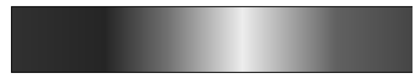
Rendering Concepts

Light
Radiance
Reflectance

Light

- The visible light is electro-magnetic radiation with wavelengths between 400 and 700 nanometers:



Optics

- Geometrical optics
- Physical optics
- Quantum mechanics

Radiant Energy

- Light can be thought of as flow of photons.
- The energy of each photon depends on its wavelength: $\frac{hc}{\lambda}$
- We are interested in the equilibrium of radiant energy in a 3D scene.

Flux and Radiance

- Flux Φ: the radiant energy crossing some surface per second [W].
- Flux density: flux per unit area [W/m^2]
- Radiance $L(x,\omega)$: the flux through x in direction ω, per unit area perpendicular to ω, per unit solid angle:

$$L(x,\omega) = \frac{d^2\Phi}{\cos\theta \, dx \, d\omega}$$

Solid Angles (1)

- When defining various radiometric quantities, we need to be able to quantify sets of directions.
- In 2D:
  - directions are points on the unit circle;
  - a simply connected set of directions is an arc, whose size corresponds to an angle [radians];
- In 3D:
  - directions are points on the unit sphere;
  - a simply connected set of directions is an area on the sphere, whose size corresponds to a ***solid angle***;
  - solid angles are measured in ***steradians*** [sr].

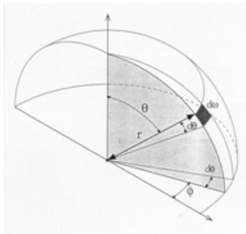
Solid Angles (2)

- Consider the spherical coordinates (r,θ,ϕ) of a point on the sphere. What is the area of a differential surface element at this point?

$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2\sin\theta\,d\theta\,d\phi$$

- Differential solid angle:

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$



Radiometric quantities

- Radiant energy [J]
- Radiant power (flux): radiant energy/sec [W]
- Irradiance (flux density): incident radiant power per unit area [W/m^2]
- Radiant intensity: radiant power per unit solid angle [W/sr]
- Radiance (angular flux density): radiant power per unit projected area per unit solid angle [W/(m^2 sr)]

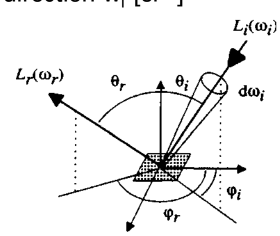
Photometric quantities

- Luminous energy [talbot]
- Luminous power [lumen = talbot/sec]
- Illuminance [Lux = lumen/m^2]
- Luminous intensity [Candela = lumen/sr]
- Luminance [Nit = lumen/(m^{2}sr)]

The BRDF

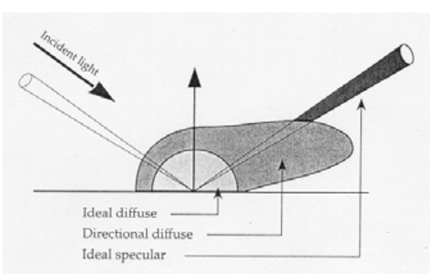
- **B**idirectional **R**eflection **D**istribution **F**unction
- The BRDF is a function describing the ratio of radiance leaving a point in direction w_r to the differential irradiance from direction w_i [sr^{-1}]

$$f_r(\omega_i \to \omega_r) \equiv \frac{L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\,d\omega_i}$$



The BRDF

- The BRDF is commonly separated into three components:
  - Ideal diffuse
  - Directional diffuse
  - Ideal specular



Reflectance Equations (1)

- The radiance emitted by a surface point in a particular direction w_r is obtained by an integral over all incident directions, weighted by the BRDF:

$$L_r(x,\omega_r) = \int_{\omega_i \in \Omega_i} f_r(x,\omega_i \to \omega_r)\,L(x,\omega_i)\,\cos\theta_i\,d\omega_i$$

Example: diffuse reflection

- Ideal diffuse (Lambertian) reflectance means that incident light is scattered uniformly in all outgoing directions.
- In this case, the BRDF is a constant:

$$f_{r,\text{diffuse}}(\omega_i \to \omega_r) = \rho$$

Example: mirror reflection

- Angle of reflectance is equal to angle of incidence
- The reflected vector is coplanar with the incident vector and the surface normal.
- For a perfect mirror, the reflected radiance is exactly equal to the incident radiance:

$$\begin{aligned}\theta_r &= \theta_i \\ \phi_r &= \phi_i \pm \pi \\ L_r(\theta_r, \phi_r) &= L_i(\theta_i, \phi_i \pm \pi)\end{aligned}$$

- The above properties are obtained with a BRDF involving Dirac's delta functions:

$$f_{r,\text{mirror}} = 2\,\delta(\sin^2\theta_i - \sin^2\theta_r)\,\delta(\phi_i - (\phi_r \pm \pi))$$

Reflectance Equations (2)

- Illumination by a finite-area light source S:

$$L_r(x,\omega_r) = \int_{\omega_i \in \Omega(S)} f_r(x, \omega_i \to \omega_r)\, L(x,\omega_i)\, \cos\theta_i\, d\omega_i$$

- Replace integral over incoming directions by an integral over the area of the light source:

$$\begin{aligned}L_r(x,\omega_r) &= \int_{y \in S} f_r(x, \omega_{xy} \to \omega_r)\, L(y,\omega_{yx})\, \cos\theta_x\, V(x,y)\, d\omega(y) \\ &= \int_{y \in S} f_r(x, \omega_{xy} \to \omega_r)\, L(y,\omega_{yx})\, \frac{\cos\theta_x \cos\theta_y}{\|x-y\|^2} V(x,y)\, dy\end{aligned}$$

The Rendering Equation

- Outgoing radiance = emitted radiance + reflected radiance:

$$\begin{aligned}L_r(x,\omega_r) &= L_e(x,\omega_r) + \\ &\quad \int_{y \in \Gamma} f_r(x, \omega_{xy} \to \omega_r)\, L(y,\omega_{yx})\, G(x,y)\, dy\end{aligned}$$

- where

$$G(x,y) = \frac{\cos\theta_x \cos\theta_y}{\|x-y\|^2} V(x,y)$$

A Formal Solution

- Define a reflection operator *R*:

$$(\mathcal{R}L)(x,\omega) = \int_{\omega_i \in \Omega} f_r(x, \omega_i \to \omega)\, L_i(x,\omega_i)\, \cos\theta_i\, d\omega_i$$

- The rendering equation becomes: $L = L_e + \mathcal{R}L$
- And the solution is:

$$\begin{aligned}L &= (1 - \mathcal{R})^{-1} L_e \\ &= \sum_{n=0}^{\infty} \mathcal{R}^n L_e\end{aligned}$$