Timing Acquisition of Wideband PPM Systems over Multipath

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***Abstract*— The acquisition, or synchronization, of a multipath channel profile for ultra-wideband pulse position modulation (PPM) communication systems is considered. The rate of increase of the number of paths as the bandwidth grows determines whether acquisition can occur. If the number of independent Gaussian paths increases without bound, but slower than the bandwidth, then the system cannot acquire in the limit. Acquisition is not possible on multipath channels with deterministic path amplitudes, if the number of paths diverges but not too fast. These results hold for exponential or uniform power delay profiles.**

I. Introduction

Ultra wideband (UWB) impulsive systems have gained recent interest for communications, as well as channel imaging and positioning. While significant diversity is achievable given the large amount of multipath in UWB channels, methods for harnessing such diversity when the channel is unknown remain a challenge. In fact, the fundamental limits of UWB signaling in the presence of channel uncertainty have not been fully established. In this paper, we consider multipath delay acquisition in the context of pulse position modulation (PPM) systems. We show that in the limit of large bandwidth, for many classes of channels, delay acquisition cannot be achieved even if all system resources are devoted to estimating the delays of the multipath.

We focus on the uncertainty in determining the multipath delays of the UWB channel. Knowledge of path delays is essential for proper operation of PPM systems, and timing errors seriously degrade PPM system performance [1], [2].

Our result implies that over many channels, the achievable rate of coherent PPM diminishes as the bandwidth increases because the receiver is unable to acquire the channel. A numerical analysis [3] also suggests a communication rate decrease with bandwidth increase; [4] underscores the importance of accurate channel estimates in a study of practical UWB systems.

UWB synchronization, or delay acquisition, is often performed by a correlation operation followed by a threshold decision [5], [6], [7]. However, we have shown that in the limit of large bandwidth, threshold synchronization fails [8], a similar conclusion is drawn in [9] for high SNR.

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	Unknown path amp., delays	Unknown amp., known delays
PPM	$L \sim \mathrm{const}$	$L/\log W \to 0$
DSSS	$\frac{L}{W/\log W} \to 0$	$L/W \to 0$

TABLE I
Multipath versus bandwidth growth rates for asymptotically achieving channel capacity.

Sensitivity to timing errors was also observed for spread-spectrum systems in [10] and references therein. With an *unknown* channel, direct sequence spread spectrum (DSSS) throughput diminishes in the limit of large bandwidth if the number of channel paths increases too fast (Table I). Information theoretic analyses of spread-spectrum systems [11], [12] show that the scenario of unknown path amplitudes with *known* delay locations allows almost the same performance as complete channel knowledge in the limit of large bandwidth.

We examine regimes where the bandwidth diverges $W \to \infty$ and the number of multipaths also grows without bound $L \to \infty$. The growth rate of the number of paths with bandwidth is of interest. Thus, we show that if $\frac{L}{W} \to 0$ (for independent Gaussian path amplitudes) or if $\frac{L}{W^{0.53}} \to 0$ (for deterministic path amplitudes), acquisition fails. These growth rates are sub-linear; recent wideband channel propagation measurements suggest that the number of channel paths grows with bandwidth, an almost linear rate is reported by [13], and a sub-linear rate is observed by [14], [11].

This work completes [11], by characterizing PPM performance over a channel that is unknown both to the transmitter and receiver. We review, in Table I, the conditions on the rate of increase of the number of channel paths that allows PPM and direct sequence spread spectrum (DSSS) systems to communicate at the channel capacity in the limit of large bandwidth, over channels with i.i.d. multipath. Note that the channel capacity in the limit equals the additive white Gaussian noise (AWGN) channel capacity, independent of L or its rate of increase. We assume that the transmitter does not know the channel realization and receiver knowledge is as listed. The shaded block shows the new result presented in this paper, the other results were shown in [11]. Our new result extends to more complex channels than the model of [11]; herein we consider both uniform delay profiles as well as the more realistic exponential delay profiles.

