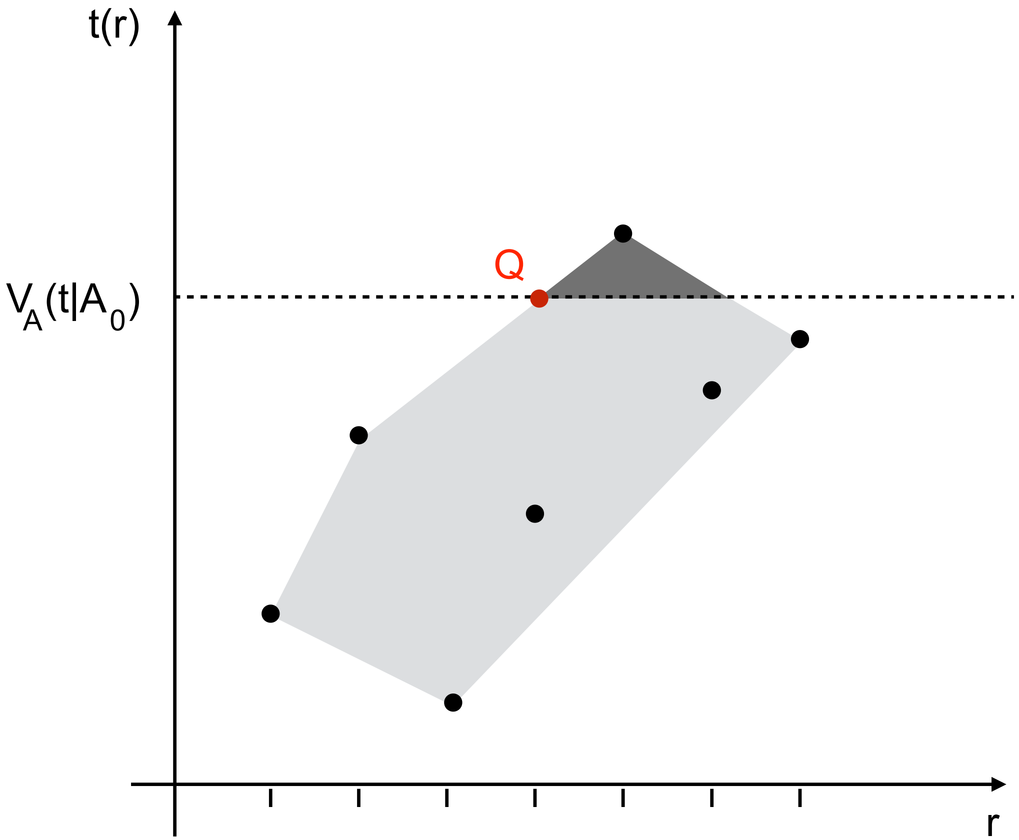
1. Argue that for any (not necessarily monotone) contract t there is an affine contract t' with the same or better worst-case guarantee (see next few slides)
2. Show that for any such affine contract t' there is an even better linear contract t''
   (see Carroll's paper for details)

Why Affine is Enough



- Fix an arbitrary contract t (black dots)
- For any action $a = (F, c)$ the agent may take, consider the point $(\mathbb{E}_F[r], \mathbb{E}_F[t(r)])$
- This point lies in the convex hull of $\left\{\left(r_j, t\left(r_j\right)\right) : 1 \leq j \leq m\right\}$ (gray area)

