Synchronization of PPM over Wideband Multipath Channels

Dana Porrat
School of Engineering and Computer Science
The Hebrew University
Jerusalem, Israel
dporrat@cs.huji.ac.il

Urbashi Mitra
Department of Electrical Engineering
University of Southern California
Los Angeles, CA USA
ubli@ucs.edu

***Abstract*— The acquisition, or synchronization, of the multi-path profile for ultra-wideband pulse position modulation (PPM) communication systems is considered. The rate of increase of the number of paths (as the bandwidth grows) determines whether synchronization can occur. It is shown that if the number of paths increases without bound, but slower than the square root of the bandwidth, then the system cannot synchronize in the limit. This result holds for an exponential or a uniform power delay profile, for IID Gaussian paths or deterministic paths.**

I. Introduction

Ultra wideband (UWB) impulsive systems gained recent interest for communications, as well as channel imaging and positioning. While significant diversity is achievable given the large amount of multi-path in UWB radio channels, methods for harnessing such diversity when the channel is unknown remain a challenge. In fact, the fundamental limits of UWB signaling in the presence of channel uncertainty have not been fully established. In this paper we consider synchronization of pulse position modulation (PPM) systems in the limit of large bandwidth and show that it cannot be achieved on many channels, even if all the system resources are devoted to it.

We focus on the uncertainty in determining the multi-path profile (including leading delay) of the UWB channel. Knowledge of the channel path delays is critical for the operation of pulse position modulation systems over large bandwidths, and timing errors as small as fractions of nanoseconds can seriously degrade system performance as reported in [5], [10]. Synchronization, or channel acquisition, is normally performed by a receiver that applies a correlator followed by a threshold decision [2], [4], [12], [3]. For a review of UWB channel acquisition see *Aedudodla et al.* [1]; the same authors suggest [11] that threshold-based synchronization does not perform well even in asymptotically high SNR, implying that the performance of pragmatic synchronization methods could limit their UWB potential. We have shown in [6] that in the limit of large bandwidth, threshold based synchronizers cannot achieve synchronization.

This work shows that pulse position modulation is very sensitive to the knowledge of path delays. In fact, even a mild increase (with bandwidth) of the number of independently distributed paths composing a channel will cause a PPM system to fail in the limit of large bandwidth. We prove that

	Unknown path gains and delays	Unknown gains, known delays
PPM	$L \sim \text{const}$	$L/\log W \to 0$
DSSS	$\frac{L}{W/\log W} \to 0$	$L/W \to 0$

TABLE I
Channel conditions for asymptotically achieving AWGN channel capacity in multipath channels for PPM and DSSS modulations.

synchronization is not possible in the limit of large bandwidth if the number of multi-path L diverges as the bandwidth increases ($\lim_{W\to\infty} L = \infty$) and in addition $\lim_{W\to\infty} \frac{L}{\sqrt{W}} = 0$. This additional condition does not appear to be inherently significant, and we conjecture that our result holds for any channel with a diverging number of multi-path.

This work completes [7] by characterizing PPM performance over a channel that is unknown both to the transmitter and receiver. Table I summarizes the conditions on the rate of increase of the number of channel paths, that allow PPM and direct sequence spread spectrum (DSSS) systems to communicate at the channel capacity in the limit of large bandwidth, over channels with IID multi-path. L is the number of identically distributed multi-path in the channel's impulse response and W is the bandwidth. Note that the channel capacity in the limit equals the additive white Gaussian noise (AWGN) channel capacity; it does not depend on the number of multi-path or on its rate of increase. The table lists the rate of increase of L that allows different systems to achieve the channel capacity, the transmitter does not know the channel realization and receiver knowledge is as listed. The shaded block shows the new result presented in this paper, the other results where shown in [7].

Note the effect of knowledge of the path delays which enables the removal of a factor of $\log W$ from the rate of increase of the number of paths that permits the system to achieve the channel capacity. Thus, DSSS with delay knowledge at the receiver achieves the channel capacity in the limit if $\frac{L}{W} \to 0$, but without this knowledge the condition acquires a $\log W$ factor and becomes $\frac{L \log W}{W} \to 0$. For PPM, the effect of knowledge of the path delays is similar, relaxing the conditions for achieving capacity from $\frac{L}{logW} \to 0$ with knowledge of the delays to $L \sim \text{const}$ without this knowledge. The underlying reasoning is that with L IID paths, the amount of information needed to describe the channel path delays is $L \log W$. This information cannot be applied to the system's achievable rate for communication. Our new result extends to more complex channels than the model of [7]; herein we consider both uniform delay profiles as well as the more realistic exponential delay profiles.

