

# On Synchronization of Wideband Impulsive Systems in Multipath

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**Abstract**—A preponderance of ultrawideband radio communication systems under current study employ the use of pulse position modulation (PPM). However, there are significant issues related to the synchronization of PPM systems in multipath. If the multipath scattering is rich *i.e.* the number of reflections (paths) increases without bound as the bandwidth increases, then synchronization is impaired. In particular, it is shown that a threshold-type detector will fail in synchronizing, for any possible threshold, in the limit of large bandwidth. The maximum likelihood detector can synchronize under some severely constrained scenarios which do not appear to reflect reality in light of recent propagation measurements.

## I. INTRODUCTION

We consider the asymptotic performance of basic synchronization schemes for pulse position modulations in the limit of large bandwidth. This study is motivated by recent interest in ultra wideband (UWB) signaling schemes for radio communications. UWB systems have received significant recent attention due to characteristics inherent to very wideband radio communication signals: very fine timing resolution, improved penetration properties, low probability of intercept, robustness to jamming, and increased diversity due to significant multipath [15], [3]. Several of the properties of UWB signaling which make such systems attractive, also make UWB systems challenging to implement. Thus, while significant diversity is achievable given the large amount of multipath, methods for harnessing such diversity when the channel is unknown remain a challenge. In fact, the fundamental limits of UWB signaling in the presence of channel uncertainty have not been fully established. In this work, we focus on the uncertainty in determining the multipath profile (including leading delay) of the UWB channel. For the purposes of this work, such uncertainty is equivalent to uncertainty in synchronization. Previous work on the effect of synchronization on data rates [12] investigated a different aspect of ‘synchronization’ namely the relative timing of users over a common channel. Although the terminology is similar, [12] is unrelated to our work.

The capacity and achievable rates of systems in the limit of infinite bandwidth have been the subject of recent work. The capacity of the multipath channel in the limit of infinite bandwidth is identical to the additive white Gaussian noise channel capacity, and can be achieved by FSK modulation and duty cycle transmission [2], [1]. The data-rates over multipath

channels with any number of paths was investigated in [10], [13] for specific modulation methods. Employing spreading modulations without duty cycles leads to low data rates if multipath is significant [10] and for fading channels with large numbers of multipath, the data rates are zero in the limit [5], [9]. This prior work assumed knowledge of the delays of the paths. If duty cycles are employed (flash signaling), spreading modulations can achieve the AWGN capacity [7] provided the growth of the multipath as a function of bandwidth is not too rapid. The results of [7] require knowledge of multipath delays for PPM, but not for the direct-sequence spread spectrum signaling. We note that physical UWB channels exhibit a growth in multipath that is sub-linear in the transmission bandwidth [7], [8].

The sensitivity of UWB PPM systems to synchronization errors has been previously observed in the context of communication theoretic research. Timing errors as small as fractions of nanoseconds can seriously degrade system performance as reported in [4], [11]. Furthermore, [14] suggests that threshold-based UWB synchronization for PPM does not perform well even in asymptotically high SNR; further implying that the performance of pragmatic synchronization could limit UWBs potential. Our interest is in synchronization performance in the limit of large bandwidth; however, we reach similarly pessimistic conclusions.

By investigating the performance of threshold detectors in the limit of large bandwidth, we show that there is no value of the threshold that yields correct detection of the positions that contain the signal. Any value we choose for the threshold is smaller than the received signal at many positions that contain only noise. The intuition of the difficulty to synchronize is as follows: Each multipath position contains a small fraction of the signal energy, that diminishes with bandwidth as the number of paths increases. The energy decrease per position is held back to some extent by the flashy nature of the signal, but the concentration of energy due to ‘flashing’ is limited by the low spectral efficiency of the pulse position modulation. The detector must handle a large number of noise positions (that grows linearly with the bandwidth), while searching for the signal positions that hold a moderately increasing amount of energy. The task is increasingly difficult as the bandwidth increases, and becomes impossible in the limit.

We also analyze the conditions that allow a maximum likelihood (ML) detector to find the channel path correctly under favorable conditions. The maximum likelihood detector compares candidate multipath profiles against each other; however it too suffers from the effects of noise as the bandwidth increases. The growth rate of the number of reflections is established such that the ML synchronization error is arbitrarily small. However, this growth rate has little physical meaning. We also study a channel where the span (in delay) of the channel impulse response diminishes as the system bandwidth increases. The system can synchronize over this channel, if the span of the impulse response diminishes in an appropriate manner.