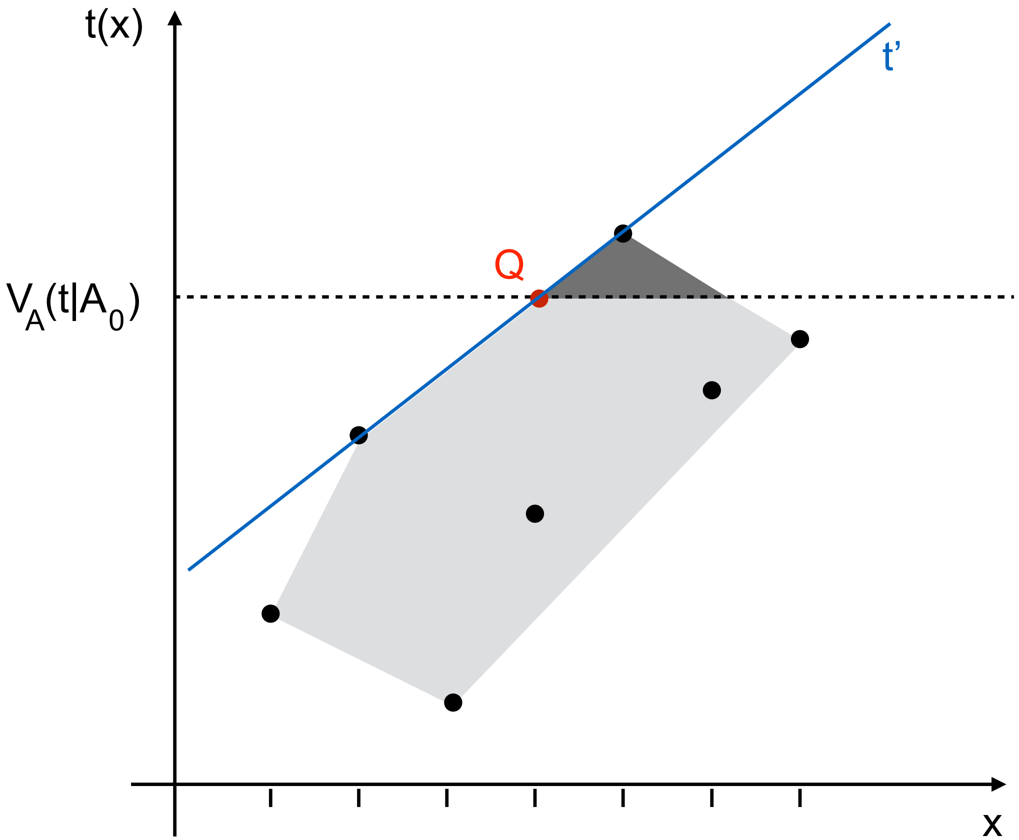
Why Affine is Enough



- Moreover, the agent will only take actions that give him payoff at least $V_A(t|\mathcal{A}_0)$ (dark gray area)
- Point Q is the point where expected payoff to the principal $\mathbb{E}[r - t(r)]$ is smallest

(bottom left of dark gray area)

Why Affine is Enough



- Support line t' to the convex hull at Q is an affine contract, whose worst-case payoff to the principal is no worse than that of contract t

Discussion

- Obviously: Not the only way in which one can formalize model uncertainty
- Standard approach in computer science in cases where input is stochastic:
  - Assume details of the distributions are unknown
  - But first moments (or first few moments) are known

[E.g., Scarf'58, …, Azar-Daskalakis-Micali-Weinberg'13, Bandi-Bertsimas'14]

New Notion of Robustness

In an EC'19 paper (with Tim Roughgarden) we explore contract design with moment information:

- Fixed set of outcomes $r_1, \ldots, r_m$
- There are n actions with costs $c_1, \ldots, c_n$
- Details of the distributions $F_1, \ldots, F_n$ are unknown
- But their expected rewards $R_i = \mathbb{E}_{r \sim F_i}[r]$ for $i = 1, \ldots, n$ are known ("compatible distributions")

New Notion of Robustness

Theorem [Dütting, Roughgarden, Talgam-Cohen’19a]

For every contract setting with known expected rewards, a linear contract maximizes the principal’s expected payoff in the worst-case over compatible distributions.

So: Carroll’s same conclusion, but under a very different hypothesis!

(Come to the EC talk!)

