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On the Epipolar Geometry of the Crossed-Slits Projection

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Abstract

The Crossed-Slits (X-Slits) camera is defined by two non-intersecting slits, which replace the pinhole in the common perspective camera. Each point in space is projected to the image plane by a ray which passes through the point and the two slits. The X-Slits projection model includes the pushbroom camera as a special case. In addition, it describes a certain class of panoramic images, which are generated from sequences obtained by translating pinhole cameras.

In this paper we develop the epipolar geometry of the X-Slits projection model. We show an object which is similar to the fundamental matrix; our matrix, however, describes a quadratic relation between corresponding image points (using the Veronese mapping). Similarly the equivalent of epipolar lines are conics in the image plane. Unlike the pinhole case, epipolar surfaces do not usually exist in the sense that matching epipolar lines lie on a single surface; we analyze the cases when epipolar surfaces exist, and characterize their properties. Finally, we demonstrate the matching of points in pairs of X-Slits panoramic images.

1 Introduction

People can obtain a very vivid sense of three dimensions by fusing two images of a scene, taken from slightly different angles; we call this phenomenon "stereo vision". It has been observed long ago that the images do not have to adhere to the precise geometry of the eyes in order to elucidate a vivid sense of 3-D; in fact, very crude image pairs will do, as Julesz' random dot stereograms clearly demonstrate [3].

Recently, a few techniques for the generation of stereo pairs of panoramic images have been proposed [6, 9, 10, 4, 11, 2]. A prominent one is the stereo panorama algorithm [5], which is designed to deal with a sequence obtained by a rotating pinhole camera. This algorithm essentially generates two oblique panoramic pushbroom images of the scene; one panorama is generated by sampling and stitching together a column from the left side of each image (say column 64 from a 255×255 image), while the second panorama is similarly generated by sampling a column from the right side of each image (say column 192). We note that this technique can obtain a single pair of panoramic images from each input sequence of images.

Unrelated to stereo, we have previously described [7] how to generate panoramic images from sequences taken by translating pinhole cameras. Our method samples columns from successive images and then stitches them together, as in traditional mosaicing. The main difference is that we sample different columns from different images according to a simple sampling function (say column 1 from the first image, column 2 from the second, 3 from the third, and so on). Thus our method allows us to generate many panoramic images from a single sequence of input images; in fact, we can generate a video corresponding to the forward motion of a camera, using as input a sequence taken by a sideways moving camera.

The geometry underlying our panoramic images, which we called X-Slits projection, is different from the geometry of the pinhole camera; some of its basic characteristics are summarized in Section 2. We can show that the pushbroom projection is a special case of the X-Slits projection model, and in some sense it is the most distorted limiting case, i.e., the deviation from perspective projection in X-Slits images is maximal in the pushbroom limiting case.

As a natural extension of this work, we can generate a X-slits stereo panorama from an input sequence. We do this by generating two X-Slits panoramic images, each obtained by a slightly different column sampling function as described in Section 3. We show a few examples, and compare them to the results obtained with the "stereo panorama" algorithm [5]. It is worth noting that the main advantage of our method is the ability to generate many panoramic stereo pairs from a single pinhole sequence, which can be used to generate a virtual walkthrough in stereo.

Our main theoretical contribution in this paper is the analysis and characterization of the epipolar geometry in the X-Slits projection, as described in Section 4. Our motivation for doing this is twofold: First, for the purpose of visualization, it may help us understand what makes a stereo pair, which is not perspective, look compelling. Second, understanding the epipolar geometry can aid image correspondence; in particular we would like to be able to match images of different kinds to each other, including X-Slits images, pushbroom images, perspective images with barrel distortion, and ideal perspective images. This can be used for such applications as 3-D reconstruction, image warping, or animation. Previously, the epipolar geometry of the pushbroom camera has been analyzed in [1].

The epipolar geometry of X-Slits cameras resembles the pinhole epipolar geometry in some ways, but has its own unique (and somewhat bizarre) properties. In analogy with the pinhole case, there exists a 6×6 matrix which we call the *fundamental matrix* and denote $\mathbf{F}$. $\mathbf{F}$ gives a relation between the Veronese mappings of corresponding image points p and p', where the Veronese mapping of an image point $p = (x, y, w)$ is defined to be $v(p) = (x^2, xy, xw, y^2, yw, w^2)^T$, and the relation is $v(p)^T \mathbf{F} v(p')$. The fundamental matrix in the X-Slits case therefore describes second order epipolar curves (or conics). Moreover, we can show that the rank of $\mathbf{F}$ is 4; this result can be used to derive constraints on $\mathbf{F}$ to be used during the computation of its components.

The main novel feature of the X-Slits epipolar geometry is the fact that the epipolar conics do not usually match. In the pinhole case, all the image points on one epipolar line correspond to points on a *single* epipolar line in the second image. In the X-Slits case this is not generally the case; typically, each point on a certain epipolar conic in one image will induce a *different* epipolar conic in the second image. There are only two special cases where epipolar conics match each other uniquely in the X-Slits projection: (i) when the two cameras have a common slit, in which case the epipolar geometry is identical to the pinhole case, i.e., all epipolar curves are lines and all epipolar surfaces are planes; (ii) when the slits of the two cameras intersect each other in 4 distinct points, in which case the epipolar surfaces are quadratic but unique.

The rest of this paper is organized as follows: After reviewing the X-Slits projection in Section 2, we describe how to generate X-Slits stereo panoramic pairs in Section 3. In Section 4 we study the X-Slits epipolar geometry, showing all the results summarized above. Examples are given in Section 3 (X-Slits stereo) and Section 5 (X-Slits epipolar geometry).

2 Crossed-Slits Projection

The Crossed-Slits (X-Slits) projection is defined by two lines (“slits”) through which all projection rays must pass, and an image plane on which neither slit lies. The image of a scene point is the intersection with the image plane of the ray passing through it and both slits. Let the slits $\mathbf{l}_1, \mathbf{l}_2$ be given by their dual Plücker matrices $\mathbf{S}_1^*, \mathbf{S}_2^*$.[1] Let $\mathbf{Q}_1,\ \mathbf{Q}_2,\ \mathbf{Q}_3$ denote the Plücker matrices corresponding to the X axis, Y axis, and the line at infinity on the image plane respectively (see Fig. 1). Then the projection of a scene point $\mathbf{p} \in \mathcal{P}^3$ to an image point $p \in \mathcal{P}^2$ is given by:

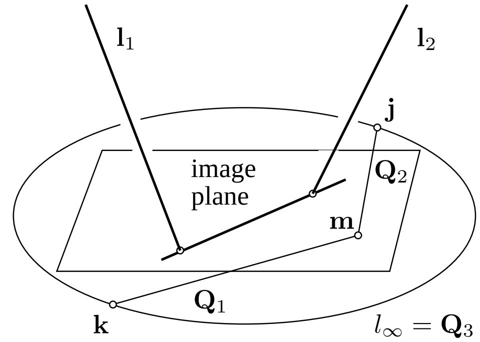


Figure 1: X-Slits projection is defined by two slits $\mathbf{l}_1, \mathbf{l}_2$, and three points $\mathbf{k}, \mathbf{l}, \mathbf{m}$

$$p = \begin{pmatrix} x \\ y \\ w \end{pmatrix} \propto \begin{pmatrix} \mathbf{p}^T \mathbf{S}_1^* \mathbf{Q}_1 \mathbf{S}_2^* \mathbf{p} \\ \mathbf{p}^T \mathbf{S}_1^* \mathbf{Q}_2 \mathbf{S}_2^* \mathbf{p} \\ \mathbf{p}^T \mathbf{S}_1^* \mathbf{Q}_3 \mathbf{S}_2^* \mathbf{p} \end{pmatrix} \equiv \begin{pmatrix} \mathbf{p}^T \mathbf{T}_1 \mathbf{p} \\ \mathbf{p}^T \mathbf{T}_2 \mathbf{p} \\ \mathbf{p}^T \mathbf{T}_3 \mathbf{p} \end{pmatrix} \tag{2.1}$$

Thus the X-Slits projection is a quadratic transformation from $\mathcal{P}^3$ to $\mathcal{P}^2$.

Let $v : \mathcal{P}^3 \to \mathcal{P}^9$ denote the Veronese mapping of degree 3, given for $\mathbf{p} = (p_1, p_2, p_3, p_4)^T$ by

$$v(\mathbf{p}) = (p_1^2, p_1p_2, p_1p_3, p_1p_4, p_2^2, p_2p_3, p_2p_4, p_3^2, p_3p_4, p_4^2)^T$$

Using this notation, the transformation in (2.1) can be concisely written as

$$p \propto \mathbf{A} v(\mathbf{p}) \tag{2.2}$$

where $\mathbf{A}$ is a 3×10 matrix determined by the two slits and the image plane (or, equivalently, the five camera matrices $\mathbf{S}_1^*,\ \mathbf{S}_2^*,\ \mathbf{Q}_1,\ \mathbf{Q}_2,\ \mathbf{Q}_3$).

Let $\mathbf{j}, \mathbf{k}, \mathbf{m} \in \mathcal{P}^3$ denote three points on the image plane such that $\mathbf{m}$ is the origin of the image and $\mathbf{j}, \mathbf{k}$ are points at infinity on the image X and Y axes respectively. By definition it follows that $\mathbf{Q}_1 = \mathbf{k}\mathbf{m}^T - \mathbf{m}\mathbf{k}^T$, $\mathbf{Q}_2 = \mathbf{m}\mathbf{j}^T - \mathbf{j}\mathbf{m}^T$, and $\mathbf{Q}_3 = \mathbf{j}\mathbf{k}^T - \mathbf{k}\mathbf{j}^T$. Let us denote

$$\mathbf{M} = [\mathbf{j}, \mathbf{k}, \mathbf{m}] \tag{2.3}$$

where the columns of $\mathbf{M}$ are $\mathbf{j}, \mathbf{k}, \mathbf{m}$. It now follows that the 3-D representation $\mathbf{p}_1 \in \mathcal{P}^3$ describing image point $p \in \mathcal{P}^2$ is

$$\mathbf{p}_1 = \mathbf{M} p \tag{2.4}$$

[1] The dual Plücker matrix can be defined as $\mathbf{S}_i^* \propto \mathbf{u}_i \mathbf{v}_i^T - \mathbf{v}_i \mathbf{u}_i^T$, where $\mathbf{u}_i, \mathbf{v}_i$ are any 2 planes that intersect in $\mathbf{l}_i$.

The X-Slits projection defined by non-intersecting slits projects general lines onto conic sections. More specifically, given two non-intersecting slits and a line denoted by the Plücker matrix $\mathbf{L}$, that does not coincide with either of the slits, it can be shown that the surface of all projection rays that intersect points on $\mathbf{L}$ is the quadric $\mathbf{S}_1^*\mathbf{L}\mathbf{S}_2^*$, which is a double-ruled surface [8]. The intersection of the quadric with the image plane, which is a conic section, is the projection of the line.

It is interesting to analyze the special case when the two slits are coplanar. As we shall show, this limit case is practically equivalent to the conventional pinhole camera. More specifically, let there be 3 planes $\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2$ defining the slits so that $\mathbf{S}_i^* = \mathbf{u}\mathbf{v}_i^T - \mathbf{v}_i\mathbf{u}^T$ (i.e., both slits lie on plane $\mathbf{u}$). Since $\mathbf{Q}_i$ are anti-symmetric, $\mathbf{u}^T\mathbf{Q}_i\mathbf{u} = 0$, and therefore

$$\mathbf{T}_i = \mathbf{S}_1^*\mathbf{Q}_i\mathbf{S}_2^* = \mathbf{u}\mathbf{v}_1^T\mathbf{Q}_i\mathbf{u}\mathbf{v}_2^T - \mathbf{u}\mathbf{v}_1^T\mathbf{Q}_i\mathbf{v}_2\mathbf{u}^T + \mathbf{v}_1\mathbf{u}^T\mathbf{Q}_i\mathbf{v}_2\mathbf{u}^T$$

It follows that

$$\mathbf{p}^T\mathbf{T}_i\mathbf{p} = \mathbf{p}^T\mathbf{u} \cdot (\mathbf{v}_1^T\mathbf{Q}_i\mathbf{u}\mathbf{v}_2^T\mathbf{p} - \mathbf{v}_1^T\mathbf{Q}_i\mathbf{v}_2\mathbf{u}^T\mathbf{p} + \mathbf{u}^T\mathbf{Q}_i\mathbf{v}_2\mathbf{v}_1^T\mathbf{p})$$

and therefore we obtain from (2.1)