This paper is organized as follows: Section II describes the transmitted signal, channel model and received signal. The consideration of the threshold detector is given in Section III, while the derivation and analysis of the maximum likelihood synchronization scheme is provided in Section IV.

## II. SIGNAL MODELS

### A. The Transmitted Signal

We consider pulse position modulation (PPM), where the transmitted signal can be written as,

$$x(t) = \sum_{n=-\infty}^{\infty} p\left(t - nT_s - \frac{1}{W}b[n]\right)$$

$$p(t) = \begin{cases} \sqrt{\frac{\mathcal{E}}{M\theta}} & t \in [0, \frac{T_s}{N}) \\ 0 & \text{else} \end{cases}$$

The symbol duration is given by  $T_s$  and the number of pulse positions is dictated by the transmission bandwidth  $W$ , *i.e.*  $N = WT_s$ . The data symbol is denoted  $b[n] \in \{0, 1, \dots, N-1\}$ .  $\mathcal{E}$  is the average transmitted energy per coherence period of the channel that is bandwidth independent,  $M$  is the number of transmitted symbols per coherence period and  $\theta$  is a flash parameter to be explained shortly. Thus, in a symbol duration, there is a single rectangular pulse of duration  $\frac{T_s}{N}$ . Our goal is to investigate performance of such a PPM system as the transmission bandwidth increases. We shall assume that the symbol duration does not diminish; however, due the use of flash signaling [13], the information rate will not grow without bound. With the use of flash signaling, transmission is bursty and communication occurs a fraction  $\theta$  of the total communication period. The *flash parameter*  $\theta$  is known at the receiver and furthermore, the receiver is aware of the on-periods of communication. The transmission frame corresponds to a coherence period of the channel and as such, one out of every  $\frac{1}{\theta}$  coherence periods is employed for transmission (an on-period). There is a distinction to be made between flashy transmission and PPM modulation. For regular data transmission, the receiver must detect which one of the  $N = WT_s$  pulse positions has been employed in each symbol; in contrast, with flashy transmission, the receiver is synchronized to the on-periods of communication. We note that if  $\theta$  is quite small, then the transmitter is predominantly silent.

The fraction of time utilized for transmission may decrease as the bandwidth  $W$  increases, but it cannot do so too fast. In order to maintain a positive (non-diminishing) data-rate, the parameter  $\theta$  must be large enough so that  $\theta \log W$  does not diminish. The reasoning for this is straightforward:  $\log_2 WT_s$  bits are transmitted per coherence time; however, only a fraction of the coherence periods are employed and thus the data rate is proportional to  $\theta \log_2 WT_s$ . The requirement on  $\theta$  can be written as:

$$\theta \geq \frac{k_1}{\log(Wk_2)} \quad (1)$$

for fixed  $k_1, k_2$  that are independent of the bandwidth.

Several features of our assumptions should be underscored. The first is that there is no limit imposed on the number of PPM positions that are employed for data signaling. Thus, a guard time can be implemented by limiting the number of employed positions. Second, we emphasize the employment of a lower bounded symbol time, where the lower bound does not depend on the signal bandwidth. Thus we do not consider schemes where the symbol time diminishes with bandwidth. As such, in our scheme, the number of bits that can be transmitted in a single coherence period depends logarithmically on the bandwidth. Note that systems that use a guard period between symbols, that depends on the channel path delays, have a natural lower bound on their symbol time.