Open Questions

- Is there a unification of Carroll's and our result?
- Study other models of uncertainty (e.g., distributions over outcomes are only known approximately [Bergemann-Schlag'11, Cai-Daskalakis '17, Dütting-Kesselheim'19])

More Generally

A rapidly growing area in economics and computer science:

- Contracts [Carroll’15, Dütting-Roughgarden-Talgam-Cohen’19a]
- Revenue maximizing auctions [Bergemann-Schlag’11, Azar-Daskalakis-Micali-Weinberg’13, Bandi-Bertsimas’14, Carroll’17, Cai-Daskalakis’17, Carrasco-et-al.’18, Gravin-Lu’18, Bei-Gravin-Lu-Tang’19]
- Posted pricing and prophet inequalities [Dütting-Kesselheim’19]

2. Approximation

A Powerful Tool from AGT

- Given a simple microeconomic mechanism, bound the worst-case performance loss relative to the optimal mechanism

- For a maximization problem: Find largest $\beta \in [0,1]$ such that for all instances

$$ALG(I) \geq \beta \cdot OPT(I)$$

Performance of simple mechanism on instance

Optimal performance on instance

Example: Linear Contracts

	$r_1 = 1$	$r_2 = 3$	
Action 1	$F_{1,1} = 1$	$F_{1,2} = 0$	$c_1 = 0$
Action 2	$F_{2,1} = 0$	$F_{2,2} = 1$	$c_2 = 4/3$

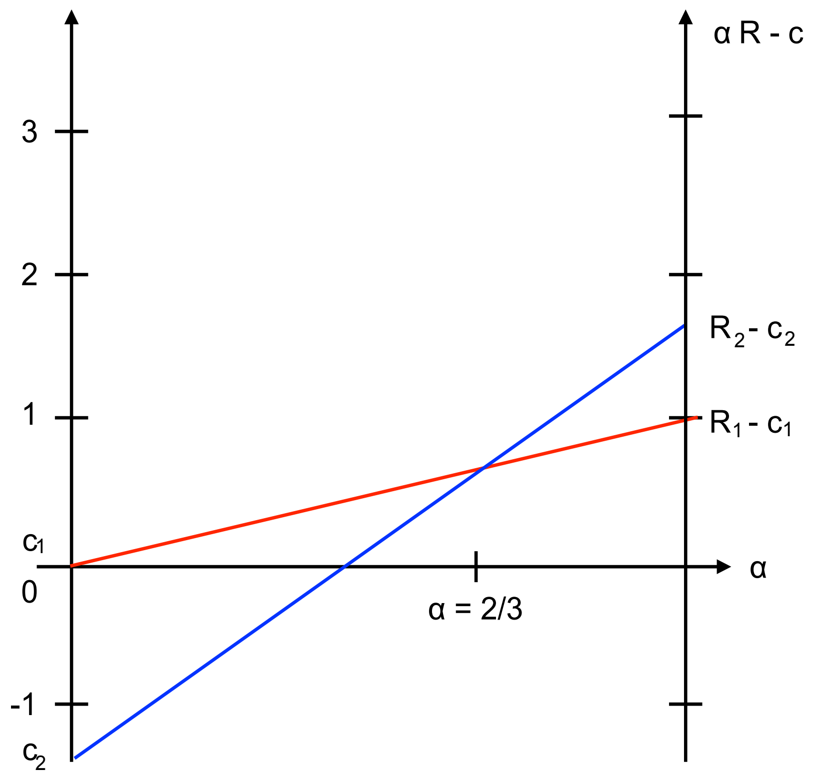
To find the optimal contract:

- The best way to incentivize action a_1 is to pay $t = (0,0)$ for an expected payoff of 1
- The best way to incentivize action a_2 is to pay $t = (0,4/3)$ for an expected payoff of $3 - 4/3 = 5/3$