Knowledge of the path delays enables the removal of a factor of $\log W$ from the rate of increase of the number of paths which allows the system to achieve the channel capacity [11] as reflected in Table I for DSSS. The sensitivity to delay information is more severe for PPM as the conditions for achieving capacity go from $\frac{L}{\log W} \rightarrow 0$ with delay knowledge to $L \sim \mathrm{const}$ without this knowledge. The underlying reasoning is that with L i.i.d. paths, the amount of information needed to describe the channel path delays is proportional to $L \log W$. This information cannot be applied to the system's achievable rate for communication.

This paper is organized as follows: Section II describes the transmitted signal, channel model and received signal. The optimal receiver is discussed in Section III. The main results are presented in Section IV, and conclusions are provided in Section V.

II. Signal Model

A. The Transmitted Signal

We consider pulse position modulation (PPM), where one transmitted symbol can be written as

$$\begin{aligned} x(t) &= p\left(t - \frac{1}{W} b[n]\right) \qquad t \in [0, T_s) \\ p(t) &= \begin{cases} \sqrt{\frac{\mathcal{E}}{\theta}} & t \in \left[0, \frac{T_s}{N}\right) \\ 0 & \text{else} \end{cases} \end{aligned}$$

The symbol duration is given by T_s and the number of pulse positions is dictated by the transmission bandwidth W, *i.e.* $N = WT_s$. The data symbol is denoted $b[n] \in \{0, 1, \cdots, N-1\}$. The average transmitted energy per symbol is $\mathcal{E}$, it ensures an average power constraint. We shall assume that the symbol duration does not diminish with increasing bandwidth.

The *flash parameter* θ is the fraction (duty cycle) of the total communication period over which communication occurs, yielding a data rate proportional to $\theta \log_2 WT_s$. The receiver is aware of the on-periods of communication. We assume a positive (non-diminishing) data rate with increasing bandwidth. Thus, the parameter θ must be large enough so that $\theta \log W$ does not diminish – the requirement on θ can be written as:

$$\theta \geq \frac{k_1}{\log(Wk_2)} \tag{1}$$

Several features of our setup should be underscored. First, we allow for guard times. Second, the symbol time is lower bounded, and the bound is independent of signal bandwidth. Thus, the number of bits that can be transmitted in a single coherence period depends logarithmically on the bandwidth. Systems that use a guard period between symbols, that depends on the channel path delays, have a natural lower bound on their symbol time.

B. The Channel

We assume a tapped delay line model for the channel $h(t)$, thus

$$h(t) = \sum_{l=1}^{L} g_l \delta\left(t - \frac{d_l}{W}\right)$$

where the channel amplitudes are given by g_l and $\delta(\cdot)$ denotes the Dirac delta function and $\{d_l\}$ represent the path delays which are assumed non-negative integers between 1 and M. We assume an average power constraint: $\mathbf{E}\left[\sum_{i=1}^{L} g_i^2\right] = 1$.

The maximal possible number of resolvable paths is given by $M = \lfloor WT_d \rfloor$, where T_d represents the delay spread of the channel, and thus the number of multipath L satisfies $L \leq M$. Given M possible values of the path delays, we assume that the realizations of the path delays are uniformly distributed over $\binom{M}{L} = \frac{M!}{L!(M-L)!}$ possibilities. The channel model is of the block-type: the channel is fixed over the channel coherence time T_c; channel realizations at different coherence periods are statistically independent. Channel variation over time is immaterial to our results, they hold as long as the average power per channel realization is finite.

In the sequel, we shall consider four different channel types which differ in their path amplitude model (deterministic versus random) and in their profile (uniform power delay profile and exponential delay profile). As such, we treat a large class of channels. While specific practical channel models as specified in the IEEE 802.15 standard are not exactly examined, many of the key features are considered herein such as "sparse" channels and those with a decaying power delay profile. We provide in Section IV a sketch of the proof for the random, exponential case, the other cases are proven in a similar fashion.