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} \propto \begin{pmatrix} \mathbf{v}_1^T\mathbf{Q}_1\mathbf{u}\mathbf{v}_2^T - \mathbf{v}_1^T\mathbf{Q}_1\mathbf{v}_2\mathbf{u}^T + \mathbf{u}^T\mathbf{Q}_1\mathbf{v}_2\mathbf{v}_1^T \\ \mathbf{v}_1^T\mathbf{Q}_2\mathbf{u}\mathbf{v}_2^T - \mathbf{v}_1^T\mathbf{Q}_2\mathbf{v}_2\mathbf{u}^T + \mathbf{u}^T\mathbf{Q}_2\mathbf{v}_2\mathbf{v}_1^T \\ \mathbf{v}_1^T\mathbf{Q}_3\mathbf{u}\mathbf{v}_2^T - \mathbf{v}_1^T\mathbf{Q}_3\mathbf{v}_2\mathbf{u}^T + \mathbf{u}^T\mathbf{Q}_3\mathbf{v}_2\mathbf{v}_1^T \end{pmatrix} \mathbf{p} \quad (2.5)$$

which is a projective transformation. Note that this is true only when $\mathbf{p}^T\mathbf{u} \neq 0$, i.e., when $\mathbf{p}$ does not lie on plane $\mathbf{u}$. We therefore obtain the following result:

Proposition 2.1. *When the slits are coplanar, the X-Slits projection agrees with a perspective or orthographic projection in all points which do not lie on the plane of the slits.*

If $\mathbf{p}$ *does* lie on plane $\mathbf{u}$, then its projection is $(0, 0, 0)^T$, which is not a point. Geometrically its projection is the line where the image plane intersects plane $\mathbf{u}$.

3 X-Slits Stereo

We can generate X-Slits panoramas from "regular" perspective images as follows. The input sequence is assumed to be captured by a pinhole camera translating along a horizontal line in 3-D space in a roughly constant speed, and without changing its orientation or internal calibration. In the simplest case, panorama synthesis is performed as follows:

- From each frame t, sample the vertical column (strip) centered on the horizontal coordinate $s(t)$.
- Paste the strips into a mosaic image, as in [6].

The geometry of the resulting panoramic image is X-Slits, corresponding to a X-Slits camera with one slit parallel to the image vertical axis, and a second slit overlapping the path of the camera. The parameters of the strip sampling function $s(t)$ determine the location of the vertical slit of the virtual camera.

In the simplest case, when the camera is moving sideways in constant speed, we can make the following general observation [7]:

> *Any* linear column sampling function $s(t) = \alpha t + \beta$ produces a valid X-Slits panoramic image.[2]

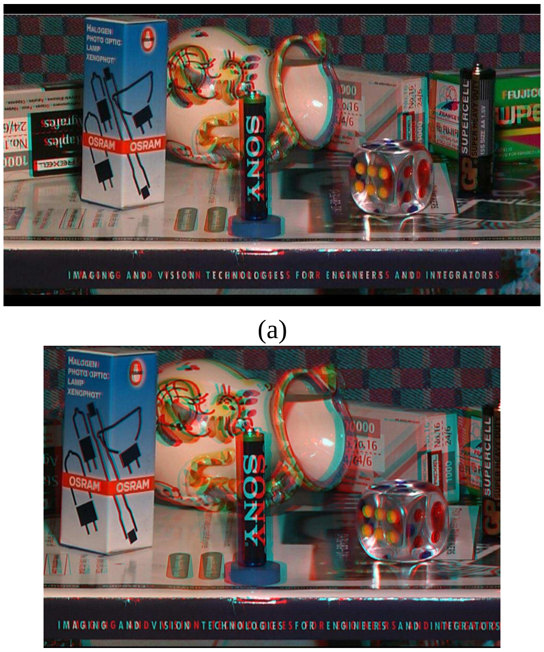
The parameters of the camera's slits are completely defined by α and β. In fact, there is a simple qualitative relation: α determines the "depth" of the vertical slit and β its "horizontal" shift. Thus a stereo pair can be generated qualitatively by using any two linear sampling functions with the same slope, and no additional calibration information is required. Moreover, we note that linear column sampling can be implemented by slicing the space-time volume of images, which can be done efficiently in a variety of ways with existing hardware and/or software.

To see the relation between α, β and the slits' parameters, consider the plane Π defined by the camera's path and optical axis. Let X denote the axis overlapping the camera path on this plane (to be called "horizontal" axis), and Z denote the orthogonal axis (to be called "depth"). One slit of the virtual camera overlaps the X axis, and we call it therefore the "horizontal" slit. Let (X_0, Z_0) denote the intersection of the vertical slit with plane Π. This point is fully determined by the column sampling function $s(t)$, and we can show that $X_0 \propto \beta$ and $Z_0 \propto \alpha$.

In order to generate a stereo pair of panoramas, we generate two X-Slits images with two different sampling functions. Let the first sampling function be $s(t) = \alpha t + \beta$ for some α, β, and let the virtual camera corresponding to the panoramic image generated by this function have its vertical axis going through (X_0, Z_0). We can now generate a second X-Slits image by using the sampling function $s'(t) = \alpha t + \beta + \delta$; the virtual camera corresponding to the panoramic image generated by this function has its vertical axis going through $(X_0 + \Delta, Z_0)$. Thus we have created two images with identical horizontal slit, and with two vertical slits shifted horizontally by Δ one with respect to the other. This is going to be our stereo pair, where the free parameter δ is tuned to allow for comfortable image fusion.

We tested our method on a sequence taken by a sideways moving pinhole camera; examples are shown in Fig. 2. We show a number of stereo pairs, obtained by moving the pair of vertical slits around. For comparison, we also show the stereo pair obtained from the same sequence by the "stereo panorama" algorithm [5].

[2] The case where the camera is not moving sideways is discussed in [7], with some discussion of what to do with varying camera speed and varying camera orientation and internal calibration.



(a)

(b)

Figure 2: Panoramic stereo pairs: (a) X-Slits and (b) pushbroom. See attached material for full resolution images and a walkthrough video sequence in stereo. The images should be viewed in color, using cyan fi lter for the right eye and red fi lter for the left eye.

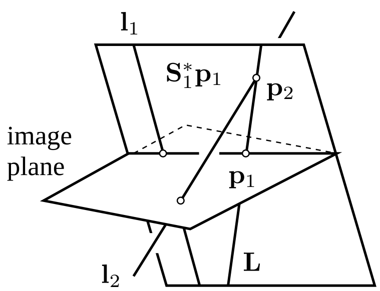


Figure 3: Illustration for the derivation of the fundamental matrix for two X-Slits cameras.

4 Epipolar Geometry

We now study the relation between two X-Slits images, taken by two X-Slits cameras $\mathbf{T}$ and $\mathbf{T}'$.