$$t_2 = \frac{c_2}{F_{2,2} - F_{1,2}}$$

$$\Longrightarrow OPT = 5/3$$

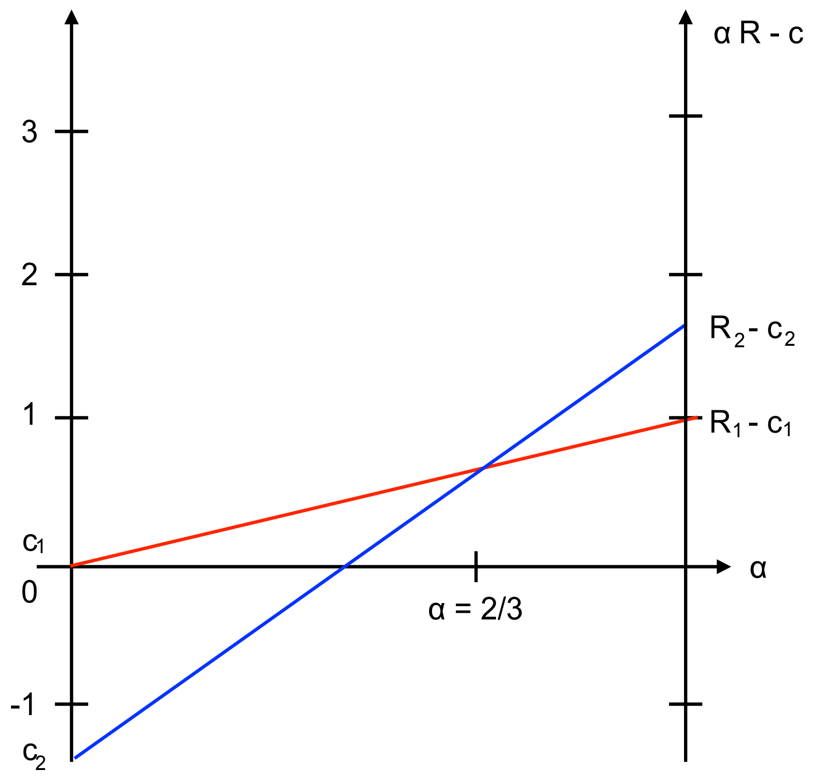
Example: Linear Contracts



To find the best linear contract:

- Draw upper envelope with α on x-axis and $\alpha R - c$ on y-axis
- Each action corresponds to a line
- For every given α, highest line corresponds to best (= chosen) action

Example: Linear Contracts



- Here smallest α at which action 1 and action 2 are implemented is $\alpha = 0$ and $\alpha = 2/3$

$\Rightarrow ALG = 1 < 5/3$

(Note: This shows that β can be at most $3/5$)

Approximation Result

Theorem (informal): [Dütting, Roughgarden, Talgam-Cohen'19a]

Linear contracts achieve good approximation except in pathological settings with simultaneously:

- many actions;
- big spread among actions of expected rewards;
- big spread among actions of costs

Example of a Pathological Setting

Let $\epsilon \to 0$

$$(R_1, R_2, R_3, \dots) = (1, \frac{1}{\epsilon}, \frac{1}{\epsilon^2}, \dots)$$

$$(c_1, c_2, c_3, \dots) = (0, \frac{1}{\epsilon} - 2 + \epsilon, \frac{1}{\epsilon^2} - 3 + 2\epsilon, \dots)$$

Formally

Theorem [Dütting, Roughgarden, Talgam-Cohen'19a]

ρ = worst-case ratio of optimal contract and best linear contract

- with n actions, $\rho = n$;
- with ratio R of highest to lowest R_i, $\rho = \Theta(\log R)$;
- with ratio C of highest to lowest c_i, $\rho = \Theta(\log C)$

- Upper bound w.r.t. to first best, lower bound w.r.t. optimal contract
- Lower bounds apply even under MLRP
- Bounds are tight, even for best monotone contract!

Open Questions

- We only scratched the surface!
- The general question is: For which classes of contracts and under which assumptions on the setting can we get good (constant factor) approximations?
- Cf. ”simple vs. optimal mechanisms” literature [Hartline and Roughgarden’09,…]

3. Computational Complexity

Motivation

- If everything is given explicitly and there is only one agent then not interesting computationally
- If there is more than one agent or if some part of the input is given implicitly things become interesting:
  - E.g. an action could consist of several binary decisions
  - E.g. outcomes could be subsets of a ground set
  - E.g. ...

