

Physically Based Modeling and Animation of Fire



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Combustion

- Chemical reactions converting fuel into hot gaseous products.
- Two distinct types of phenomena:
 - detonations: high-speed events, such as explosions (shock waves and other compressible effects play an important role).
 - deflagrations: low-speed events, such as fire and flames.

Deflagrations

- Can be modeled using the equations for incompressible flow;
- Viscous effects are small; can use the incompressible inviscid Euler equations.

Momentum $\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mathbf{f}$

Continuity $\nabla \cdot \mathbf{u} = 0$

Physically Based Model

- Three distinct visual phenomena:
 - Blue core
 - Emission from intermediate chemical species produced during the reaction
 - Black-body radiation
 - Emission from the hot gaseous products (at high temperatures)
 - Smoke and soot
 - Becomes visible at lower temperatures.

Physically Based Model

- First step: heating of solid/liquid fuel until it undergoes a phase change to a gaseous state.
- Next, gas heats up until it reaches its ignition temperature (blue core)
- Temperature continues to increase (yellowish-orange black-body radiation), and then decreases

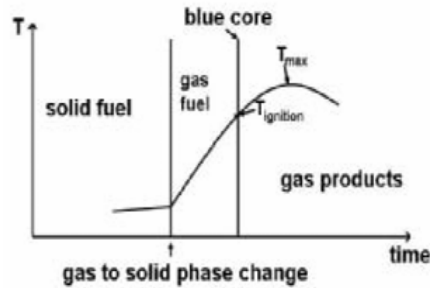


Figure 3: The hot gaseous products and soot emit blackbody radiation that illuminates the smoke.

Blue Core

- Modeled by an implicit surface separating the gaseous fuel from the hot gaseous products and surrounding air.
- The approximate surface area of the blue core is estimated with:

$$v_f A_f = S A_S$$

fuel injected into the core = fuel leaving the core

Blue Core

$$v_f A_f = S A_S$$



- v_f : speed at which fuel is injected
- A_f : injection surface area
- S : speed at which fuel turns into gaseous products (flame speed, e.g. .44m/s for propane)
- A_S : the surface area of the blue core

Blue Core

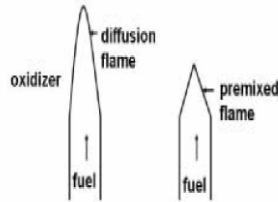


Figure 5: Location of the blue reaction zone core for a premixed flame versus a diffusion (non-premixed) flame

Hot Gaseous Products

- For proper appearance, we must track individual elements of the flow and follow them through their temperature histories.
- Because of expansion, flow trajectories are not straight.

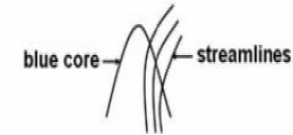


Figure 6: Streamlines illustrating the path of individual fluid elements as they cross the blue reaction zone core. The curved path is caused by the expansion of the gas as it reacts.

Expansion

- ρ_f : fuel density (about 1 kg/m^3 for air)
- ρ_h : gaseous product density
- Expansion factor: ρ_f / ρ_h

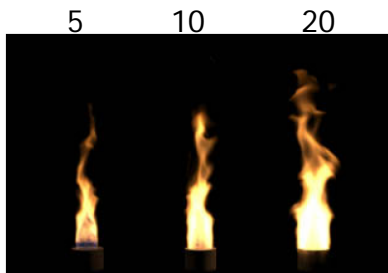


Figure 7: Comparison of flame shapes for differing degrees of gaseous expansion. The amount of expansion increases from left to right making the flame appear fuller and more turbulent.

Flow Equations

- Use two sets of incompressible flow equations:
 - For the fuel
 - For the gaseous products and surrounding airflow.
- Must couple these two sets of equations across the interface in a manner that models expansion, while preserving mass and momentum.

Balance Equations

$$\rho_h(V_h - D) = \rho_f(V_f - D)$$

$$\rho_h(V_h - D)^2 + p_h = \rho_f(V_f - D)^2 + p_f$$

- V_f, V_h : normal velocities of fuel/gaseous products
- p_f, p_h : pressures of fuel/gaseous products
- $D = V_f - S$: the speed of the implicit surface in the normal direction.

Solid Fuels

- Expansion also takes place when solid fuel becomes a gaseous fuel.
- The boundary surface of the solid fuel is used as the reaction front.

$$\rho_f(V_f - D) = \rho_s(V_s - D)$$

$$D = V_s - S$$

$$V_f = V_s + (\rho_s / \rho_f - 1)S$$



Reaction Front Tracking

- The reaction front is modeled as an implicit surface, $\phi(x,y,z) = 0$.
 - $\phi > 0$ inside the gaseous fuel region
 - $\phi < 0$ outside the fuel region
- The implicit surface moves with velocity \mathbf{w} :

$$\mathbf{w} = \mathbf{u}_f + S \mathbf{n}$$
- \mathbf{u}_f is the velocity of the gaseous fuel
- $S\mathbf{n}$ governs the conversion of fuel into gaseous products, \mathbf{n} is the normal to ϕ .

Reaction Front Tracking

- The motion of the implicit surface is defined through

$$\frac{\partial \phi}{\partial t} = -\mathbf{w} \cdot \nabla \phi$$
- and solved at each grid point using

$$\phi^{new} = \phi^{old} - \Delta t (w_1 \phi_x + w_2 \phi_y + w_3 \phi_z)$$

Simulating Two Flows

- Each set of flow equations might need to use velocity values on the other side of the interface. This is resolved by computing "ghost values" using the balance equation.
- Example: ghost velocity for the gaseous products:

$$V_f = \mathbf{u}_f \cdot \mathbf{n}$$

$$V_h^G = V_f + (\rho_f / \rho_h - 1)S$$

$$\mathbf{u}_h^G = V_h^G \mathbf{n} + \mathbf{u}_f - V_f \mathbf{n}$$

Buoyancy

- To simulate buoyancy effects, an external force proportional to the temperature is added into the flow equations:

$$\mathbf{f}_{buoy} = \alpha(T - T_{air})\mathbf{z}$$

- where \mathbf{z} is the upward vector, T_{air} is the air temperature, and α is a buoyancy constant.

Temperature

- In order to assign each flame element a temperature we need to track each element that crosses the blue core boundary.
- Temperature is assigned according to the time that passed since the crossing.
- Tracking is done using another scalar field Y :

$$\frac{\partial Y}{\partial t} = -(\mathbf{u} \cdot \nabla)Y - k$$

- Setting $Y(0)=1$ inside the blue core and $k=1$, the time since the crossing is given by $(1 - Y)$.

Additional Results

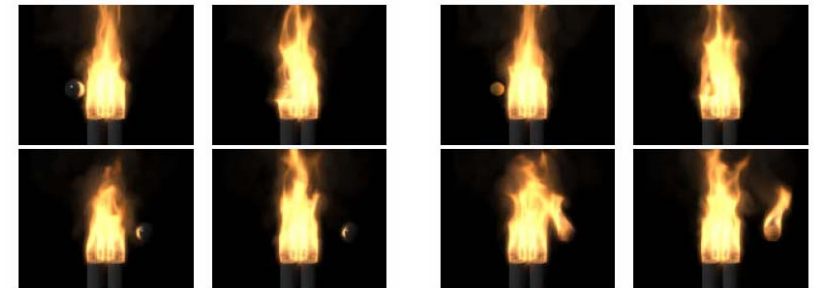


Figure 9: A metal ball passes through and interacts with a gas flame.

Figure 10: A flammable ball passes through a gas flame and catches on fire.