### B. The Channel and Received Signal

We assume an tapped delay line model for the channel  $h(t)$ , thus

$$h(t) = \sum_l^L g_l \delta\left(t - \frac{d_l}{W}\right)$$

where the channel gains are given by  $g_i$  and we further assume that  $\sum g_i^2 = 1$ ;  $\delta(\cdot)$  denotes the Kronecker delta function and  $d_i$  represent the path delays which are assumed to be non-negative integers. For simplicity of exposition we shall assume a uniform profile for the path gains and therefore  $g_i = \frac{1}{\sqrt{L}} \forall i$ . The number of possible resolvable paths is given by  $M = WT_d$ , where  $T_d$  represents the maximum delay of the channel, thus  $L \leq M$ . Recent wideband channel propagation measurements suggest that the number of channel paths grows sub-linearly with bandwidth [8], possibly satisfying

$$\lim_{W \rightarrow \infty} L = \infty$$

$$\lim_{W \rightarrow \infty} \frac{L}{W} = 0$$

Given  $M$  possible values of the path delays, we assume that the realizations of the path delays are uniformly distributed over  $\binom{M}{L} = \frac{M!}{L!(M-L)!}$  possibilities. The channel model assumed is a block-type: the channel is fixed over the channel coherence time  $T_c$ ; channel realizations at different coherence periods are statistically independent.

The received signal is given by,

$$\begin{aligned} y(t) &= h(t) \otimes x(t) + z(t) \\ &= \sum_{l=1}^L g_l x\left(t - \frac{d_l}{W}\right) + z(t), \end{aligned}$$

where  $z(t)$  is a zero-mean, white Gaussian noise process.

At the receiver, the received signal is matched filtered with the pulse shape and sampled at  $\frac{1}{W}$  yielding the following discrete time equivalent signal:

$$\begin{aligned} y_i &= \frac{1}{\sqrt{L}} \sum_{l=1}^L X_{i-d_l} + Z_n \\ x_i &= \begin{cases} \sqrt{\frac{\mathcal{E}}{\theta}} & \text{if } \exists n : i \div N = n \\ & \text{and } i \bmod N = b[n] \\ 0 & \text{else} \end{cases} \end{aligned}$$

$i \div N$  signifies the largest integer  $k$  such that  $kN \leq i$ . The signal  $X_i$  is zero-valued except at the positions corresponding to the transmitted PPM pulse; recall that  $N = WT_s$  is the number of possible positions. The amplitude of the signal at the non-zero position is normalized so the noise samples  $\{Z_n\}$  are zero-mean and unit variance.

### C. Additional Simplifying Conditions

In order to assess the challenges of synchronization of PPM in multipath, we analyze a further simplified system that operates under two additional conditions:

- The system employs an additional guard time between transmitted symbols with duration  $T_d$ . This guard time ensures that there is no inter-symbol interference. The parameters of the new system are just as defined above, however, the channel coherence time is longer:

$$\tilde{T}_c = \frac{T_s + T_d}{T_s} T_c \quad (2)$$

The lack of inter-symbol interference (ISI) simplifies analysis. We can show that synchronization in the presence of ISI is more challenging than in ISI-free channels; thus our results bound those for ISI channels.

- We make another assumption that simplifies synchronization: that the receiver knows the transmitted signal value - which can be interpreted as training information. Synchronization is naturally easier with this assumption in place, thus our result that the system cannot achieve synchronization, holds also for realistic systems that do not know the transmitted values. The receiver sums over all the symbols per coherence period before it begins processing. After summation and proper normalization, the received signal is given by

$$\begin{aligned} Y_i &= \begin{cases} \sqrt{\frac{\mathcal{E}}{\theta L}} + Z_i & \text{if } \exists l : d_l = i \\ Z_i & \text{else} \end{cases} \\ i &= 1, \dots, M \end{aligned} \quad (3)$$

where  $\{Z_i\}$  are standard normals.

## III. THRESHOLD SYNCHRONIZATION

In this section we consider a synchronization scheme based on threshold detection. The determination of multipath locations is done independently position by position by assessing whether the signal exceeds a threshold. That is,

$$Y_i > B \rightarrow \text{declare a path location}$$

$$Y_i | \text{path location} \sim \mathcal{N}\left(\sqrt{\frac{\mathcal{E}}{\theta L}}, 1\right) \quad (4)$$

$$Y_i | \text{noise only position} \sim \mathcal{N}(0, 1)$$

Given that we know the PPM symbol, the observation vector is of length  $M$  and of the  $M$  possible positions,  $L$  correspond to the transmitted signal. The remaining  $M - L$  correspond to noise.

We examine the ability of this method to synchronize by investigating its performance with various threshold values. We show that for any threshold  $B = \beta \sqrt{\frac{\mathcal{E}}{\theta L}}$ , the number of signal positions above the threshold  $B$  is much smaller than the number of noise positions above it, so correct detection of all  $L$  signal positions is not possible. An intuitive extension applies this conclusion to the optimal receiver.