Prior Work

- A paper which was way ahead of its time:

  Combinatorial Agency paper of Babaioff-Feldman-Nisan [2006, 2012] (and follow-up work)
- Studies a setting with multiple agents, in which each agent can take a binary action

New Approach

In ongoing work (with Tim Roughgarden) we consider the following succinct single-agent model:

- There are μ items, $m = 2^{\mu}$ possible outcomes
- Given action a_i, each item k is included in the outcome independently wp $F_{i,k}$
- The principal's reward is the sum of rewards r_k for each item k included in the outcome

Example from Part I

Additive

	No visitor $r_1 = 0$	General visitor $r_2 = 3$	Targeted visitor $r_3 = 7$	Both visitors $r_4 = 10$
Low effort $c_1 = 0$	0.72	0.18	0.08	0.02
Medium effort $c_2 = 1$	0.12	0.48	0.08	0.32
High effort $c_3 = 2$	0	0.4	0	0.6

Product

E.g. $\Pr[general \mid a_3] = 1, \Pr[targeted \mid a_3] = 0.6$

Goal

Use succinct structure to exponentially speed-up finding the optimal contract in comparison to the naïve LP-based method

Recall: Naïve LP-based Approach

- Based on solving n instances of the "MIN-PAY" problem
- Given action a_i, find optimal contract that implements a_i

$$\text{minimize} \sum_j F_{i,j} t_j$$

$$\text{s.t.} \sum_j F_{i,j} t_j - c_i \geq \sum_j F_{i',j} t_j - c_{i'} \quad \forall i' \neq i \quad \text{(IC)}$$

There are polynomial in m, exponential in μ many variables, but only n constraints – Ellipsoid to the rescue?

The Dual

$$\text{maximize} \sum_{i' \neq i} \lambda_{i'} (c_i - c_{i'})$$

$$\text{s.t.} \sum_{i' \neq i} \lambda_{i'} - 1 \leq \boxed{\frac{\sum_{i' \neq i} \lambda_{i'} F_{i',j}}{F_{i,j}}} \quad \forall j \in [m]$$

A separation oracle boils down to finding an item subset with minimum likelihood in the combination distribution $\sum_{i' \neq i} \lambda_{i'} F_{i'}$ relative to F_i

Computational Hardness

- Solving the separation oracle exactly is NP-hard
- In fact computing the optimal expected payoff in succinct contract settings in time polynomial in μ turns out to be NP-hard

Approximate IC

A solution from AGT: Relax the IC constraints!

Definition: Given a contract t, action a_i is δ-IC if

$$(1+\delta)\sum_j F_{i,j}t_j - c_i \geq \sum_j F_{i',j}t_j - c_{i'} \quad \forall i' \neq i$$

In normalized settings, the agent loses $\leq \delta$ by choosing a δ-IC action

[By δ-IC contract we mean a contract t and δ-IC action a_i that pleases the principal]

Theorem

Let OPT be the expected payoff of the optimal (IC) contract.

Theorem [Dütting, Roughgarden, Talgam-Cohen’19b]

There is an Ellipsoid-based algorithm that given a succinct contract setting with μ items and a parameter $\delta > 0$, returns a δ-IC contract with expected payoff $\geq \mathrm{OPT}$ in time polynomial in μ and $1/\delta$.

(Recall: Running time of naïve method is exponential in μ)

Ellipsoid-Based Algorithm

- Strengthened dual:

$$\text{maximize} \sum_{i' \neq i} \lambda_{i'} (c_i - c_{i'})$$

$$\text{s.t. } (1+\delta) \left(\sum_{i' \neq i} \lambda_{i'} - 1 \right) \leq \frac{\sum_{i' \neq i} \lambda_{i'} F_{i',j}}{F_{i,j}} \quad \forall j \in [m]$$

- Run Ellipsoid calling an FPTAS for the separation oracle
- FPTAS runs in time polynomial in μ and $\frac{1}{\delta}$, and exponential in n

Additional Results

In the paper [Dütting, Roughgarden, Talgam-Cohen’19b] we also show:

- Hardness of approximation for exactly IC contracts
- Constant factor δ-IC contracts
- ….

