Proving Incompleteness

- {NAND} is a complete system
- Is {XOR,0} a complete system?

Intuition

X	Y	XOR(X,Y)
0	0	0
0	1	1
1	0	1
1	1	0

X'Y + Y'X

Intuition

X	Y	XOR(X,Y)
0	0	0
0	1	1
1	0	1
1	1	0

xor(X,Y) = xor(Y,X)

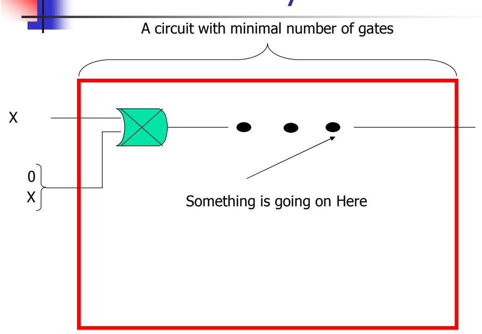
Intuition

- xor(X,Y) = xor(Y,X)
- xor(x,x) = 0
- xor(x,0) = x

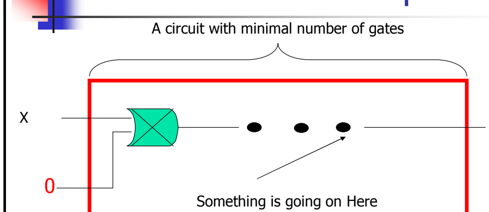
X	Y	XOR(X,Y)
0	0	0
0	1	1
1	0	1
1	1	0

A **single layer** circuit that includes {XOR,0} cannot produce the gate not(X)

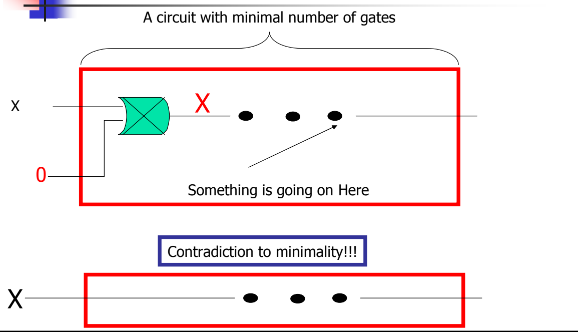
Proof for n-layered circuit



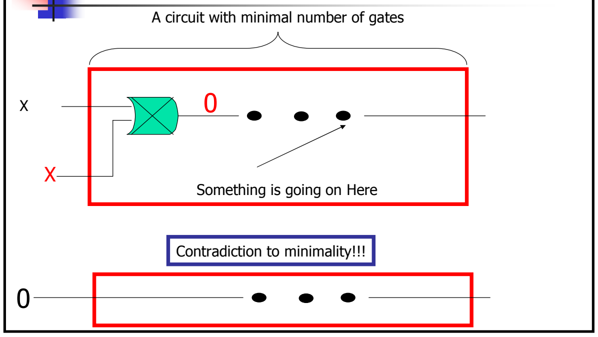
Proof for n-layered circuit Case A: the other input is 0



Proof for n-layered circuit
Case A: the other input is 0



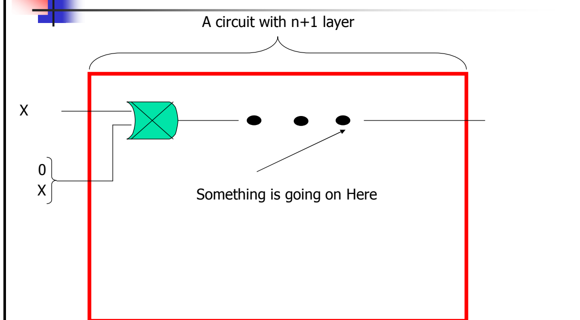
Proof for n-layered circuit
Case B: the other input is X



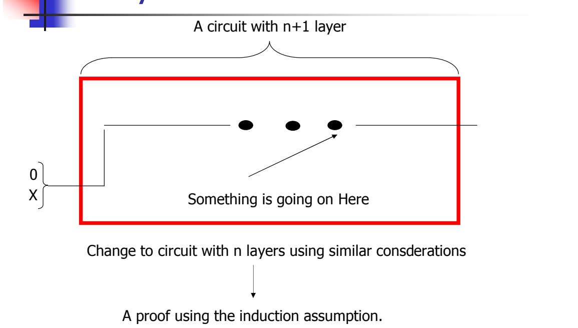
Proof for n-layered circuit (II)
Proof in induction

- For circuit with 1 layer we already prooved.
- Induction assumption:
  - There is not circuit with n layers that can produce not with xor and 0.
- Proof that there is no circuit with n+1 layers that implements not with xor.

Proof in induction for
n-layered circuit



Proof in induction for
n-layered circuit



Minimizing to sum of products
and product of sums

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

→ How to write in minimal form?

When do we minimize?

ABC + ABC′ = AB(C+C′) = AB

When there are two terms that differ in only one literal!!

Minimizing to sum of products

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

= X′Y′Z + X′YZ′ + XY′Z′ + XYZ

Nothing to minimize!

Minimizing to product of sums

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

F′ = X′Y′Z′ + X′YZ+XY′Z + XYZ′

Nothing to minimize!

The table Method: Example

Minimize :

F = w′x′y′z′ + w′x′y′z + w′x′yz′ + wx′y′z′ +
wx′yz′ + wx′yz + wxyz′ + wxyz

Very difficult!!