To highlight the differences between the four channel models, we list the statistics of their channel taps as follows:

(ud): A uniform (**u**) average power delay profile, with equal and deterministic (**d**) path amplitudes

$$g_l^{(\mathbf{ud})} = \frac{1}{\sqrt{L}} \qquad l = 1, \ldots, L \tag{2}$$

(ur): A uniform (**u**) average power delay profile, with random (**r**) i.i.d. independent Gaussian path amplitudes

$$g_l^{(\mathbf{ur})} \sim \mathcal{N}\left(0, \frac{1}{L}\right) \qquad l = 1, \ldots, L \tag{3}$$

(ed): An exponential (**e**) average power delay profile, with deterministic (**d**) path amplitudes, that depend on the delay

$$\begin{aligned} g_l^{(\mathbf{ed})} &= \frac{F^{(\mathbf{e})}}{\sqrt{L}} e^{-\frac{d_l T_d}{M\tau}} \qquad l = 1, \ldots, L \\ F^{(\mathbf{e})} &= \sqrt{M \frac{1 - e^{-\frac{2T_d}{M\tau}}}{1 - e^{-\frac{2T_d}{\tau}}} e^{\frac{T_d}{M\tau}}} \qquad \tau > 0 \end{aligned}$$

note that $F^{(\mathbf{e})}$ is a normalization constant.

(er): An exponential **(e)** average power delay profile, with random **(r)** Gaussian path amplitudes

$$g_l^{(\mathbf{er})} \sim \mathcal{N}\left(0, {\sigma_l^{(\mathbf{er})}}^2\right) \qquad l = 1, \ldots, L \quad (4)$$

$${\sigma_l^{(\mathbf{er})}}^2 = \frac{{F^{(\mathbf{e})}}^2}{L} e^{-\frac{2d_l}{W\tau}} \qquad \tau > 0 \qquad (5)$$

where $F^{(\mathbf{e})}$ is as above.

The notation $^{(\mathbf{ud})}$, $^{(\mathbf{ur})}$, $^{(\mathbf{ed})}$ and $^{(\mathbf{ur})}$ is used throughout to denote the four channel models. Note that a positive exponential parameter τ ensures that at a large enough bandwidth, energy is spread over many received positions.

C. The Received Signal

The received signal is given by

$$y(t) = h(t) \otimes x(t) + z(t) = \sum_{l=1}^{L} g_l x\left(t - \frac{d_l T_d}{M}\right) + z(t),$$

where $z(t)$ is a zero-mean, white Gaussian noise process. At the receiver, the received signal is pulse matched filtered and sampled at $\frac{1}{W}$ yielding the following discrete time equivalent signal:

$$Y_i = \sum_{l=1}^{L} g_l X_{i-d_l} + Z_i \qquad i = 1, \ldots, T_c W \qquad (6)$$

$$X_i = \begin{cases} \sqrt{\frac{\mathcal{E}}{\theta}} & \text{if } \exists n : i \div N = n \\ & \text{and } i \bmod N = b[n] \\ 0 & \text{else} \end{cases} \qquad (7)$$

$i \div N$ signifies the largest integer κ such that $\kappa N \leq i$. The signal X_i is zero-valued except at the positions corresponding to the transmitted PPM pulse; recall that $N = WT_s$ is the number of positions within a symbol. The noise samples $\{Z_i\}$ are zero-mean with unit variance, and the average signal to noise ratio is given by $\mathcal{E}$.

In order to assess the challenges of acquisition of PPM in multipath, we analyze a further simplified system. We assume no intersymbol interference (a guard time) and *knowledge of the PPM symbols* at the receiver (training). Under these idealized conditions, we show a failure to acquire the channel, implying statements about more practical systems.