4.1 Fundamental Matrix

Given a point $p \in \mathcal{P}^2$ on the image of camera $\mathbf{T}$, recall from (2.4) that the 3-D point on the image plane corresponding to p is given by $\mathbf{p}_1 = \mathbf{M}p$, where $\mathbf{M}$ is the 4×3 matrix defined in (2.3). The plane that joins $\mathbf{p}_1$ to slit $\mathbf{l}_1$ is given by $\mathbf{S}_1^*\mathbf{p}_1$, see Fig. 3. This plane intersects $\mathbf{l}_2$ at the point $\mathbf{p}_2 = \mathbf{S}_2\mathbf{S}_1^*\mathbf{p}_1$ (note that $\mathbf{S}_2$ is the Plücker matrix representing $\mathbf{l}_2$ and $\mathbf{S}_1^*$ is the *dual* Plücker matrix representing $\mathbf{l}_1$). The point $\mathbf{p}_2$, which is not seen by $\mathbf{T}$ since it lies on $\mathbf{l}_2$, lies on the ray $\mathbf{L}$ that projects $\mathbf{p}_1$ to p. This ray is given by the Plücker matrix

$$\mathbf{L} = \mathbf{p}_1\mathbf{p}_2^T - \mathbf{p}_2\mathbf{p}_1^T = \mathbf{p}_1\mathbf{p}_1^T\mathbf{S}_1^*\mathbf{S}_2 - \mathbf{S}_2\mathbf{S}_1^*\mathbf{p}_1\mathbf{p}_1^T \quad (4.1)$$

As explained in Section 2, the projection of a line represented by the Plücker matrix $\mathbf{L}$ on $\mathbf{T}'$ is a conic given by the intersection of the image plane with the quadric $\mathbf{S'}_1^*\mathbf{L}\mathbf{S'}_2^*$. It follows from Eq. (2.4) that the projection of the ray through p on the image of camera $\mathbf{T}'$ is given by:

$$\begin{aligned} 0 &= p'^T\mathbf{M}'^T\mathbf{S'}_1^*\mathbf{L}\mathbf{S'}_2^*\mathbf{M}'p' \quad (4.2)\\ &= p'^T\mathbf{M}'^T\mathbf{S'}_1^*(\mathbf{M}pp^T\mathbf{M}^T\mathbf{S}_1^*\mathbf{S}_2 - \mathbf{S}_2\mathbf{S}_1^*\mathbf{M}pp^T\mathbf{M}^T)\mathbf{S'}_2^*\mathbf{M}'p' \end{aligned}$$

This equation defines a bi-quadratic relation between corresponding points p and p' in cameras $\mathbf{T}$ and $\mathbf{T}'$ respectively.

Using the Veronese mapping of degree 2 $v : \mathcal{P}^2 \rightarrow \mathcal{P}^5$, given by

$$v(\mathbf{p}) = (p_1^2, p_1p_2, p_1p_3, p_2^2, p_2p_3, p_3^2)^T$$

(4.2) can be rewritten as

$$v(p')^T\mathbf{F}v(p) = 0 \quad (4.3)$$

where $\mathbf{F}$ is a 6×6 matrix whose components depend on the values of the camera matrices $\mathbf{M}, \mathbf{S}_1^*, \mathbf{S}_2^*$ and $\mathbf{M}', \mathbf{S'}_1^*, \mathbf{S'}_2^*$. For each image point p in $\mathbf{T}$, $\mathbf{F}$ defines the conic on which the image point p' in $\mathbf{T}'$ must lie, and vice versa. We shall refer to such conics as *visibility curves*.

Definition 4.1. *The two conics on which two corresponding image points p, p' must lie, as determined by Eq. (4.3), are called* **visibility curves**.

As will be shown below, these curves play a role similar to the epipolar lines in the perspective model.

In analogy with the pinhole camera, we define the *fundamental matrix* of a pair of X-Slits cameras:

Definition 4.2. *Matrix F in Eq. (4.3) is called the* **fundamental matrix** *of a pair of X-Slits cameras* $\mathbf{T}$ *and* $\mathbf{T}'$.

Clearly, $\mathbf{F}$ always exists. It is similar to the conventional fundamental matrix in the sense that it captures the relative position of two X-Slits cameras, and that it makes it possible to get the visibility curves in one image from points in the other image.

Since $\mathbf{F}$ depends on cameras $\mathbf{T}$ and $\mathbf{T}'$, it is determined by 34 free parameters at most. The real number of free parameters is, however, much smaller. To see this, suppose $\mathbf{A}$ is a 4×4 matrix representing a projective transformation $\mathbf{p} \mapsto \mathbf{Ap}$. It can easily be verified that this transformation, when applied to a Plücker and a dual Plücker matrix, is given by $\mathbf{S} \mapsto \mathbf{ASA}^T$ and $\mathbf{S}^* \mapsto \mathbf{A}^{-T}\mathbf{S}^*\mathbf{A}^{-1}$, respectively. By substituting these mappings into Eq. (4.2) one obtains the same equation, and therefore Eq. (4.2) is independent of the choice of the coordinate system. Note, however, that due to the construction of matrix $\mathbf{M}$ with 2 points on the plane at inifinity, we are only free to choose a 3-D affine transformation to change the coordinate system; this removes 12 degrees of freedom, leading us to the conclusion that $\mathbf{F}$ is fully determined by 22 free parameters.

We can derive $\mathbf{F}$ directly from the camera parameters as follows. Let p and p' denote the corresponding projections of 3-D point $\mathbf{p}$ in the two images. As discussed above, the projection ray of $\mathbf{p}$ in camera $\mathbf{T}$ is defined by the intersection of the two planes $\mathbf{S_1 M}p$ and $\mathbf{S_2 M}p$. It follows that $\mathbf{p}$ must lie on the two planes, namely, $\mathbf{p}^T\mathbf{S_1 M}p = 0$ and $\mathbf{p}^T\mathbf{S_2 M}p = 0$. A similar argument regarding camera $\mathbf{T}'$ allows us to conclude that $\mathbf{p}^T\mathbf{S'_1 M'}p' = 0$ and $\mathbf{p}^T\mathbf{S'_2 M'}p' = 0$.

Let us define the 4×4 matrix $\mathbf{B}$ whose columns are the vector representations of the 4 planes, namely:

$$\mathbf{B} = [\mathbf{S_1 M}p, \mathbf{S_2 M}p, \mathbf{S'_1 M'}p', \mathbf{S'_2 M'}p']$$

Clearly $\mathbf{p}^T \cdot \mathbf{B} = 0$. This implies that the null space of $\mathbf{B}$ is not empty, and thus the determinant of $\mathbf{B}$ must be 0. The equation $det(\mathbf{B}) = 0$ gives us another expression for the bi-quadratic relation between the image points described by the fundamental matrix $\mathbf{F}$. With some algebraic manipulations of $det(\mathbf{B}) = 0$, we can arrive at the following form:

$$0 = det(\mathbf{B}) = v(p)^T \cdot \mathbf{H} \cdot \mathbf{G} \cdot v(p')$$

where $\mathbf{H}$ and $\mathbf{G}$ are two 6×6 matrices which depend each *only* on the cameras $\mathbf{T}$ and $\mathbf{T}'$ respectively.

