

1 Digital Systems small exercise 2

Submission date: 26/3/2006.

1.1 Question 1 (10%)

Which of the following 6 basic rules

1. Existence of identity (neutral element).
2. Associative
3. Commutative
4. Closure
5. Distributive
6. Existence of a complement that is, if I is the identity of operator $*$, the if $X*Y = I$ then Y is X complement.

does not apply for the operands:

+		0	1	2
0		0	0	0
1		0	1	1
2		0	1	2

.		0	1	2
0		0	1	2
1		1	1	2
2		2	2	2

1.2 Question 2 (20%)

Prove the following equalities using the boolean axioms only:

1. $x \cdot (x + y) = x$
2. $x + (x'y) = x + y$

3. $X' + Y' = X \oplus Y + X'Y'$

Where \oplus is the xor operator:

A	B	\oplus
0	0	0
0	1	1
1	0	1
1	1	0

4. Find a similar expression for $X' + Y' + Z'$

1.3 Question 3 (20%)

Let F_1

x	y	z	F_1
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

1. Express F_1 as a canonical products and canonical sums.
2. Express the function as a sum of products with the minimal number of terms possible.
3. Express the function as a products of sums with the minimal number of products possible.

1.4 Question 4 - (50%)

Implement the following functions using NAND (and followed by not) alone. Assume that you can use the literal, its complement (its negation), and the values 0,1. $Nand(a,b,c)$ has the following formula $(a*b*c)'$.

1. $BD + BCD + AB'C'D' + A'B'CD'$ Use up to 6 gates with 3 entries each

2. $(AB + A'B')(CD' + C'D)$ Use gates with 2 entries each.