Given our assumptions about the channel noise and the multipath coefficients, each observation sample is Gaussian. If we define $\mathbf{MP}$ as the event that a multipath component is present at a sample of the received signal and $\overline{\mathbf{MP}}$ is the complementary event, that is the event of noise only, then under the four channel models we have $Y_i^{(xx)} | \mathbf{MP} \sim \mathcal{N}\left(m^{(xx)}, v^{(xx)}\right)$, where

$$\begin{array}{ll} m^{(\mathbf{ud})} = \sqrt{\frac{\mathcal{E}}{\theta L}} & v^{(\mathbf{ud})} = 1 \\ m^{(\mathbf{ur})} = 0 & v^{(\mathbf{ur})} = \frac{\mathcal{E}}{\theta L} + 1 \\ m_i^{(\mathbf{ed})} = \sqrt{\frac{\mathcal{E}}{\theta}} g_i^{(\mathbf{ed})} & v^{(\mathbf{ed})} = 1 \\ m^{(\mathbf{er})} = 0 & v_i^{(\mathbf{er})} = \frac{\mathcal{E}}{\theta} {\sigma_i^{(\mathbf{er})}}^2 + 1 \end{array} \qquad (8)$$

and $Y_i | \overline{\mathbf{MP}} \sim \mathcal{N}(0, 1)$.

Given that we know the PPM symbol, the observation vector is of length M and of the M possible positions, L correspond to the transmitted signal. The remaining $M - L$ positions correspond to noise. In the sequel, constants, independent of the bandwidth are denoted by k_n.

III. Maximum Likelihood Acquisition

Recall that the position of the transmitted PPM symbol is known; however, the multipath initial delay and path delays are unknown. The acquisition problem can be posed as a multiple hypothesis testing problem for which there are $\binom{M}{L}$ hypotheses with L non-zero channel taps out of $M = \lfloor WT_d \rfloor$ possible positions. The received signal under each hypothesis can be written as

$$\underline{Y}^{()} | H_i = \sqrt{\frac{\mathcal{E}}{\theta}} \underline{s}_i^{()} + \underline{Z} \quad \text{where } \underline{Z} \sim \mathcal{N}(\underline{0}, \mathbf{I})$$

$$\underline{s}_i^{()} = \Bigg[\underbrace{0, \cdots, g_1^{()}, 0, \cdots, g_2^{()}, \cdots, g_L^{()}, \cdots, 0}_{L \text{ amplitudes at corresponding delays}}\Bigg]^T$$

To facilitate the description of the optimal, maximum likelihood (ML) detector we define the multipath location vector, that is, for each particular multipath profile vector $\underline{s}_i^{()}$, we have

$$\underline{p}_i = \Bigg[\underbrace{0, \cdots, 1, \cdots, 1, \cdots, 1}_{L \text{ ones at corresponding delays}}\Bigg]^T \quad i = 1, 2, \ldots, \binom{M}{L}$$

Due to the fact that the signal vectors in the **(ud)** case are equal energy and each non-zero path has equal amplitude, the maximum likelihood **(ud)** detector is a correlator:

$$\hat{i}^{(\mathbf{ud})} = \arg\max_i \underline{p}_i^T \underline{Y}^{(\mathbf{ud})}$$

Given the form of $\underline{p}_i$, we can see the following equivalence. Let $\tilde{Y}_1, \tilde{Y}_2, \cdots, \tilde{Y}_L$ be the L largest components of $\underline{Y}$. Then,

$$\max \underline{p}_i^T \underline{Y} = \sum_{j=1}^{L} \tilde{Y}_j$$

Thus the maximum likelihood detector is equivalent to determining the multipath locations by selecting the L positions with the L largest signal values.

For the uniform random **(ur)** path scenario, the ML detector is an energy detector: the vector observations are zero-mean Gaussian vectors with covariance matrices, $C_i = \frac{\mathcal{E}}{L\theta} \text{diag}(\underline{p}_i) + \mathbf{I}$ for $i = 1, \ldots, \binom{M}{L}$. Thus we have,

$$\arg\max \mathrm{p}(\underline{y}^{(\mathbf{ur})} | H_i) = \arg\min \underline{y}^T C_i^{-1} \underline{y}$$

where we exploit the fact that, $\det C_i = \left(1 + \frac{\mathcal{E}}{L\theta}\right)^L \;\; \forall\, i$.