Let us take a closer look at $v(p)^T \cdot \mathbf{H}$. For any Matrix $\mathbf{X}$, we use the notation $\mathbf{X}^k$ to denote the k-th column of $\mathbf{X}$. By construction we have:

$$\begin{aligned}
v(p)^T\mathbf{H}^1 &= p^T\mathbf{M}^T[(\mathbf{S}_1^1)(\mathbf{S}_2^2)^T - (\mathbf{S}_2^1)(\mathbf{S}_1^2)^T]\mathbf{M}p \\
v(p)^T\mathbf{H}^2 &= p^T\mathbf{M}^T[(\mathbf{S}_1^1)(\mathbf{S}_2^3)^T - (\mathbf{S}_2^1)(\mathbf{S}_1^3)^T]\mathbf{M}p \\
v(p)^T\mathbf{H}^3 &= p^T\mathbf{M}^T[(\mathbf{S}_1^1)(\mathbf{S}_2^4)^T - (\mathbf{S}_2^1)(\mathbf{S}_1^4)^T]\mathbf{M}p \\
v(p)^T\mathbf{H}^4 &= p^T\mathbf{M}^T[(\mathbf{S}_1^2)(\mathbf{S}_2^3)^T - (\mathbf{S}_2^2)(\mathbf{S}_1^3)^T]\mathbf{M}p \\
v(p)^T\mathbf{H}^5 &= p^T\mathbf{M}^T[(\mathbf{S}_1^2)(\mathbf{S}_2^4)^T - (\mathbf{S}_2^2)(\mathbf{S}_1^4)^T]\mathbf{M}p \\
v(p)^T\mathbf{H}^6 &= p^T\mathbf{M}^T[(\mathbf{S}_1^3)(\mathbf{S}_2^4)^T - (\mathbf{S}_2^3)(\mathbf{S}_1^4)^T]\mathbf{M}p
\end{aligned}$$

This defined matrix $\mathbf{H}$; moreover, it can be shown that the rank of $\mathbf{H}$ is at most 4 given that the 4×4 matrices $\mathbf{S}_1$ and $\mathbf{S}_2$ are anti-symmetric and of rank 2. $\mathbf{G}$ is define in a similar way for camera $\mathbf{T}'$, and its rank is therefore also 4. Since the fundamental matrix $\mathbf{F} = \mathbf{H} \cdot \mathbf{G}$, we can conclude the following:

Proposition 4.1. *The rank of the fundamental matrix of the X-Slits projection is 4 at most.*

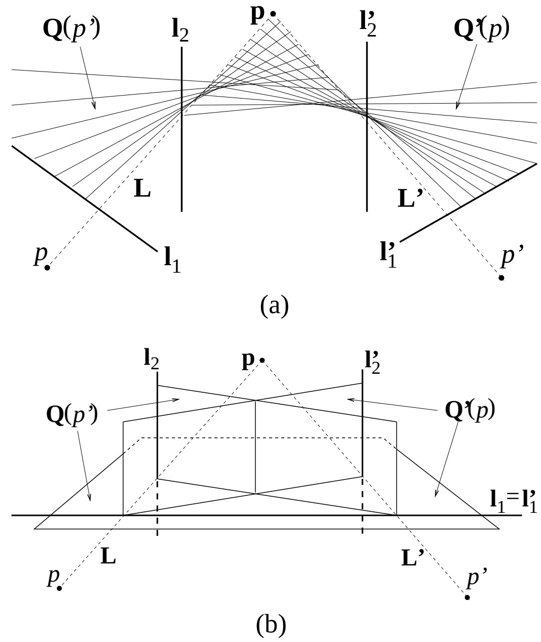


Figure 4: visibility quadrics: (a) $\mathbf{Q}'(p)$ is the collection of projection rays of $\mathbf{T}'$ passing through line $\mathbf{L}$, while $\mathbf{Q}(p')$ is the collection of projection rays of $\mathbf{T}$ passing through $\mathbf{L}'$. (b) When the cameras have a shared slit ($\mathbf{l}_1 = \mathbf{l'}_1$), the visibility quadrics intersect in a plane.

This proposition immediately gives us 4 independent constraints on the elements of $\mathbf{F}$. For example, we can choose four different 5×5 sub-matrices of $\mathbf{F}$, and require that the determinant of each equals 0.

4.2 Visibility Quadrics

Let $\mathbf{L}$ denote the projection ray of camera $\mathbf{T}$ passing through image point p. Let $\mathbf{Q}'(p)$ denote the quadric $\mathbf{S'}_1^*\mathbf{L}\mathbf{S'}_2^*$ where $\mathbf{L}$ is defined in (4.1); thus $\mathbf{Q}'(p)$ is the projection of $\mathbf{L}$ in camera $\mathbf{T}$'. This quadric is a double-ruled surface that is ruled by the family of all rays of camera $\mathbf{T}'$ passing through the line $\mathbf{L}$ (see Fig. 4a). Similarly, let $\mathbf{L}'$ denote the projection ray of camera $\mathbf{T}'$ passing through image point p', and let the quadric $\mathbf{Q}(p') = \mathbf{S}_1^*\mathbf{L}'\mathbf{S}_2^*$ denote the projection of $\mathbf{L}'$ in camera $\mathbf{T}$.

Definition 4.3. *The quadric $\mathbf{Q}'(p)$ (resp. $\mathbf{Q}(p')$) for any image points p (resp. p') is called a visibility quadric.*

Visibility quadrics play a role similar to epipolar planes in the pinhole camera. However, unlike the perspective model, these quadrics are not necessarily symmetric with respect to the two cameras. For a given scene point $\mathbf{p}$ that is projected to p and p' in $\mathbf{T}$ and $\mathbf{T}'$ respectively, the corresponding surfaces of rays of $\mathbf{T}$ and $\mathbf{T}'$ are $\mathbf{Q}'(p)$ and $\mathbf{Q}(p')$ respectively. These quadrics do not usually coincide.

