QUANTUM COMPUTATION - FALL 2006 - EXERCISE 1

Due: to be published (submit to Ido's mailbox in Ross).

1. Bell Basis

The Bell states are:

$$\begin{aligned}
\left|\Phi^+\right\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\
\left|\Phi^-\right\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\
\left|\Psi^+\right\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\
\left|\Psi^-\right\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)
\end{aligned}$$

1.1. Show that the Bell states form an orthonormal basis to the Hilbert space of two qubits.

1.2. Choose (any) one of the Bell states, and show that it is *entangled* (i.e. it cannot be written in the form of a tensor product of two one-qubit states).

2. Order Of Measurements

2.1. Give an example of a quantum state of two qubits, such that when measuring the left qubit in the $\{|+\rangle, |-\rangle\}$ basis the probability to get $|+\rangle$ is $\frac{3}{4}$, and when measuring the right qubit the probability to get $|+\rangle$ is $\frac{1}{2}$. Express your state in both the standard basis and the $\{|+\rangle, |-\rangle\}$ basis.

2.2. Give an example of an orthonormal basis to the Hilbert space of one qubit which is neither the standard basis, nor the $\{|+\rangle, |-\rangle\}$ basis.

2.3. Show that the probability distribution induced by measuring the left qubit of your state in your basis, and then the right qubit of your state in the standard basis, is the same as when you first measure the right qubit in the standard basis, and then the left qubit in your basis.

3. Optimal Measurements

You are given one of two states, $|0\rangle$ or $|+\rangle$, with equal probabilities, $\frac{1}{2}$. You are allowed to perform a measurement, and then guess, according to the measurement outcome, which state it was. Your goal is to maximize your probability of success, by choosing the basis in which to measure the state wisely. Try to find the best basis for this purpose (do the best you can- no need to prove you found the optimal basis). What is your success probability with the basis you chose?