After some manipulation it can be shown that the ML detector is equivalent to,

$$\hat{i}^{(\mathbf{ur})} = \arg\max_i \underline{p}_i^T \left|\underline{Y}^{(\mathbf{ur})}\right|$$

where $|\underline{x}|$ is a vector whose components correspond to the absolute values of the components of the vector $\underline{x}$. With this perspective of the maximum likelihood detector, we can develop a method for evaluating the likelihood of an error through order statistics. In order to upper-bound the detection performance in the exponential power delay profile (PDP) case, we will analyze a uniform PDP; the acquisition performance over this profile bounds that of the exponential PDP.

In analyzing the random path amplitudes, we apply order statistics on the signed values $\left\{Y_i^{(\mathbf{ur})}\right\}$ rather than the absolute values. The final result is shown by exploiting the symmetry of the order statistics about the mean [15].

IV. Results

We present the main theorem of the work, that applies to the (**er**) channel. Similar theorems apply to the (**ud**), (**ur**) and (**ed**) cases, they are omitted due to limited space. The proof is merely sketched.

Theorem 1: (**er**) case. Consider M independent, Gaussian random variables with the following distributions:

$$\begin{aligned} Y_i^{(\mathbf{er})} &\sim \mathcal{N}\left(0, v_i^{(\mathbf{er})}\right) \quad i = 1, 2, \cdots, L \\ v_i^{(\mathbf{er})} &= \frac{\mathcal{E}}{\theta} \sigma_i^{(\mathbf{er})2} + 1 \\ Z_i &\sim \mathcal{N}(0,1), \; i = 1, 2, \cdots, M-L \end{aligned}$$

where $\sigma_i^{(\mathbf{er})}$ are defined in (5); $L \leq M$.

We order the $Y_i^{(\mathbf{er})}$ such that $B_{1:L}^{(\mathbf{er})} = \max Y_i^{(\mathbf{er})}$ and $B_{L:L}^{(\mathbf{er})} = \min Y_i^{(\mathbf{er})}$; similarly, $S_{1:M-L} = \max Z_i$ and $S_{M-L:M-L} = \min Z_i$. Then,

$$\lim_{M,L\to\infty} P\left[S_{L:M-L} > B_{1:L}^{(\mathbf{er})}\right] = 1 \quad \text{if} \quad \frac{L}{M} \to 0 \tag{9}$$

Theorem 1 shows that even with very modest growth rates on the number of multipath with respect to bandwidth, the maximum likelihood detector fails to detect any of the positions corresponding to a non-zero path in the limit of large bandwidth. This statement holds irrespective of whether we consider a uniform or an exponential profile. A similar statement holds for deterministic path amplitudes if the number of paths L does not grow too fast, namely $\frac{L}{M^{0.53}} \to 0$.

A. Proof Sketch

1) A Bounding Uniform Profile: We start by presenting a uniform PDP, where acquisition is easier than over the exponential profile. The proof is based on showing that even on this uniform profile, acquisition is not possible in the limit.

Our new uniform profile has Gaussian i.i.d. amplitudes, that are equivalent to the strongest tap of the original exponential random profile. Thus, we look at the path of the exponential profile at delay $d = 1$ and get from (5)

$$Y_i^{(\max)} \sim \mathcal{N}\left(0, k\frac{M\log M\left(1 - e^{-\frac{2T_d}{M\tau}}\right)}{L}\right) \quad i = 1, 2, \ldots, L$$

Thus the new profile has greater energy than the original exponential profile – despite this additional energy, delay acquisition will fail.