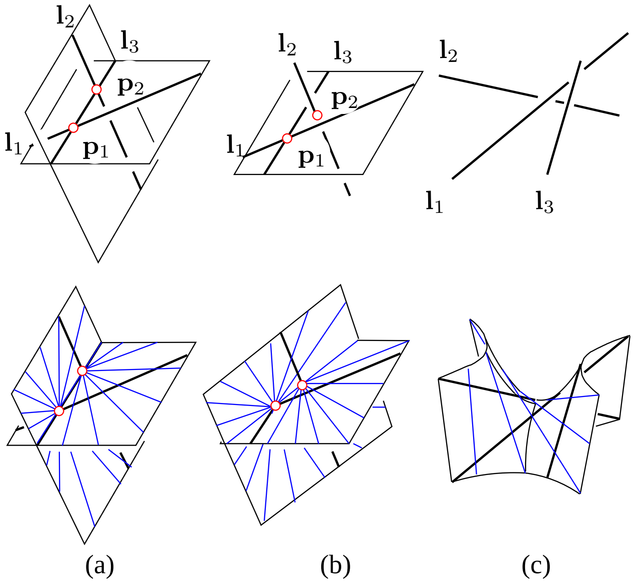


Figure 5: Illustrations for the proof of Lemma 4.2.

When $\mathbf{Q}'(p)$ and $\mathbf{Q}(p')$ do coincide, we refer to this quadric as an *epipolar quadric*. The property of an epipolar quadric is that all points on it are projected to a single conic in each camera, and the corresponding conics can be used for matching in the same way as epipolar lines are used in perspective images. We shall describe next the camera configurations in which this occurs for all scene points; in these cases the *visibility curves* in both cameras can be matched with each other, similarly to epipolar lines in the perspective model. This notion will be made more precise next.

Epipolar Quadrics

In this section we assume that *slits are not visible by their camera*, because otherwise epipolar quadrics are not well defined (the explanation is omitted for lack of space). To make the discussion precise, let us start with a few **definitions**: We say that two *lines intersect* if they have a common point. We say that lines are *disjoint* if they do not intersect. Let L be a set of lines. Denote by $\mathcal{T}(L)$ the set of lines that intersect every line in L and call $\mathcal{T}(L)$ *the transversal* of lines in L. Adopt that a set R of lines is called *regulus* if there are three pairwise disjoint lines $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3$ such that $R = \mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3\})$.

Let $\mathbf{T}$ denote a X-Slits camera with slits $\mathbf{l}_1, \mathbf{l}_2$. We say that line $\mathbf{l}$ is a *projector* of a point $\mathbf{p}$ in camera $\mathbf{T}$ if $\mathbf{l}$ is transversal to slits $\mathbf{l}_1, \mathbf{l}_2$, and $\mathbf{p}$ is in $\mathbf{l} \setminus (\mathbf{l}_1 \cup \mathbf{l}_2)$. A nonempty quadric Q is said to be an *epipolar quadric* of two X-Slits cameras $\mathbf{T}, \mathbf{T}'$ if for every point $\mathbf{q} \in Q$ all projectors of $\mathbf{q}$ in $\mathbf{T}$ and $\mathbf{T}'$ are contained in Q.

Lemma 4.2. *Let $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3$ be three distinct lines, out of which at least two are disjoint. Then transversal $\mathcal{R} = \mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3\})$ is either the union of two planar pencils of lines that have one line in common, or a regulus.*

Proof. Let w.l.o.g. $\mathbf{l}_1, \mathbf{l}_2$ be disjoint. Then one of the following 3 options hold: (1) $\mathbf{l}_3$ intersects both $\mathbf{l}_1, \mathbf{l}_2$, see Fig. 5(a); (2) $\mathbf{l}_3$ intersects only $\mathbf{l}_1$, see Fig. 5(b); (3) $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3$ are pairwise disjoint, see Fig. 5(c).

case 1: Denote $\mathbf{l}_1 \cap \mathbf{l}_3$ by $\mathbf{p}_1$ and $\mathbf{l}_2 \cap \mathbf{l}_3$ by $\mathbf{p}_2$. The set of lines transversal to $\mathbf{l}_1, \mathbf{l}_3$ is the union of the set L_1 of all lines passing through $\mathbf{p}_1$ and the set L_2 of all lines in the plane spanned by $\mathbf{l}_1, \mathbf{l}_3$. The lines in L_1 that intersect $\mathbf{l}_2$ form the pencil of lines with center $\mathbf{p}_1$ in the plane spanned by $\mathbf{l}_2, \mathbf{l}_3$. The lines in L_2 that intersect $\mathbf{l}_2$ form the pencil of lines with center $\mathbf{p}_2$ in the plane spanned by $\mathbf{l}_1, \mathbf{l}_3$. Clearly line $\mathbf{l}_3$ lies in both pencils.

case 2: Denote $\mathbf{l}_1 \cap \mathbf{l}_3$ by $\mathbf{p}_1$. Line $\mathbf{l}_2$ intersects the plane spanned by $\mathbf{l}_1, \mathbf{l}_3$ in a point $\mathbf{p}_2$. Now, the set of lines transversal to $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3$ is the union of two planar pencils of lines that have one line in common by the same argument as in the previous case.

case 3: In this case $\mathcal{R}$ is a regulus, see [8, page 42] for the proof. $\square$

Theorem 4.3. *Let $\mathbf{T}$ (resp. $\mathbf{T}'$) be a X-Slits camera with disjoint slits $\mathbf{l}_1, \mathbf{l}_2$ (resp. disjoint slits $\mathbf{l}'_1, \mathbf{l}'_2$). Then, every point in the set $\mathcal{V}$ of all points that have a projector in both cameras is contained in an epipolar quadric $\iff$ the cameras either share a slit, or slits $\mathbf{l}_1, \mathbf{l}_2$ intersect with slits $\mathbf{l}'_1, \mathbf{l}'_2$ in four pairwise distinct points.*

Proof. ($\Longrightarrow$) One of the following must be true: (1) all projectors of $\mathbf{T}$ intersect at least one of the slits $\mathbf{l}'_1$ or $\mathbf{l}'_2$; (2) there is a projector $\mathbf{l}$ of $\mathbf{T}$ so that $\mathbf{l}, \mathbf{l}'_1, \mathbf{l}'_2$ are pairwise disjoint.

case 1: The set of projectors of $\mathbf{T}$ is the union of two sets of lines

$$A = \{\mathbf{m} \text{ is a projector of } \mathbf{T} |\ \mathbf{m} \cap \mathbf{l}'_1 \neq \emptyset\} = \mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}'_1\})$$
$$B = \{\mathbf{m} \text{ is a projector of } \mathbf{T} |\ \mathbf{m} \cap \mathbf{l}'_2 \neq \emptyset\} = \mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}'_2\})$$

