

Some Vector Derivatives

Let \mathbf{x} , \mathbf{u} be vectors of length N and let \mathbf{A} be a matrix of size $N \times N$

$$\frac{\partial}{\partial \mathbf{x}}(\mathbf{u}^T \mathbf{x}) = \frac{\partial}{\partial \mathbf{x}}(\mathbf{x}^T \mathbf{u}) = \mathbf{u}^T \quad (1)$$

$$\frac{\partial}{\partial \mathbf{x}}(\mathbf{A}\mathbf{x}) = \mathbf{A} \quad (2)$$

$$\frac{\partial}{\partial \mathbf{x}}(\mathbf{x}^T \mathbf{A}\mathbf{x}) = \mathbf{x}^T(\mathbf{A} + \mathbf{A}^T) \quad (3)$$

$$\frac{\partial^2}{\partial \mathbf{x}^2}(\mathbf{x}^T \mathbf{A}\mathbf{x}) = \mathbf{A} + \mathbf{A}^T \quad (4)$$

Note that if \mathbf{A} is a symmetric matrix $\mathbf{A} + \mathbf{A}^T = 2\mathbf{A}$.