2) The L^{th} Largest Noise Variable: We look at the L^{th} largest of the noise variables in the limit of large M, L. We use order statistics [15] over $G = M - L$ i.i.d. standard Gaussian variables, and consider the L^{th} largest variable, *i.e.* there are $L-1$ variables larger than the one we investigate. Its mean is given by

$$\begin{aligned} \mathbf{E}_{L:M-L} &= \sqrt{2\ln(M-L)} \\ &\quad - \frac{\ln\ln(M-L) + \ln 4\pi + 2\left(Q_1(L) - C\right)}{2\sqrt{2\ln(M-L)}} \\ &\quad + O\left(\frac{1}{\ln(M-L)}\right) \end{aligned}$$

and the variance is

$$\begin{aligned} \mathbf{var}_{L:M-L} &= \frac{1}{2\ln(M-L)}\left(\frac{\pi^2}{6} - Q_2(L)\right) \\ &\quad + O\left(\frac{1}{\ln^2(M-L)}\right) \end{aligned}$$

where $C \approx 0.5772$ and

$$Q_1(L) = \sum_{l=1}^{L-1} \frac{1}{l} \approx \ln L \quad \text{and} \quad Q_2(L) = \sum_{l=1}^{L-1} \frac{1}{l^2}$$

We observe that $Q_2(L)$ is finite for any L, thus $\lim_{L,M\to\infty} \mathbf{var}_{L:M-L} = 0$. Therefore, in the limit of large M and L, the L^{th} largest variable approaches its mean.

To further investigate the mean, we exploit the simple approximation for $Q_1(L)$ above, for large L:

$$\mathbf{E}_{L:M-L} \approx \sqrt{2\ln(M-L)} - \frac{\ln L}{\sqrt{2\ln(M-L)}}$$

Observe that $\lim_{M,L\to\infty} \mathbf{E}_{L:M-L} = \infty$, and $\lim_{M,L\to\infty} \mathbf{var}_{L:M-L} = 0$. The L^{th} noise variable does not strictly converge in the mean square sense because its mean is not defined.

3) The Largest Signal Variable with Random Path Gains: We use Gaussian order statistics [15] again, with $G = L$ random variables, and examine the largest variable of the uniform PDP presented earlier. For a large bandwidth and a large number of paths we get:

$$\mathbf{E}_{1:L}^{(\mathbf{er})} \approx \sqrt{k_5 \ln L\left(\frac{\log M}{L} + k_6\right)} \tag{10}$$

$$\mathbf{var}_{1:L}^{(\mathbf{er})} \approx k_4 \frac{M\log M\left(1 - e^{-2/M \; T_d/\tau}\right)}{L\ln L} \tag{11}$$

4) Conditions for Dominance with Random Path Gains: We show that $\mathbf{E}_{L:M-L} > \mathbf{E}_{1:L}^{(\mathbf{er})}$. The inequality of interest is equivalent to

$$\begin{aligned} \sqrt{2\ln(M-L)} \quad \gg ? \quad & \sqrt{k_5 \ln L\left(\frac{\log M}{L} + k_6\right)} \\ & + \frac{\ln L}{\sqrt{2\ln(M-L)}} \end{aligned} \tag{12}$$

Via algebraic manipulation, we can show that the inequality is achieved if $\frac{L}{M} \to 0$.

Considering the negative paths, the situation is symmetric: the mean of the smallest (most negative) signal variable is higher than the the L^{th} most negative noise variable. To complete the proof, note that the L^{th} noise variable converges to its mean $\mathbf{E}_{L:M-L}$ because its variance diminishes. This mean dominates over the mean of the largest signal variable $\mathbf{E}_{1:L}^{(\mathbf{er})}$, and over any constant multiple of it, in particular $2\mathbf{E}_{1:L}^{(\mathbf{er})}$. We proceed to show that the probability of the largest signal variable exceeding $2\mathbf{E}_{1:L}^{(\mathbf{er})}$ diminishes even though its variance may diverge (it can be shown that $\mathbf{var}_{1:L}^{(\mathbf{er})} \approx k_4 \frac{\log M}{L \ln L}$).