We define the following probabilities,

$$p_s = \Pr(Y_i \geq B | \text{signal}) = Q\left((\beta - 1)\sqrt{\frac{\mathcal{E}}{\theta L}}\right) \quad (5)$$

$$p_n = \Pr(Y_i \geq B | \text{noise}) = Q(B) = Q\left(\beta\sqrt{\frac{\mathcal{E}}{\theta L}}\right) \quad (6)$$

Next define two integer random variables, that count the number of signal and noise positions above the threshold:

$$\begin{aligned} X_s &= \{\text{number of signal positions} \geq B\} \leq L \\ X_n &= \{\text{number of noise positions} \geq B\} \end{aligned}$$

Given the probabilities defined in (5) and (6), we can determine the average number of noise and signal positions above the threshold:

$$\begin{aligned} \mu_s &= Lp_s = \mathbf{E}[X_s] \\ \mu_n &= (M - L)p_n = \mathbf{E}[X_n] \end{aligned}$$

Our approach is to investigate the ratio  $\frac{X_s}{X_n}$  and show that it diminishes as the bandwidth increases, so false detections due to noise overwhelm the detection of signal paths. We study this ratio for three different possibilities of parameter choices:

- 1)  $\beta \rightarrow \infty$ : In this case we get  $p_s \approx p_n$  (compare (5) and (6)), and for a sub-linear increase of the number of paths ( $L/W \rightarrow 0$ )

$$\frac{X_s}{X_n} \approx \frac{\mu_s}{\mu_n} = \frac{L}{M - L} \rightarrow 0 \quad (7)$$

- 2)  $\beta$  finite,  $\theta L > 0$ : This parameter choice yields a finite  $p_n$  (that does not diminish as the bandwidth increases,

see (6)) and the average number of detected noise positions increases linearly with the bandwidth ( $\mu_n \sim W$ ).

$$\frac{X_s}{X_n} \approx \frac{\mu_s}{\mu_n} \rightarrow 0$$

- 3)  $\beta$  finite,  $\theta L \rightarrow 0$ : The probability  $p_n$  diminishes, and in this case, we use the approximation  $Q(B) \approx \frac{1}{\sqrt{2\pi}B} e^{-B^2/2}$ :

$$\begin{aligned} p_n &= Q(B) \approx \frac{1}{\sqrt{2\pi}B} e^{-B^2/2} \\ &= \frac{\sqrt{\theta L}}{\sqrt{2\pi}\beta\sqrt{\mathcal{E}}} \exp\left[-\frac{\beta^2}{2} \frac{\mathcal{E}}{\theta L}\right] \\ &\geq \frac{\sqrt{k_1 L}}{\sqrt{2\pi}\beta\sqrt{\mathcal{E} \log W k_2}} \exp\left[-\frac{\beta^2}{2} \frac{\mathcal{E} \log W k_2}{k_1 L}\right] \\ &= \text{const} \frac{\sqrt{L}}{\sqrt{\log W k_2}} (W k_2)^{-\frac{\beta^2}{2} \frac{\mathcal{E}}{k_1 L}} \end{aligned}$$

$$\begin{aligned} (\text{for } L \rightarrow \infty) \quad p_n &\geq \text{const} \frac{\sqrt{L}}{\sqrt{\log W k_2}} W^{-\frac{\beta^2}{2} \frac{\mathcal{E}}{k_1 L}} \\ \mu_n &= (W T_d - L) p_n \\ &\geq \text{const} \frac{\sqrt{L}}{\sqrt{\log W k_2}} W^{1 - \frac{\beta^2}{2} \frac{\mathcal{E}}{k_1 L}} \\ (\text{for } L \rightarrow \infty) &\rightarrow \infty \end{aligned}$$

We next show that for any constant  $\beta$ , the ratio  $\frac{X_s}{X_n}$  is very small, so the number of paths detected above the threshold is very small compared to the number of false detections due to noise.

