Gradient Domain High Dynamic Range Compression

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In A Nutshell...
Dynamic Range

- Quantity defined as the ratio between highest and lowest value a signal can attain:
Dynamic Range of Light

dim interior

daylit interior, x1500

outdoor shade, x16

outdoor sunlit, x4

the Sun

$1 : 7,500,000,000$
High Dynamic Range Scenes

1/1000  1/500

1/250  1/60

1/30  1/15  1/8  1/4
High Dynamic Range Images
High Dynamic Range Images

high

low
High Dynamic Range Images
High Dynamic Range Images

high

low
High Dynamic Range Images

- Large ratio between brightest and darkest intensities in the image.
- Small magnitude, local variations in intensity are present across the entire range:
Where do they come from?

• Physically-based illumination simulations.

• Digital photography:
  • Camera RAW images (12 bits per pixel).
  • HDR digital cameras.
  • Combining several differently exposed LDR images into a single HDR image (Debevec & Malik 1997).
  • HDR panoramic video mosaics (Schechner & Nayar 2001).
Our Goal

• Compression of dynamic range to enable rendering HDR images on LDR devices.

• Desirable features:
  • Avoid large over/under exposed regions.
  • Preserve visibility of fine details (local contrasts).
  • Avoid introducing artifacts to the image.
Previous Solutions

• Manually combining several exposures into a single image using an interactive tool, such as Adobe Photoshop:
Previous Solutions

- Spatially invariant tone mapping operators
  - Image-independent curves:
    - Linear scaling, Gamma correction, logarithmic mappings...
  - Image-dependent curves:
    - Histogram equalization
    - Visibility matching tone reproduction (Ward et al. 97)
Linear Scaling
Gamma Correction
Exposure + Gamma
Histogram Equalization
Previous Solutions

- Spatially invariant tone mapping operators
  - Image-independent curves:
    - Linear scaling, Gamma correction, logarithmic mappings...
  - Image-dependent curves:
    - Histogram equalization
    - Visibility matching tone reproduction (Ward et al. 97)
- Problem: monotonic mapping leads to loss of local contrast!
Previous Solutions

• Spatially variant tone mapping operators
  • Homomorphic filtering (Stockham 72, Horn 74)
  • Retinex-based operators (Jobson et al. 97)
  • Adaptive histogram equalization (Pizer et al. 87)
  • Multi-scale operators (Pattanaik et al. 98)

• Problem: “halo” artifacts
Example: multi-scale operator
Local Adaptation
Our Approach – Overview

• Observations:
  • High dynamic range results from strong luminance changes
  • Absolute change magnitude is not important

• Method:
  • Examine gradients to identify luminance changes
  • Attenuate high luminance gradients
  • Reconstruct a low-dynamic range image
The Method in 1D

2500:1

log

derivative

7.5:1

exp

integrate

attenuate

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The Method in 2D

• Given: a log-luminance image $H(x, y)$

• Compute an *attenuation map* $\Phi(\|\nabla H\|)$

• Compute an attenuated gradient field $G$:

$$G(x, y) = \nabla H(x, y) \cdot \Phi(\|\nabla H\|)$$

• Problem: $G$ is not integrable!
Aside: Conservative Fields

- In vector calculus an *irrotational* or *conservative* vector field is a vector field whose curl is zero:
  \[ \text{curl} \mathbf{v} = \nabla \times \mathbf{v} = 0 \]

- Any gradient field is conservative.

- Any conservative field \( \mathbf{v} \) may be expressed as a gradient of a scalar potential:
  \[ \mathbf{v} = \nabla \phi \]
Solution

• Look for image $I$ with gradient closest to $G$ in the least squares sense.

• Two possible, but equivalent, formulations:
  
  • Continuous formulation via the Euler-Lagrange equation
  
  • Discrete formulation
Continuous Formulation

• Look for a 2D function $I$ whose gradient is the closest to $G$ in the least squares sense.

• Image $I$ minimizes the integral:

$$\int \int F(\nabla I, G) \, dx \, dy$$

• where:

$$F(\nabla I, G) = \| \nabla I - G \|^2 = \left( \frac{\partial I}{\partial x} - G_x \right)^2 + \left( \frac{\partial I}{\partial y} - G_y \right)^2$$
Euler - Lagrange Equation

I must satisfy:
\[
\frac{\partial F}{\partial I} - \frac{d}{dx} \frac{\partial F}{\partial I_x} - \frac{d}{dy} \frac{\partial F}{\partial I_y} = 0
\]

Substitute \( F \):
\[
F(\nabla I, G) = \left( \frac{\partial I}{\partial x} - G_x \right)^2 + \left( \frac{\partial I}{\partial y} - G_y \right)^2
\]

\[
\frac{\partial F}{\partial I} = 0
\]
\[
\frac{\partial F}{\partial I_x} = 2 \left( \frac{\partial I}{\partial x} - G_x \right)
\]
\[
\frac{d}{dx} \frac{\partial F}{\partial I_x} = 2 \left( \frac{\partial^2 I}{\partial x^2} - \frac{\partial G_x}{\partial x} \right)
\]
Euler - Lagrange Equation

- $I$ must satisfy:
  \[ \frac{\partial F}{\partial I} - \frac{d}{dx} \frac{\partial F}{\partial I_x} - \frac{d}{dy} \frac{\partial F}{\partial I_y} = 0 \]

\[ 2 \left( \frac{\partial^2 I}{\partial x^2} - \frac{\partial G_x}{\partial x} \right) + 2 \left( \frac{\partial^2 I}{\partial y^2} - \frac{\partial G_y}{\partial y} \right) = 0 \]

\[ \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} \]

The Poisson equation: \[ \nabla^2 I = \text{div } G \]
Discrete Formulation

• Look for an image $I^*$ whose gradient (at each pixel) is the closest to $G$ in the least squares sense:

$$J(I) = \sum_{x,y} \| \nabla I(x,y) - G(x,y) \|^2$$

$$I^* = \arg\min_I J(I)$$

• Use finite differences for approximating the derivatives of $I$:

$$\nabla I(x,y) = \begin{bmatrix} I(x+1,y) - I(x,y) \\ I(x,y+1) - I(x,y) \end{bmatrix}$$
Discrete Formulation

• $J$ is a quadratic functional, with a unique minimum, attained at: $\nabla J = 0$

$$\nabla J = \begin{bmatrix}
\frac{d}{dI(x,y)} \sum_{x,y} (I_x(x,y) - G_x(x,y))^2 + (I_y(x,y) - G_y(x,y))^2
\end{bmatrix} = 0$$

The Poisson equation: $\nabla^2 I = \text{div} \ G$
Gradient Attenuation

• Strong luminance changes may occur at different rates:

• Must examine gradients at multiple scales!
Multiscale Gradient Attenuation

log(Luminance)  Gradient magnitude  Attenuation map
Multiscale Gradient Attenuation

Interpolate

\[ x = \]

Interpolate

\[ x = \]
Final Gradient Attenuation Map
Performance

• Measured on 1.8 GHz Pentium 4:
  • 512 x 384: 1.1 sec
  • 1024 x 768: 4.5 sec

• Can be accelerated using processor-optimized libraries.
Results (Ward et al. 1997)
Results (LCIS)
Results (our method)
Local Adaptation
Image Enhancement
Image Enhancement
Summary

• New method for detail-preserving compression of dynamic range.
• Also useful for enhancing ordinary images.

• Future work:
  • Better handling of color
  • Incorporate psychophysical properties of the HVS
  • Explore other applications of gradient field manipulations.
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