QUANTUM COMPUTATION - EXERCISE 3

(1) Define two types of unitary operations - a translation T_l, and a phase rotation P_l:

$$T_l \left| x \right\rangle = \left| (x + l) \mod N \right\rangle ,$$
$$P_l \left| x \right\rangle = w^{xl} \left| x \right\rangle .$$

Show that

$$FT_N \cdot T_l \cdot FT_N^{-1} = P_l.$$

(2) Consider two functions, f and h, from the Abelian group modulo N to the complex numbers. Define a third function g, called the *convolution* of f and h, and denoted $g = f * h$, by

$$g(x) = f * h(x) = \sum_{y=0}^{N-1} f(y)h(x - y).$$

Denote by $\hat{f}$ the Fourier transform of f, i.e. $\hat{f} = FT_N \cdot f$. Prove that for every x it holds that

$$(f \hat{*} h)(x) = \hat{f}(x) \cdot \hat{h}(x).$$

(3) let a, b such that $a|q$ and $b|q$ and $\gcd(a, b) = 1$. Let $S = \{0 \le x < q \text{ s.t. } a|x \text{ or } b|x\}$, and define the state

$$\left| \phi \right\rangle = \frac{1}{\sqrt{|S|}} \sum_{x \in S} \left| x \right\rangle .$$

What is the Fourier transform modulo q of $\left| \phi \right\rangle$?

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