An information theoretic perspective is shown in

$$I(\underline{X};\underline{Y}) = I(\underline{X},\underline{D};\underline{Y}) - I(\underline{D};\underline{Y}|\underline{X}) \qquad (1)$$

where $\underline{X}$ is the transmitted signal, $\underline{Y}$ is the received signal and $\underline{D}$ holds the multi-path delays. The first term on the right of (1) describes the mutual information of an artificial system which can optimize both the source signal $\underline{X}$ and the path delays $\underline{D}$ of a multipath, additive white Gaussian noise channel. This can be viewed as a 'multi-position' PPM system where each symbol corresponds to L non-zero pulses versus a single non-zero pulse in a standard PPM system.

The second term on the right is the mutual information between the input and output of a system which optimizes the channel path delays alone conditioned on the source. This is another 'multi-position' PPM system, with L pulses per symbol. However, in this case, the number of possible symbols is smaller than in the first case due to the conditioning of the source. The original input signal $\underline{X}$ serves as a delay to the entire multi-path profile, that is known both to the transmitter and the receiver. The difference in (1) is between the rates of two similar systems, the second has a lower rate because of the lower cardinality of the effective alphabet of the source. We are interested in this difference, and in particular in its behavior as the bandwidth increases. Our result essentially shows that in the limit of large bandwidth and large L, $I\left(\underline{X};\underline{Y}\right) \to 0$ and communication is not possible. The limiting rates of the two multi-position systems represented in the right hand side of (1) are identical.

This paper is organized as follows: Section II describes the transmitted signal, channel model and received signal and the main results are provided in Section III.

II. Signal Model

A. The Transmitted Signal

We consider pulse position modulation (PPM), where one transmitted symbol can be written as

$$\begin{aligned} x(t) &= p\left(t - \frac{1}{W}b[n]\right) \qquad t \in [0, T_s) \\ p(t) &= \begin{cases} \sqrt{\frac{\mathcal{E}}{\theta}} & t \in \left[0, \frac{T_s}{N}\right) \\ 0 & \text{else} \end{cases} \end{aligned}$$

The symbol duration is given by T_s and the number of pulse positions is dictated by the transmission bandwidth W, *i.e.* $N = WT_s$. The data symbol is denoted $b[n] \in \{0, 1, \cdots, N-1\}$. $\mathcal{E}$ is the average transmitted energy per symbol that is bandwidth independent (thus an average power constraint is maintained). Thus, in a symbol duration, there is a single rectangular pulse of duration $\frac{T_s}{N}$. The symbol θ represents a flash (duty cycle) parameter to be explained shortly. Our goal is to investigate performance of such a PPM system as the transmission bandwidth increases. We shall assume that the symbol duration does not diminish. With the use of flash signaling, transmission is bursty and communication occurs over a fraction θ of the total communication period. The *flash parameter* θ is known at the receiver and furthermore, the receiver is aware of the on-periods of communication.

Note the distinction between flashy transmission and PPM modulation. For regular data transmission, the receiver must detect which one of the $N = WT_s$ pulse positions has been employed in each symbol; in contrast, with flashy transmission, the receiver is synchronized to the on-periods of communication. We note that if θ is small, then the transmitter is predominantly silent.

The fraction of time utilized for transmission may decrease as the bandwidth W increases, but it cannot do so too fast. In order to maintain a positive (non-diminishing) data-rate, the parameter θ must be large enough so that $\theta \log W$ does not diminish. The reasoning for this is straightforward: up to $\log_2 WT_s$ bits are transmitted per symbol, but only a fraction of the possible symbol periods are employed and thus the data rate is proportional to $\theta \log_2 WT_s$. The requirement on θ can be written as:

$$\theta \geq \frac{k_1}{\log\left(Wk_2\right)} \qquad (2)$$

with fixed $k_1,\ k_2$ that are independent of the bandwidth.