It follows from Lemma 4.2 that transversal $\mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}\})$ for a line $\mathbf{l} \neq \mathbf{l}_1, \mathbf{l}_2$ is either a planar pencil of lines or a regulus. Therefore, from $\mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2\}) = A \cup B = \mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}'_1\}) \cup \mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}'_2\})$, it follows that $\mathbf{l}'_1 \in \{\mathbf{l}_1, \mathbf{l}_2\}$ or $\mathbf{l}'_2 \in \{\mathbf{l}_1, \mathbf{l}_2\}$. This is because $\mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2\})$ is not a surface (but rather a volume), since for every point $\mathbf{p}$ there is a line in $\mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2\})$ passing through $\mathbf{p}$.

case 2: Take $\mathbf{p} \in \mathbf{l} \cap \mathcal{V}$. There is an epipolar quadric Q containing $\mathbf{p}$. Thus $\mathbf{l}$ is in Q and also the regulus $\mathcal{R} = \mathcal{T}(\{\mathbf{l}'_1, \mathbf{l}'_2, \mathbf{l}\})$ is in Q by which Q is a regular double ruled quadric. Either (2.1) there is a line $\mathbf{s} \in \mathcal{R}$ which does *not* intersect both $\mathbf{l}_1$ and $\mathbf{l}_2$, or (2.2) $\mathbf{l}_1, \mathbf{l}_2$ intersect all lines in $\mathcal{R}$.

(case 2.1) Regulus $\mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{s}\})$ is in Q and therefore lines $\mathbf{l}_1, \mathbf{l}_2$ are in Q. Lines $\mathbf{l}_1, \mathbf{l}_2$ are disjoint and consequently are in the same regulus. The same holds for $\mathbf{l}'_1, \mathbf{l}'_2$. Line s intersects $\mathbf{l}'_1, \mathbf{l}'_2$ but does not intersect $\mathbf{l}_1, \mathbf{l}_2$, and thus $\mathbf{l}_1, \mathbf{l}_2$ are in the opposite regulus to the regulus containing $\mathbf{l}'_1, \mathbf{l}'_2$ (in other words, they are in different rulings on the surface). Consequently, slits $\mathbf{l}_1, \mathbf{l}_2$ intersect with slits $\mathbf{l}'_1, \mathbf{l}'_2$ in four pairwise distinct points.

(case 2.2) Lines $\mathbf{l}_1, \mathbf{l}_2$ are in regulus $\mathcal{T}(\mathcal{R})$. Since $\mathbf{l}'_1, \mathbf{l}'_2 \in \mathcal{T}(\mathcal{R})$, all four $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}'_1, \mathbf{l}'_2$ are in the same regulus and are pairwise distinct because $\mathbf{l}$ intersects $\mathbf{l}_1, \mathbf{l}_2$ but does not intersect $\mathbf{l}'_1, \mathbf{l}'_2$. Then, however, no point $\mathbf{q} \in \mathcal{V} \setminus Q$ is contained in an epipolar quadric due to the following argument. Denote by $\mathbf{n}$ (resp. $\mathbf{n}'$) the line from $\mathbf{T}$ (resp. $\mathbf{T}'$) that passes through a point $\mathbf{q} \in \mathcal{V}$. Assume that there is an epipolar quadric Q' containing $\mathbf{q}$. Then both $\mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{n}'\})$ and $\mathcal{T}(\{\mathbf{l}'_1, \mathbf{l}'_2, \mathbf{n}\})$ are in Q', and thus all $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}'_1, \mathbf{l}'_2$ are in Q'. However, now $Q = Q'$ since every four distinct lines from a regulus are exactly in one regulus. Therefore, $\mathbf{q} \in Q$.

($\Longleftarrow$) By the assumption one of the following 3 options hold: (1) the cameras share exactly one slit; (2) the cameras share both slits; (3) the cameras intersect in four distinct points.

case 1: Let w.l.o.g. $\mathbf{l}_1 = \mathbf{l}'_1$. A point $\mathbf{q} \in \mathcal{V}$ is not contained in $\mathbf{l}_1$ and therefore there is exactly one plane π through $\mathbf{l}_1$ and $\mathbf{q}$. Every projector from $\mathbf{T}$ or $\mathbf{T}'$, which contains $\mathbf{q}$, intersects $\mathbf{l}_1$ and is therefore in π; thus π is an epipolar quadric.

case 2: Let w.l.o.g. $\mathbf{l}_1 = \mathbf{l}'_1$ and $\mathbf{l}_2 = \mathbf{l}'_2$. Then for every point $\mathbf{q} \in \mathcal{V}$ there is a projector $\mathbf{l}$ disjoint with both $\mathbf{l}_1$ and $\mathbf{l}_2$. Therefore, every $\mathbf{q}$ is in the regular epipolar quadric that contains regulus $\mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}\})$.

case 3: Every point $\mathbf{q} \in \mathcal{V}$ is contained in exactly one projector $\mathbf{l}$ from $\mathbf{T}$ and in exactly one projector $\mathbf{l}'$ from $\mathbf{T}'$. We assumed that $\mathbf{l}_1, \mathbf{l}_2$ are transversal to $\mathbf{l}'_1, \mathbf{l}'_2$. Line $\mathbf{l}$ (resp. $\mathbf{l}'$) is transversal to $\mathbf{l}_1, \mathbf{l}_2$ (resp. $\mathbf{l}'_1, \mathbf{l}'_2$). Line $\mathbf{l}$ is transversal to $\mathbf{l}'$ since both contain $\mathbf{q}$. Lines $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}'$ (resp. $\mathbf{l}'_1, \mathbf{l}'_2, \mathbf{l}$) are pairwise disjoint. Therefore, $\mathbf{q}$ is contained in a regular epipolar quadric that contains regulus $\mathcal{T}(\{\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}'\})$. □

We can now conclude the following:

Corollary 1. *If two X-Slits cameras share a slit, then every point is contained in an epipolar plane, see Fig. 6(b). Moreover, the epipolar planes form a pencil of planes.*

Vice versa, when the epipolar quadrics are planes in general, the cameras must have one common slit.

Corollary 2. *If the slits intersect in four pairwise distinct points, then every point is contained in a regular epipolar quadric, see Fig. 6(a). Moreover, the epipolar quadrics form a pencil of quadrics.*

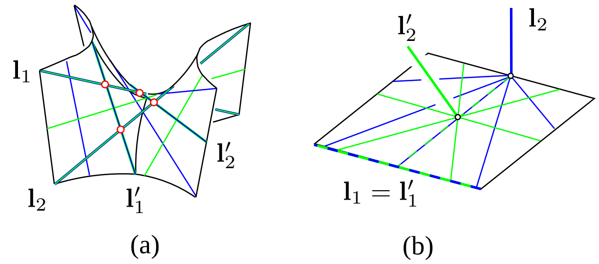


Figure 6: Epipolar quadrics of a pair of X-Slits cameras with nonintersecting slits. (a) The slits intersect in four pairwise disjoint points. (b) The cameras share a slit.