The table method for minimizing

ABC + ABC′

ABC + AB′C′

The table method for minimizing

ABC + ABC′

111 110

ABC + AB′C′

11 1 1 0 0

The table method for minimizing

AB C + ABC′
↓↓↓ ↓↓↓
111 110

7 - 6 = 1

AB C + AB′C′
↓↓↓ ↓↓↓
11 1 1 0 0

7 - 4 = 3

The table method for minimizing

ABC + ABC′
↓↓↓ ↓↓↓
111 110

7 - 6 = 1
2^0

AB C + AB′C′
↓↓↓ ↓↓↓
11 1 1 0 0

7 - 4 = 3

We can minimize only if the difference is **a power of 2**

The table method for minimizing

ABC + ABC′
↓↓↓ ↓↓↓
111 110

7 - 6 = 1
2^0

AB C + AB′C′
↓↓↓ ↓↓↓
11 1 1 0 0

7 - 4 = 3

We can minimize only if the difference is **a power of 2**

IS IT SUFFICIENT? No

The table method for minimizing

AB′C + A′BC
↓↓↓ ↓ ↓↓
101 0 11

5 - 3 = 2
2^1

The table method for minimizing

AB′C + A′BC
↓↓↓ ↓ ↓↓
101 0 11

5 - 3 = 2
2^2

We can minimize only if the difference is **a power of 2** and **the number of 1 is different!**

The table Method: Example

Minimize :

F = w′x′y′z′ + w′x′y′z + w′x′yz′ + wx′y′z′ + wx′yz′ + wx′yz + wxyz′ + wxyz

= Σ(0,1,2,8,10,11,14,15)

The table method

	w x y z
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
8	1 0 0 0
10	1 0 1 0
11	1 0 1 1
14	1 1 1 0
15	1 1 1 1

The table method

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
10	1 0 1 0	✓
11	1 0 1 1	✓
14	1 1 1 0	✓
15	1 1 1 1	✓

→

	w x y z
0,1	0 0 0 -
0,2	0 0 – 0
0,8	- 0 0 0
2,10	- 0 1 0
8,10	1 0 - 0
10,11	1 0 1 -
10,14	1 - 1 0
11,15	1 - 1 1
14,15	1 1 1 -

The table method

	w x y z	
0,1	0 0 0 -	
0,2	0 0 – 0	✓
0,8	- 0 0 0	✓
2,10	- 0 1 0	✓
8,10	1 0 - 0	✓
10,11	1 0 1 -	✓
10,14	1 - 1 0	✓
11,15	1 - 1 1	✓
14,15	1 1 1 -	✓

	w x y z
0,2,8,10	- 0 – 0
0,8,2,10	- 0 - 0
10,11,14,15	1 - 1 -
10,14,11,15	1 - 1 -

The table method

- The minimal term:

F = w'x'y' + x'z' + wy

The table method - faster

0	✓
1	✓
2	✓
8	✓
10	✓
11	✓
14	✓
15	✓

0,1 (1)	
0,2 (2)	✓
0,8 (8)	✓
2,10 (8)	✓
8,10 (2)	✓
10,11 (1)	✓
10,14 (4)	✓
11,15 (4)	✓
14,15 (1)	✓

0,2,8,10 (2,8)
0,8,2,10 (2,8)
10,11,14,15 (1,4)
10,14,11,15 (1,4)

Choosing Minimal term

F= Σ(1,4,6,7,8,9,10,11,15)

The minimal terms

0000	1	✓
0100	4	✓
1000	8	✓
0110	6	✓
1001	9	✓
1010	10	✓
0111	7	✓
1011	11	✓
1111	15	✓

1,9 (8)	
4,6 (2)	
8,9 (1)	✓
8,10 (2)	✓
6,7(1)	
9,11 (2)	✓
10,11(1)	✓
7,15 (8)	
11,15 (4)	

8,9,10,11 (1,2)
8,9,10,11 (1,2)

The minimal function

F = x′y′z + w′xz′ + w′xy + xyz + wyz + wx′

Is it really the minimum ? No

The minimal function

All the three account for
Minterms 7,15 – maybe we can dispose one of them?

F = x′y′z + w′xz′ + w′xy + xyz + wyz + wx′

Is it really the minimum ? No

Essential Primary Element

		1	4	6	7	8	9	10	11	15
x′y′z	1,9	X					X			
w′xz′	4,6		X	X						
w′xy	6,7			X	X					
Xyz	7,15				X					X
Wyz	11,15								X	X
Wx′	8,9, 10,11					X	X	X	X	

Essential Primary Element

		1	4	6	7	8	9	10	11	15
x′y′z	1,9	X					X			
w′xz′	4,6		X	X						
w′xy	6,7			X	X					
Xyz	7,15				X					X
Wyz	11,15								X	X
Wx′	8,9, 10,11					X	X	X	X	

Essential Primary Element

		1	4	6	7	8	9	10	11	15
x′y′z	1,9	X					X			
w′xz′	4,6		X	X						
w′xy	6,7			X	X					
Xyz	7,15				X					X
Wyz	11,15								X	X
Wx′	8,9, 10,11					X	X	X	X	
		V					V			

Choosing the other Essential Primary Element

		1	4	6	7	8	9	10	11	15
x'y'z	1,9	X					X			
w'xz'	4,6		X	X						
w'xy	6,7			X	X					
Xyz	7,15				X					X
Wyz	11,15								X	X
Wx'	8,9, 10,11					X	X	X	X	
		V	V	V		V	V	V	V	

The minimal function is

F = x'y'z + w'xz' + wx' + xyz