Several features of our setup should be underscored. The first is that there is no limit imposed on the number of PPM positions that are employed for data signaling. Thus, a guard time can be implemented by limiting the positions employed. Second, we emphasize the employment of a lower bounded symbol time, where the lower bound does not depend on the signal bandwidth. We do not consider schemes where the symbol time diminishes with bandwidth. Thus, the number of bits that can be transmitted in a single coherence period (that is equivalent to the spectral efficiency) depends logarithmically on the bandwidth. Note that systems that use a guard period between symbols, that depends on the channel's delay spread, have a natural lower bound on their symbol time.

B. The Channel and Received Signal

We assume a tapped delay line model for the channel $h(t)$, thus

$$h(t) = \sum_{l=1}^{L} g_l \delta\left(t - \frac{d_l}{W}\right)$$

where the channel amplitudes are given by g_l, $\delta(\cdot)$ denotes the Dirac delta function and $\{d_l\}$ represent the path delays which are assumed non-negative integers between 1 and M.

The maximal possible number of resolvable paths is given by $M = WT_d$, where T_d represents the delay spread of the channel. Thus the actual number of paths L must satisfy

$L \leq M$. Recent wideband channel propagation measurements suggest that the number of channel paths grows with bandwidth, an almost linear rate is reported by [9], and a sub-linear rate is observed in [8], [7]. Given M possible values of the path delays, we assume that the realizations of the path delays are uniformly distributed over $\binom{M}{L} = \frac{M!}{L!(M-L)!}$ possibilities. The channel model is of the block-type: the channel is fixed over the channel coherence time T_c; channel realizations at different coherence periods are statistically independent. Channel variation over time is immaterial to our result, as long as the average power per channel realization is finite.

We assume an average power constraint:

$$\mathbf{E}\left[\sum_{l=1}^{L} g_l^2\right] = 1 \tag{3}$$

and consider four different models for the path amplitudes:

(ud): A uniform (u) average power delay profile, with equal and deterministic (d) path amplitudes

$$g_l^{(ud)} = \frac{1}{\sqrt{L}} \qquad l = 1, \ldots, L \tag{4}$$

(ur): A uniform (u) average power delay profile, with random (r) IID Gaussian path amplitudes

$$g_l^{(ur)} \sim \mathcal{N}\left(0, \frac{1}{L}\right) \qquad l = 1, \ldots, L \tag{5}$$

(ed): An exponential (e) average power delay profile, with deterministic (d) path amplitudes, that depend on the delay

$$g_l^{(ed)} = \frac{F^{(ed)}}{\sqrt{L}} e^{-\frac{d_l}{W\tau}} \qquad l = 1, \ldots, L \tag{6}$$

where $F^{(ed)} = \sqrt{M \frac{1-e^{-2/W\tau}}{1-e^{-2T_d/\tau}}} e^{1/W\tau}$ is a constant that maintains the normalization (3).

(er): An exponential (e) average power delay profile, with random (r) Gaussian path amplitudes

$$g_l^{(er)} \sim \mathcal{N}\left(0, {\sigma_i^{(er)}}^2\right) \quad \text{with } {\sigma_i^{(er)}}^2 = \frac{{F^{(er)}}^2}{L} e^{-\frac{2d_l}{W\tau}} \tag{7}$$

where $F^{(er)}$ is a normalization constant equivalent to $F^{(ed)}$ above.

The notation $^{(ud)}$, $^{(ur)}$, $^{(ed)}$ and $^{(ur)}$ is used throughout the paper to denote the four channel models.

The received signal is given by,

$$y(t) \;=\; h(t) \otimes x(t) + z(t) = \sum_{l=1}^{L} g_l x\left(t - \frac{d_l}{W}\right) + z(t),$$

where $z(t)$ is a zero-mean, white Gaussian noise process.

At the receiver, the received signal is matched filtered with the pulse shape and sampled at $\frac{1}{W}$ yielding the following discrete time equivalent signal:

$$\begin{aligned} Y_i &= \sum_{l=1}^{L} g_l X_{i-d_l} + Z_i \qquad (8) \\ X_i &= \begin{cases} \sqrt{\frac{\mathcal{E}}{\theta}} & \text{if } \exists n : i \div N = n \\ & \text{and } i \bmod N = b[n] \\ 0 & \text{else} \end{cases} \end{aligned}$$

$i \div N$ signifies the largest integer κ such that $\kappa N \leq i$. The signal X_i is zero-valued except at the positions corresponding to the transmitted PPM pulse; recall that $N = WT_s$ is the number of positions within a symbol. The noise samples $\{Z_i\}$ are zero-mean with unit variance, and the average signal to noise ratio is given by $\mathcal{E}$.