The number of noise positions above the threshold is larger than some number (that depends on a constant  $\epsilon_n$ ) with a probability that is bounded using the Chernoff bound in (8).

$$\begin{aligned} P(X_n \leq (1 - \epsilon_n)\mu_n) &\leq \exp[-\epsilon_n^2 \mu_n / 2] \rightarrow 0 \implies \\ P(X_n > (1 - \epsilon_n)\mu_n) &\geq 1 - \exp[-\epsilon_n^2 \mu_n / 2] \rightarrow 1 \quad (8) \end{aligned}$$

The convergence occurs for any constant  $\epsilon_n$ . We conclude that with high probability,  $X_n > (1 - \epsilon_n)\mu_n$ . Thus,

$$\begin{aligned} \frac{X_s}{X_n} &\leq \frac{L}{(1 - \epsilon_n)\mu_n} \\ &\leq \text{const} \sqrt{L \log W k_2} W^{-1 + \frac{\beta^2}{2} \frac{\mathcal{E}}{k_1 L}} \end{aligned}$$

This ratio diminishes for any  $L \rightarrow \infty$ .

We see that the noise positions above the threshold overwhelm the signal positions above it, for any threshold. To better understand this conclusion, look at the threshold value  $100\sqrt{\frac{\mathcal{E}}{\theta L}}$ , that is a hundred times the mean value at a signal position (4). The number of signal positions above this high threshold is naturally small, and we could hope that the positions that surpass the threshold are mostly signal positions. In fact, our analysis above shows that as the bandwidth increases, the number of noise positions above the threshold increases, preventing the correct detection of the signal positions.

#### IV. MAXIMUM LIKELIHOOD SYNCHRONIZATION

In order to appreciate the conditions that enable synchronization, we study the performance of the optimal synchronizer. This section shows the conditions on the number of paths  $L$  and the length of the channel  $M$  that enable synchronization (under the favorable conditions II-C). Recall that the position of the PPM symbol is known; however, the initial delay and multipath profile are unknown. The synchronization problem can be posed as a multiple hypothesis testing problem for which there are  $\binom{M}{L}$  hypotheses given a delay spread of  $M = W T_d$  and  $L$  non-zero channel taps. The received signal under each hypothesis can be written as, (recall (3)),

$$\underline{Y}|H_i = [Y_1, Y_2 \cdots Y_M]^T \quad (9)$$

$$= \sqrt{\frac{\mathcal{E}}{L\theta}} \underline{s}_i + \underline{Z} \quad (10)$$

$$\underline{s}_i = \begin{bmatrix} 1, 1, 0, \dots, 1, 0, \\ \underbrace{\hspace{10em}} \\ L \text{ 1's over } M \text{ positions} \end{bmatrix}^T \quad (11)$$

$$\underline{Z} \sim \mathcal{N}(\underline{0}, \mathbf{I}) \quad (12)$$

The optimal detector for such a scenario is a simple correlator:

$$\hat{i} = \arg \max_i \underline{s}_i^T \underline{Y} \quad (13)$$

The statistics of the decision statistic  $T_i(Y|H_j) = \underline{s}_i^T \underline{Y}|H_j$  are Gaussian with the distribution,

$$T_i(Y|H_j) \sim \mathcal{N}\left(\rho(i, j) \sqrt{\frac{\mathcal{E}}{L\theta}}, L\right) \quad (14)$$

$$\rho(i, j) \doteq \underline{s}_i^T \underline{s}_j \quad (15)$$

Assuming that each delay profile is equally likely and without further optimizing the union bound, simple bounds on the probability of detection error are given by:

$$\frac{1}{\binom{M}{L}} Q\left(\frac{d_{\min}}{L} \sqrt{\frac{\mathcal{E}}{\theta}}\right) \leq P_e \leq \binom{M}{L} Q\left(\frac{d_{\min}}{L} \sqrt{\frac{\mathcal{E}}{\theta}}\right) \quad (16)$$

$$d_{\min}^2 = \min_{i, j, i \neq j} \|\underline{s}_i - \underline{s}_j\|^2 = 2 \quad (17)$$