Recall that $B_{1:L}^{(\mathbf{er})}$ is the largest signal variable. We have,

$$\begin{aligned} & \mathrm{Prob}\left[\left|B_{1:L}^{(\mathbf{er})} - \mathbf{E}_{1:L}^{(\mathbf{er})}\right| \geq \mathbf{E}_{1:L}^{(\mathbf{er})}\right] \\ = \ & \mathrm{Prob}\left[B_{1:L}^{(\mathbf{er})} \leq 0 \text{ or } B_{1:L}^{(\mathbf{er})} \geq 2\mathbf{E}_{1:L}^{(\mathbf{er})}\right] \\ \approx \ & \mathrm{Prob}\left[B_{1:L}^{(\mathbf{er})} \geq 2\mathbf{E}_{1:L}^{(\mathbf{er})}\right] \end{aligned} \qquad (13)$$

Apply the Chebyshev inequality to (13) to get

$$\mathrm{Prob}\left[B_{1:L}^{(\mathbf{er})} \geq 2\mathbf{E}_{1:L}^{(\mathbf{er})}\right] \leq \frac{\mathbf{var}_{1:L}^{(\mathbf{er})}}{{\mathbf{E}_{1:L}^{(\mathbf{er})}}^2}$$

Simple manipulation yields

$$\begin{aligned} \lim_{L,M\to\infty} \frac{\mathbf{var}_{1:L}^{(\mathbf{er})}}{{\mathbf{E}_{1:L}^{(\mathbf{er})}}^2} & \leq \lim_{L,M\to\infty} \frac{k_1 \log M}{k_2 \log M \ln^2 L + k_3 L \ln^2 L} \\ & = 0 \end{aligned}$$

And the desired result is shown: for a modified uniform random profile, acquisition is not achieved. This modified uniform profile achieves performance superior to that of the exponential random profile and thus we cannot achieve acquisition for the exponential profile scenario for the conditions outlined. We note that the value of the positive decay parameter τ does not affect the results. The result would be unchanged even if a fixed fraction of paths were to be acquired rather than all of the paths.

The relationships between the strongest signal and the L^{th} noise are illustrated in Figure 1. Note that the noise variance diminishes while the strongest signal variance diverges slowly. The mean of the L^{th} noise diverges faster than the mean of the strongest signal.

V. Conclusions

We have considered the problem of channel acquisition for the PPM modulation for ultra-wideband communication systems. Even under idealized conditions of no inter-symbol interference and perfect knowledge of transmitted symbols and channel gains, the optimal synchronizer fails when the number of multipath grows without bound as the bandwidth increases. We have shown that with Gaussian paths, if $L \to \infty$ and $\frac{L}{W} \to 0$, the maximum likelihood synchronizer fails to capture any of the paths for either a uniform or an exponential channel. Similar results hold for channels with deterministic path amplitudes, with an additional condition $\frac{L}{W^{0.53}} \to 0$.

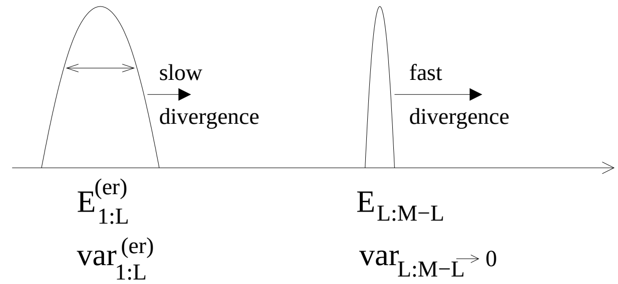

Fig. 1. An illustration of the distribution of the strongest signal position in the exponential PDP with random amplitudes case $B_{1:L}^{(\mathbf{er})}$ and the L^{th} strongest noise position $S_{L:M-L}$.

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