In order to assess the challenges of synchronization of PPM in multi-path, we analyze a further simplified system that operates under two additional conditions. We assume a sufficiently large guard time that ensures that *no inter-symbol interference* takes place. And we assume *knowledge of the PPM symbols* at the receiver, essentially assuming training information. The receiver sums over all the symbols per coherence period before it begins processing. Given that we show a failure to synchronize for an optimal detector under these idealized conditions, we effectively make statements about more practical systems as well.

The statistics of the signal positions and the noise positions are Gaussian and given by,

$$\begin{aligned} Y_i^{(ud)}|\text{signal location} &\sim \mathcal{N}\left(\sqrt{\frac{\mathcal{E}}{\theta L}}, 1\right) && (9) \\ Y_i^{(ur)}|\text{signal location} &\sim \mathcal{N}\left(0, \frac{\mathcal{E}}{\theta L} + 1\right) && (10) \\ Y_i^{(ed)}|\text{signal location} &\sim \mathcal{N}\left(\sqrt{\frac{\mathcal{E}}{\theta}} g_i^{(de)}, 1\right) && (11) \\ Y_i^{(er)}|\text{signal location} &\sim \mathcal{N}\left(0, \frac{\mathcal{E}}{\theta} {\sigma_i^{(er)}}^2 + 1\right) && (12) \\ Y_i|\text{noise position} &\sim \mathcal{N}(0, 1) \end{aligned}$$

Given that we know the PPM symbol, the observation vector is of length M and of the M possible positions, L correspond to the transmitted signal. The remaining $M - L$ positions correspond to noise.

III. Maximum Likelihood Synchronization

Recall that the position of the PPM symbol is known; however, the multi-path initial delay and multi-path profile are unknown. The synchronization problem can be posed as a multiple hypothesis testing problem for which there are $\binom{M}{L}$ hypotheses with L non-zero channel taps out of $M = WT_d$ possible positions. The received signal under each hypothesis can be written as

$$\underline{Y}^{()}|H_i = \sqrt{\frac{\mathcal{E}}{\theta}}\underline{s}^{()} + \underline{Z} \quad (13)$$

$$\underline{s}^{()} = \left[\underbrace{0,\cdots,g_1^{()},0,\cdots,g_2^{()},\cdots,g_L^{()},\cdots,0}_{L \text{ gains at corresponding delays}}\right]^T \quad (14)$$

$$\underline{Z} \sim \mathcal{N}(\underline{0},\mathbf{I}) \quad (15)$$

To facilitate the description of the optimal, maximum likelihood (ML) detector we define the multipath location vector, that is, for a particular mulitpath profile vector $\underline{s}_i^{()}$, we have

$$\underline{p}_i = \left[\underbrace{0,\cdots,1,\cdots,1,\cdots,1}_{L \text{ ones at corresponding delays}}\right]^T \quad (16)$$

$$= \text{sign}\left(\underline{s}_i^{()}\right) \quad (17)$$

Due to the fact that the signal vectors for the (ud) case are equal energy and each non-zero path has equal gain, the maximul likelihood (ud) detector can be simplified to,

$$\hat{i}^{(ud)} = \arg\max_i \underline{p}_i^T \underline{Y}^{(ud)} \quad (18)$$

For the uniform random (ur) path scenario, the ML detectors are energy detectors, that is

$$\hat{i}^{(ur)} = \arg\max_i \underline{p}_i^T \left|\underline{Y}^{(ur)}\right| \quad (19)$$

where $|\underline{x}|$ is a vector whose components correspond to the absolute values of the components of the vector $\underline{x}$. The maximum likelihood detector in the uniform PDP case is equivalent to determining the multi-path locations by selecting the L positions with the L largest signal values for the deterministic case, or the L largest absolute values for the random case. With this perspective of the maximum likelihood detector, we can develop a method for evaluating the likelihood of an error through order statistics. In order to upper-bound the detection performance in the exponential PDP case, we analyze a uniform PDP, where the path gains are equivalent to the strongest path gain for the deterministic case or are IID paths with energy equivalent to that of the strongest path gain for the (ur) scenario.

We present the main theorem of the work, that applies to the (case) channels. Similar theorems apply to the (ud), (ur) and (ed) cases, they are omitted due to limited space. The proof is also omitted.