We first investigate whether a rate on  $L$  exists such that the upper bound converges to 0 as  $W \rightarrow \infty$ . As  $\lim_{W \rightarrow \infty} M = \infty$ , Stirling's approximation for the factorial function will be of utility,  $M! \approx \sqrt{2\pi} M^{M+\frac{1}{2}} e^{-M}$ . Employing Stirling's approximation and noting that the maximal value of the choose operator occurs when  $L = \frac{M}{2}$  (assuming  $M$  is even), we have

$$\binom{M}{L} \approx \frac{1}{\sqrt{2\pi}} \frac{M^{M+\frac{1}{2}}}{L^{L+\frac{1}{2}} (M-L)^{M-L+\frac{1}{2}}} \lesssim \frac{2^{M+1}}{\sqrt{2\pi} M}$$

We next bound the Q-function as in Section III,  $Q(x) \approx \frac{1}{\sqrt{2\pi}x} e^{-\frac{x^2}{2}}$ , and recalling that the duty cycle is defined such that  $\theta \approx \frac{k_3}{\log(W k_2)}$  where  $k_3 > k_1$  in (1). Combining our

bounds yields the following approximate upper bound on the probability of synchronization error,

$$P_e \lesssim \underbrace{\frac{1}{\pi} \sqrt{\frac{k_3}{2\mathcal{E}}}}_{\text{const}} K(W) e^{-\frac{\mathcal{E} \log(Wk_2)}{k_3 L^2} + (\log 2)^M} \quad (18)$$

$$\text{where } K(W) = \frac{L}{\sqrt{M \log(Wk_2)}} \quad (19)$$

If the first term in the exponential dominates, the exponential term will converge to 0 as  $W \rightarrow \infty$ . We recall that  $M = WT_d$  and thus the desired relationship on  $L$  is given by

$$L < \sqrt{\frac{\mathcal{E} \log(Wk_2)}{k_3 (\log 2) WT_d}}$$

Note that this implies that we want the behavior of  $L \sim o\left(\sqrt{\frac{\log W}{W}}\right)^1$ . With this rate, we have  $\lim_{W \rightarrow \infty} K(W) = 0$  (see (19)), and thus the approximate bound on  $P_e$  converges to zero. However, this rate implies that the amount of multipath is actually shrinking with increasing bandwidth, which does not appear to match physical reality ( $L \rightarrow \infty$  and  $\frac{L}{W} \rightarrow 0$ ) as suggested by [8].

We next consider an alternative variation on the problem. That is, rather than have the delay spread constant and thus the number of possible multipath positions linearly growing with the bandwidth ( $M = WT_d$ ), we consider  $M$  proportional to  $L$ , with the same growth rate. Thus we assume  $M = \alpha L$  where  $\alpha > 1$ . Replacing  $M$  with  $\alpha L$  in (18) yields the following desirable relationship to ensure that the asymptotic value of the exponential term in (18) is zero:

$$L < \left(\frac{\mathcal{E} \log(Wk_2)}{k_3 (\log 2) \alpha}\right)^{\frac{1}{3}}$$

that is  $L \sim o\left((\log W)^{\frac{1}{3}}\right)$ . Similarly with this rate, it is straightforward to show that  $\lim_{W \rightarrow \infty} K(W) = 0$ . In contrast to the case of constant delay spread, we now have multipath growth that is sublinear in the bandwidth ( $\frac{L}{W} \rightarrow 0$ ) and increasing  $L \rightarrow \infty$ . However, again, there is no physical justification for the decrease of delay spread  $T_d$  with bandwidth which is the implication of  $M \sim o\left((\log W)^{\frac{1}{3}}\right)$ . The delay spread is a function of the geometry of the scattering environment and thus typically fixed.

## V. CONCLUSIONS

In this paper, we have investigated the asymptotic performance of synchronization for pulse position modulation in multipath channels in the limit of large bandwidth. We have shown that for a pragmatic threshold based detection scheme, synchronization can never be achieved in the limit of large bandwidth if the scattering is rich, that is the amount of multipath grows without bound as the bandwidth increases. For the optimal maximum likelihood detector, synchronization

can be achieved if the scattering is sufficiently sparse, which suggests that synchronization also cannot be achieved for practical growth rates on the multipath. An area for future study is to characterize the relationship between coherence time and multipath profiles and to investigate how additional training, possibly over *multiple* coherence times affects synchronization. For wireless channels, delay profiles often change much more slowly than multipath coefficients.

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<sup>1</sup>This is *little-o notation*. That is,  $f(n) = o(g(n))$  if  $\exists k \exists, f(n) < cg(n) \forall n \geq k$ .