4.3 Visibility Curves

Recall that visibility curves are the 'X-Slits equivalent' of epipolar lines, since lines are projected in X-Slits projection onto conic sections. One property of visibility curves is that they must all intersect two specific points, which are the points where the slits intersect the image plane.

Proposition 4.4. *In camera* $\mathbf{T}$*, denote the image points where slits* $\mathbf{l}_1$ *and* $\mathbf{l}_2$ *intersect with the image plane as* c_1 *and* c_2 *respectively; then all scene lines are projected into conics that pass through* c_1 *and* c_2*.*

Proof. The line $\mathbf{L}$ is projected into the conic given by

$$p^T \mathbf{M}^T \mathbf{S}_1^* \mathbf{L} \mathbf{S}_2^* \mathbf{M} p = 0 \tag{4.4}$$

For $i = 1, 2$, the 3-D point corresponding with c_i is $\mathbf{M}c_i$, and since this point lies on slit $\mathbf{l}_i$, it follows that $\mathbf{S}_i^* \mathbf{M} c_i = 0$. Therefore, (4.4) holds for $p = c_i$, which means c_1, c_2 both lie on the conic which is the projection of the line $\mathbf{L}$. □

Corollary 3. c_1 *and* c_2 *lie on all visibility curves.*

The projection of a ray is the intersection of the image plane with a subset of a quadric double-ruled surface containing the slits of the camera and the ray of the other camera through the scene point, as discussed above and in Section 4.2. When this set is a plane, the visibility curve degenerates into a line. This gives us the following result:

Proposition 4.5. *When two X-Slits cameras share a slit, visibility curves are lines and can be matched, i.e., points on a visibility line of one camera can be matched to points on the corresponding visibility line of the other camera.*

This proposition shows that in the case of a shared slit, there is great similarity to the epipolar geometry of the perspective projection. The following lemma characterizes this similarity:

Lemma 4.6. *For two cameras* $\mathbf{T}$, $\mathbf{T}'$*, if the cameras have a common slit, then each visibility curve is composed of a pair* *of lines, one of which is the projection of a singular point; excluding singular points, the remaining family of lines is the family of lines induced by the perspective fundamental matrix.*

Proof. Since a slit is shared, let us assume w.l.o.g. that $\mathbf{S}_1^* = \mathbf{S'}_1^*$. It can easily be shown that $\mathbf{S}_1^*\mathbf{S}_2\mathbf{S}_1^* = \mu\mathbf{S}_1^*$, for some $\mu \in \mathcal{R}$. Therefore, for each scene point projected to p, p' in $\mathbf{T}$, $\mathbf{T}'$ respectively,

$$\begin{aligned} 0 &= p'^T\mathbf{M}'^T\mathbf{S'}_1^*(\mathbf{M}pp^T\mathbf{M}^T\mathbf{S}_1^*\mathbf{S}_2 - \mathbf{S}_2\mathbf{S}_1^*\mathbf{M}pp^T\mathbf{M}^T)\mathbf{S'}_2^*\mathbf{M}'p' \\ &= p'^T\mathbf{M}'^T\mathbf{S}_1^*(\mathbf{M}pp^T\mathbf{M}^T\mathbf{S}_1^*\mathbf{S}_2 - \mu\mathbf{M}pp^T\mathbf{M}^T)\mathbf{S'}_2^*\mathbf{M}'p' \\ &= p'^T\mathbf{M}'^T\mathbf{S'}_1^*\mathbf{M}pp^T\mathbf{M}^T(\mathbf{S}_1^*\mathbf{S}_2 - \mu I)\mathbf{S'}_2^*\mathbf{M}'p' \\ &\equiv p'^T\mathbf{A}p \cdot p^T\mathbf{B}p' \end{aligned}$$

Since $\mathbf{M}p$ and $\mathbf{M}'p'$ are 3-D points on the image planes of $\mathbf{T}$ and $\mathbf{T}'$ corresponding to p and p' respectively, $p'^T\mathbf{A}p = 0$ if and only if p and p' are coplanar with the common slit.

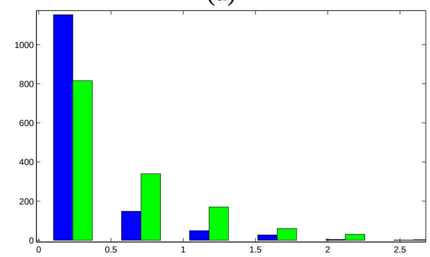
Imagine that instead of the two X-Slits cameras $\mathbf{T}$ and $\mathbf{T}'$ we have two perspective cameras with focal centers on $\mathbf{l}_1$; clearly the same relationship would exist between corresponding image points. This means that the first constraint, denoted as $p'^T\mathbf{A}p$ is equivalent to the constraint on matching points between two perspective cameras that lie on the line $\mathbf{l}_1$.

On the other hand, $(\mathbf{S}_1^*\mathbf{S}_2 - \mu I)\mathbf{S}_1^* = (\mu\mathbf{S}_1^* - \mu\mathbf{S}_1^*) = 0$. Therefore $p^T\mathbf{M}^T(\mathbf{S}_1^*\mathbf{S}_2 - \alpha I)\mathbf{S}_1^* = 0$, which means that $p^T\mathbf{M}^T(\mathbf{S}_1^*\mathbf{S}_2 - \alpha I)$ is a point on slit $\mathbf{l}_1$, and therefore the ~~epipolar~~ visibility line $\mathbf{B}^Tp$ is a projection of a point on a slit. This projection is singular (a point is projected to a line), and therefore we exclude it from the set of ~~feasible~~ points feasible for matching. □

5 Experimental Results

We generated X-Slits images from 3 different sequences of the same scene. In each sequence the input camera moved along a different trajectory, and therefore each of the synthesized panoramic X-Slits images had a different pair of slits. Using 70 manually matched points, we retrieved the X-Slits and perspective fundamental matrices, and produced visibility curves and epipolar lines (see Fig. 7a). Our experiments show that the retrieved visibility curves are closer to their corresponding image points than epipolar lines computed assuming epipolar lines rather than epipolar curves (see Fig. 7b).

(a)



(b)

Figure 7: Experimental results: (a) the visibility curves and epipolar lines retrieved from matching points in a pair of X-Slits images; (b) a histogram of the distances (in pixels) between each image point and the corresponding visibility curve (left columns) and epipolar line (right column).

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