Theorem 1: Consider M independent, Gaussian random variables with the following distributions:

$$Y_i^{(er)} \sim \mathcal{N}\left(0, k\frac{M\log M\left(1-e^{-2/M\ T_d/\tau}\right)}{L}+1\right) \quad (20)$$
$$i = 1,2,\cdots,L$$

$$W_i \sim \mathcal{N}(0,1),\ i = L+1, L+2,\cdots,M \quad (21)$$

k is a constant that does not depend on L or M; $L \leq M$. We mark the largest of the $\left\{Y_i^{(er)}\right\}$ by $B_{1:L}^{(er)} = \max Y_i^{(er)}$. The $\{W_i\}$ are ordered so that $S_{1:M-L} = \max W_i$ and $S_{M-L:M-L} = \min W_i$. Then,

$$\lim_{M,L\to\infty} P\left[S_{L:M-L} > B_{1:L}^{(er)}\right] = 1 \qquad \text{if } \frac{L}{\sqrt{M}} \to 0 \quad (22)$$

A. Discussion of the Result

Theorem 1 shows that even with very modest growth rates on the number of multipath with respect to bandwidth, the maximum likelihood detector fails to detect any of the positions corresponding to a non-zero path. Thus, the maximum likelihood detector will always detect noise variables and none of the correct paths will be detected in the limit if $\frac{L}{\sqrt{M}} \to 0$ as $L, M \to \infty$. This statement holds irrespective of whether we consider deterministic or random path gains; uniform or exponential delay profiles.

We conjecture that a similar result holds for any $L \to \infty$, as it is more difficult for the communication system to handle a large number of paths, where each is weak.

Acknowledgment

The authors thank Alex Samorodnitsky for his participation and help on statistical matters.

References

[1] Sandeep Aedudodla, Saravanan Vijayakumaran, and Tan F. Wong. Timing acquisition in ultra-wideband communication systems, March 2005. Draft available at http://www.wireless.ece.ufledu/twong/Preprints/uwbacqvt.pdf.

[2] J. L. Richards et al. System for fast lock and acquisition of ultra-wideband signals, April 2003. US Patent 6556621.

[3] Sinan Gezici, Eran Fishler, Hisashi Kobayashi, and H. Vincent Poor. A rapid acquisition technique for impulse radio. In *Pacific Rim Conference on Communications, Computers and Signal Processing (PACRIM)*. IEEE, August 2003.

[4] E. A. Homier and R. A. Scholtz. Rapid acquisition of ultra-wideband signals in the dense multipath channel. In *Conference on Ultra Wideband Systems and Technologies*. IEEE, 2002.

[5] W. M. Lovelace and J. K. Townsend. The effects of timming jitter on the performance of impulse radio. *IEEE JSAC*, 20(9):1646 – 1651, Dec. 2002.

[6] D. Porrat and U. Mitra. On Synchronization of Wideband Impulsive Systems in Multipath. In *Proc. IEEE ISIT 2005*, September 2005.

[7] Dana Porrat, David Tse, and Serban Nacu. Channel uncertainty in ultra wideband communication systems. In Preparation, available at http://wireless.stanford.edu/ dporrat/ChannelUncertainty.pdf.

[8] Rachid Saadane, Aawatif Menouni, Raymond Knopp, and Driss Aboutajdine. Empirical eigenanalysis of indoor uwb propagation channels. *Proc. IEEE Globecom*, 5:3215–3219, October 2004.

[9] Ulrich Schuster and Helmut Bölcskei. Ultra-wideband channel modeling on the basis of information-theoretic criteria. *IEEE Journal on Selected Areas in Communications*, 2005. Submitted.

[10] Z. Tian and G.B. Giannakis. BER sensitivity to mis-timing in correlation-based UWB receivers. *Proc. IEEE Globecom*, 2:441–445, Dec. 2003. San Francisco, US.

[11] S. Vijayakumaran, T. F. Wong, and S. Aedudodla. On the Asymptotic Performance of Threshold-based Acquisition Systems in Multipath Fading Channels. In *Proc. IEEE Information Theory Workshop*, October 2004.

[12] Liuquing Yang. Timing ppm-uwb signals in ad hoc multi-access. *IEEE Journal on Selected Areas in Communications*, 2006. To Appear.