Analyzing games with ambiguous player types using the MINTHENMAX decision model
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Cooperation game with ambiguous player types

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>Preferences: {A, B, C, D} \succ \phi and each player has a preference (his type) over the locations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>A</td>
<td>\phi</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>\phi</td>
<td>C</td>
</tr>
<tr>
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<td>\phi</td>
<td>D</td>
</tr>
</tbody>
</table>

Hence, in an equilibrium \(L\),

- All players choose the same location set.
- Each location is chosen either by everyone or by no one.

Omitting locations: If \(L' \subseteq L\), then \(L'\) is also an equilibrium.

Increasing ambiguity: If for all players \(T' \subseteq T\), then \(L\) is also an equilibrium for the game defined by the type sets \(T'\).

Gale's MinMax principle
Gale, A. 1950. Statistical decision functions.

Wald's MinMax principle

The maximin value of the scenario:

\[\max_{\alpha\in\Delta(T)} \min_{\omega\in\Omega} \alpha(\omega)\]

\[\min_{\alpha\in\Delta(T)} \max_{\omega\in\Omega} \alpha(\omega)\]

Equilibrium: MIN-NE & MIN-NE

\[\text{MIN-NE in mixed strategies always exists.}\]

\[\text{(for general games with ambiguity)}\]

\[\text{Finding a MIN-NE is in PPAD.}\]

\[\text{(It is a PPAD-complete problem.)}\]

\[\text{There are games for which there is no MINTHENMAX-NE.}\]

MINTHENMAX-NE of coordination games

Lemma. For a set of locations \(L\), there is a MINTHENMAX-NE profile \(a\) s.t.

\[\{t \mid \exists! t \exists t \in T^t \ a(t) = t\} = L\]

if and only if

\[\forall i \text{ the mapping } f^i : T^i \rightarrow L, \text{ that maps a type to his best location in } L, \text{ is onto.}\]

Moreover, this action profile is the unique pure MINTHENMAX-NE that satisfies

\[\{t \mid \exists! t \exists t \in T^t \ a(t) = t\} = L\]

Hence, in an equilibrium \(L\),

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Omitting locations: If \(L' \subseteq L\), then \(L'\) is also an equilibrium.

Increasing ambiguity: If for all players \(T' \subseteq T\), then \(L\) is also an equilibrium for the game defined by the type sets \(T'\).