(Watch out for the paper!)

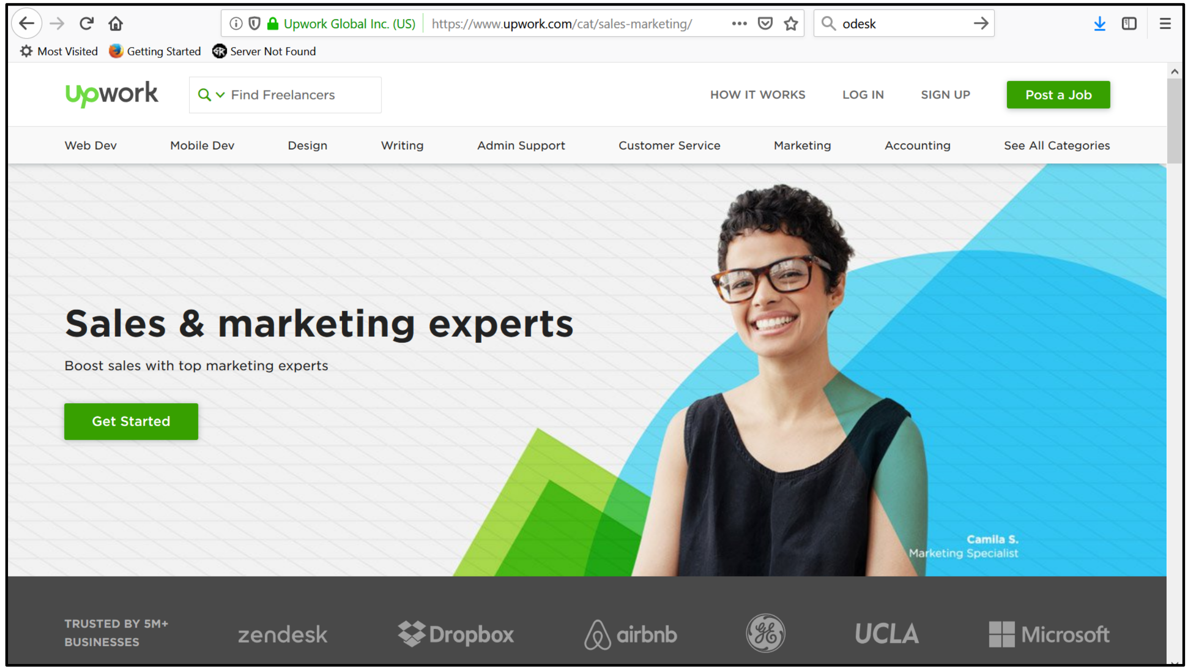
Open Questions

- Many interesting computational questions
- Approximation probably even more natural than in the mechanism design world
- Mostly unexplored …!

4. Concluding Remarks

Important Applications

- Freelancing and crowdsourcing platforms
- Start-up funding platforms
- Blockchain and smart contracts
- Venture capital contracts
- Government procurement
- …



Growing Momentum

- Combinatorial agency [Babaioff-Feldman-Nisan'12,…]
- Contract complexity [Babaioff and Winter'14,…]
- Incentivizing exploration [Frazier-Kempe-Kleinberg-Kleinberg'14,…]
- Robustness [Carroll'15,…]
- Adaptive design [Ho-Slivkins-Vaughan'16,…]

- Delegated search [Kleinberg and Kleinberg'18,…]
- Information acquisition [Azar and Micali'18,…]
- Robustness [Dütting-Roughgarden-Talgam-Cohen'19a,…]
- Succinct models [Dütting-Roughgarden-Talgam-Cohen'19b,…]
- VCG contracts [Lavi-Shamash'19,…]
- Strategic classification [Kleinberg-Raghavan'19,…]

(At this year's EC)

Many Open Problems

- There are lost of interesting open questions even in the most basic/classic models!
- The algorithmic perspective could be a powerful tool to complement the classic econ approach

Tutorial website:
http://personal.lse.ac.uk/act/index.htm

Thanks! Questions?

References

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